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Applications of three methods for obtaining optical soliton solutions for the Lakshmanan–Porsezian–Daniel model with Kerr law nonlinearity

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Abstract. This paper examines new travelling wave solutions to the Lakshmanan–Porsezian–Daniel (LPD) model with Kerr nonlinearity using Bäcklund transformation method based on Riccati equation, Kudryashov method and a new auxiliary ordinary differential equation (ODE). The three methods are adequately utilised, and some new rational-type hyperbolic and trigonometric function solutions are derived in different shapes for the aforementioned model. We confirm that our methods are more efficient than the other methods and it might be used in many other such types of nonlinear equations arising in the basic fabric of communications network technology and nonlinear optics.

Keywords. Bäcklund transformation method; Kudryashov method; new auxiliary ordinary differential equation; Lakshmanan–Porsezian–Daniel equation; Kerr nonlinearity; exact solutions.

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1. Introduction

For the last few decades, solitons and other solutions of nonlinear partial differential equations (NPDEs) are some of the main focal points in the field of mathematical physics, optical fibres, nonlinear optics, plasma physics and engineering. In this context, optical solitons have created a revolutionary effect in the electronic communication system and social media, and other such types of communications [1–26]. Moreover, it is essential to identify the sustainable significance of soliton theory in the sense of communications network technology and optical fibre. Soliton is a self-reinforcing single wave, which moves at a constant velocity while maintaining its shape. It may explain the solutions of nonlinear dispersive wave equations, which are associated with physical systems exhibiting nonlinear and dispersive effects in the medium. Precisely, solitons play a central role in many physical systems, and they exhibit numerous forms such as kink, singular, combined singular, combo-singular and kink, topological, non-topological, combo topological and non-topological, breather, cusp, rouge, combined and multiple solitons, and many others. However, many researchers have already proved extensively that solitons are interesting entities because of their localised and stable nature in the applied fields like basic fabric of communications network technology, electronic engineering, plasma physics, fluid mechanics, ocean engineering, signal processing and so on. Thus, investigation of soliton solutions of the NPDEs and the determination of their actual physical properties are tough research topics before the invention of

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computer-based symbolic softwares like Maple, Mathematica, MATLAB, etc. But, nowadays researchers can efficiently plan and systematically execute their ideas to look for solitons and their actual characteristics for any NPDEs with integer or fractional order.

This paper studies a well-known model which is the Kerr nonlinearity included Lakshmanan–Porsezian–Daniel (LPD) equation. A couple of decades ago, the model was first described in the context of Heisenberg spin chain equation [27,28]. Later on, this model was widely studied in the context of fibre optics. In this regard, we consider the LPD model with Kerr law nonlinearity in the following form [1–7]:

$$\begin{aligned} & iq_t + aq_{xx} + bq_{xt} + c|q|^2q \\ & \quad = \rho q_{xxxx} + \alpha(q_x)^2q^* + \beta|q_x|^2q \\ & \qquad +\gamma|q|^2q_{xx} + \lambda q^2q^*_{xx} + \delta|q|^4q. \end{aligned} \tag{1}$$

In eq. (1), the dependent complex-valued function $q(x,t)$ consists of two independent variables x and t that represent space and time, respectively. The first term on the left side of eq. (1) represents the temporal evolution of the optical pulse, while the coefficients a and b are the group velocity dispersion (GVD) and spatio-temporal dispersion (STD), respectively. The terms proportional to c in eq. (1) on its left-hand side is the nonlinear term. On the right-hand side of eq. (1), ρ is the fourth-order dispersion and δ is the two-photon absorption. The coefficients α, β, γ and λ indicate the dispersion of other perturbation terms with nonlinear forms.

In the past, a few researchers [1–6] have studied the LPD model (1) with three forms of nonlinearity, namely Kerr, parabolic and power laws of nonlinearity. In this paper, we shall discuss the LPD model and utilise various analytical methods of interest. For instance, the method of undetermined coefficients is applied by Guzman *et al* [1] to look for bright, dark and singular soliton solutions of the LPD model. On the other hand, bright solitons, dark solitons, periodic solitary wave, rational function and elliptic function solutions have been derived for the model (1) with Kerr and power laws of nonlinearity by Manafian *et al* [2] with the aid of extended trial equation method. Alqahtani *et al* [3] secured the bright soliton to the LPD equation with Kerr and power laws of nonlinearity via the semi-inverse variational principle. Dark and singular optical solitons have been retrieved for the LPD Kerr law nonlinearity equation by using two integration schemes, namely the extended Jacobi elliptic function approach and $\exp(-\phi(\xi))$-expansion method which was utilised by Biswas *et al* [4]. Biswas *et al* [5] have again performed the modified simple equation method of the LPD model and extracted dark and singular soliton solutions. Very recently, Bansal *et al* [6] adopted the Lie symmetry analysis for acquiring the optical and singular solitons from Kerr and power law nonlinearity based LPD model.

In addition to the aforementioned methods, many more powerful analytical methods have been built up and executed for generating new exact solutions of NPDEs such as sub-equation method [29], first integral method [30–33], functional variable method [32,33], Riccati sub-equation method [34], Kudryashov method [35], trial equation method [36–38], sine–cosine method [37], (G'/G)-expansion method [37], extended trial equation method [38], modified Kudryashov method [39–42], sine-Gordon equation expansion method [41–43], extended sinh-Gordon equation expansion method [42,44,45], Bäcklund transformation method [46], and so on.

The main aim of this study is to explore travelling wave solutions of the LPD model with Kerr law nonlinearity by using three efficient distinct methods such as Bäcklund transformation method based on Riccati equation [46], Kudryashov method [39] and a new auxiliary ordinary differential equation (ODE) method [47].

2. Mathematical analysis

In order to study eq. (1), the wave profile is split into amplitude and phase components as

$$q(x,t) = U(\eta)e^{i\phi(x,t)}, \tag{2}$$

where the wave variable η is given by

$$\eta = x - vt. \tag{3}$$

Here, $P(\eta)$ is the amplitude component of the wave profile and v is the speed of the soliton, while $\phi(x,t)$ is the phase component of the profile, where

$$\phi(x,t) = -kx + \omega t + \theta_0. \tag{4}$$

Here k is the frequency of the soliton, ω is the wave number of the soliton and θ_0 is the extra phase function depending upon the travelling variable η.

Substituting eq. (2) into eq. (1), the real and imaginary parts of the resulting equation respectively read as

$$\begin{aligned} & \rho U_{\eta\eta\eta\eta} - (a - bv + 6\rho\kappa^2)U_{\eta\eta} \\ & \quad - (b\kappa\omega - \omega - a\kappa^2 - \rho\kappa^4)U \\ & \quad -(\alpha + \gamma + \lambda - \beta)\kappa^2U^3 + \delta U^5 - cU^3 \\ & \quad +(\alpha + \beta)UU_\eta^2 + (\lambda + \gamma)U^2U_{\eta\eta} = 0 \end{aligned} \tag{5}$$

and

$$(b\kappa v - v - 2a\kappa - 4\rho\kappa^3 + b\omega)U_\eta$$
$$+2(\alpha+\gamma-\lambda)\kappa U^2 U_\eta + 4\rho\kappa U_{\eta\eta\eta} = 0. \quad (6)$$

From (5) and (6), setting the factors of the linearly independent functions to zero gives

$$\lambda = \alpha + \gamma, \quad (7)$$

$$\alpha = -\beta, \quad (8)$$

$$\rho = 0, \quad (9)$$

$$\lambda = -\gamma \quad (10)$$

and

$$v = \frac{b\omega - 2a\kappa}{1 - b\kappa}, \qquad b\kappa \neq 1. \quad (11)$$

On substituting eqs (7)–(9) along with eq. (10) in eq. (5), one obtains

$$(a - bv)U_{\eta\eta} + (b\kappa\omega - \omega - a\kappa^2)U$$
$$+(c - 4\gamma\kappa^2)U^3 - \delta U^5 = 0. \quad (12)$$

In order to obtain closed form solutions, we use the transformation

$$U = V^{1/2}, \quad (13)$$

that will reduce eq. (12) into the ODE

$$(a - bv)(-(V_\eta)^2 + 2VV_{\eta\eta}) + 4(b\kappa\omega - \omega - a\kappa^2)V^2$$
$$+4(c - 4\gamma\kappa^2)V^3 - 4\delta V^4 = 0. \quad (14)$$

By balancing between $VV_{\eta\eta}$ and V^4, we obtain $N = 1$.

3. Soliton solutions for the LPD model with Kerr law nonlinearity

3.1 *Bäcklund transformation method of Riccati equation*

The purpose of this subsection is to present the algorithm of the Bäcklund transformation method based on Riccati equation [46], and to find exact solutions of the LPD model with Kerr law nonlinearity.

Now we suppose that eq. (14) has the following solution:

$$V(\eta) = A_0 + A_1\psi(\eta), \quad (15)$$

where

$$\psi(\eta) = \frac{-\sigma B + D\varphi(\eta)}{D + B\varphi(\eta)}, \quad (16)$$

$$\phi'(\eta) = \sigma + \phi^2(\eta). \quad (17)$$

Equation (17) gives the following solutions:

$$\varphi(\eta) = \begin{cases} -\sqrt{-\sigma}\ \tanh(\sqrt{-\sigma}\eta), & \sigma < 0, \\ -\sqrt{-\sigma}\ \coth(\sqrt{-\sigma}\eta), & \sigma < 0, \\ -\frac{1}{(\eta+\bar{\omega})}, \quad \bar{\omega} = \text{const.} & \sigma = 0, \\ \sqrt{\sigma}\ \tan(\sqrt{\sigma}\eta), & \sigma > 0, \\ -\sqrt{\sigma}\ \cot(\sqrt{\sigma}\eta), & \sigma > 0. \end{cases} \quad (18)$$

Case 1.

$$A_1 = \pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}, \qquad A_0 = \pm\frac{1}{2}\sqrt{-\frac{3L\sigma}{\delta}},$$
$$\omega = \frac{L\sigma + a\kappa^2}{b\kappa - 1}, \qquad c = 4\gamma\kappa^2 \pm \frac{4}{3}\sqrt{-3\delta L\sigma},$$

where

$$L = (a - bv).$$

In view of Case (1), we obtain new types of exact solutions of eq. (1) as follows:

(I) When $\sigma < 0$ then,

$$q_1(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}\left[\sqrt{-\sigma} + \frac{-\sigma B - D\sqrt{-\sigma}\ \tanh\left(\sqrt{-\sigma}(x - vt)\right)}{D - B\sqrt{-\sigma}\ \tanh\left(\sqrt{-\sigma}(x - vt)\right)}\right]\right)^{1/2}$$
$$\times \exp\left[i\left(-\kappa x + \left(\frac{L\sigma + a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_2(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}\left[\sqrt{-\sigma} + \frac{-\sigma B - D\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}(x - vt)\right)}{D - B\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}(x - vt)\right)}\right]\right)^{1/2}$$
$$\times \exp\left[i\left(-\kappa x + \left(\frac{L\sigma + a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right].$$

(II) When $\sigma = 0$ then,

$$q_3(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}\left[\sqrt{-\sigma} + \frac{-\sigma B - \frac{D}{(x-vt)+\bar{\omega}}}{D - \frac{B}{(x-vt)+\bar{\omega}}}\right]\right)^{1/2}$$
$$\times \exp\left[i\left(-\kappa x + \left(\frac{L\sigma + a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

(III) When $\sigma > 0$ then,

$$q_4(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}\left[\sqrt{-\sigma} + \frac{-\sigma B + D\sqrt{\sigma}\tan\left(\sqrt{\sigma}(x-vt)\right)}{D + B\sqrt{\sigma}\tan\left(\sqrt{\sigma}(x-vt)\right)}\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x + \left(\frac{L\sigma + a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

and

$$q_5(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}\left[\sqrt{-\sigma} + \frac{-\sigma B - D\sqrt{\sigma}\cot\left(\sqrt{\sigma}(x-vt)\right)}{D - B\sqrt{\sigma}\cot\left(\sqrt{\sigma}(x-vt)\right)}\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x + \left(\frac{L\sigma + a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right].$$

3.2 *Kudryashov method*

Assume the formal solution of Kudryashov method (eq. (14)) as

$$V(\eta) = a_0 + a_1 Q(\eta), \tag{19}$$

where $Q(\eta)$ satisfies the following auxiliary equation:

$$Q'(\eta) = Q^2(\eta) - Q(\eta). \tag{20}$$

Equation (20) gives the following solutions:

$$Q(\eta) = \frac{1}{1 + K\mathrm{e}^{\eta}}. \tag{21}$$

Riccati equation (20) also admits the following exact solutions:

$$Q_1(\eta) = \frac{1}{2}\left(1 - \tanh\left[\frac{\eta}{2} - \frac{\varepsilon \ln \eta_0}{2}\right]\right), \qquad \eta_0 > 0,$$

$$Q_2(\eta) = \frac{1}{2}\left(1 - \coth\left[\frac{\eta}{2} - \frac{\varepsilon \ln \eta_0}{2}\right]\right), \qquad \eta_0 < 0, \tag{22}$$

Case 1.

$$a_1 = \pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}, \quad a_0 = \pm\frac{1}{2}\sqrt{\frac{3L}{\delta}},$$

$$\omega = -\frac{1}{4}\frac{L - 4a\kappa^2}{b\kappa - 1}, \quad c = 4\gamma\kappa^2 \pm \frac{2}{3}\sqrt{3L\delta},$$

Case 2.

$$a_1 = \pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}, \quad a_0 = 0,$$

$$\omega = -\frac{1}{4}\frac{L - 4a\kappa^2}{b\kappa - 1}, \quad c = 4\gamma\kappa^2 \pm \frac{2}{3}\sqrt{3L\delta},$$

where

$$L = (a - bv).$$

In view of Case (1), we obtain new types of exact solutions of eq. (1) as follows:

$$q_6(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}\left[1 + \frac{1}{1 + K\mathrm{e}^{(x-vt)}}\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{L - 4a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_7(x,t) = \left(\mp\frac{1}{4}\sqrt{\frac{3L}{\delta}}\left[2 + \tanh\left(\frac{(x-vt)}{2} - \frac{\varepsilon \ln \eta_0}{2}\right)\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{L - 4a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right]$$

and

$$q_8(x,t) = \left(\mp\frac{1}{4}\sqrt{\frac{3L}{\delta}}\left[2 + \coth\left(\frac{(x-vt)}{2} - \frac{\varepsilon \ln \eta_0}{2}\right)\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{L - 4a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right].$$

According to Case (2), we obtain a new type of exact solutions of eq. (1) as follows:

$$q_9(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{3L}{\delta}}\frac{1}{1 + K\mathrm{e}^{(x-vt)}}\right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{L - 4a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_{10}(x,t) = \left(\pm\frac{1}{4}\sqrt{\frac{3L}{\delta}}\left(1 - \tanh\left[\frac{(x-vt)}{2} - \frac{\varepsilon \ln \eta_0}{2}\right]\right)\right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{L - 4a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right]$$

and

$q_{11}(x,t)$

$$= \left(\pm\frac{1}{4}\sqrt{\frac{3L}{\delta}}\left(1-\coth\left[\frac{(x-vt)}{2}-\frac{\varepsilon\ln\eta_0}{2}\right]\right)\right)^{1/2} \times \exp\left[i\left(-\kappa x-\left(\frac{1}{4}\frac{L-4a\kappa^2}{b\kappa-1}\right)t+\theta_0\right)\right].$$

3.3 *A new auxiliary ordinary differential equation*

The purpose of this subsection is to present the algorithm of a new auxiliary ordinary differential equation [47] to find exact solutions of the LPD model with Kerr law nonlinearity. As we know, $N=1$. Now we suppose that eq. (14) has a solution in the form

$$U(\eta)=n_0+n_1\Phi(\eta), \tag{23}$$

where n_0, n_1 are constants to be determined later and the new variable $\Phi(\eta)$ satisfies the following ODE:

$$\left(\Phi'(\eta)\right)^2=m_1\Phi^2(\eta)+m_2\Phi^4(\eta)+m_3\Phi^6(\eta). \tag{24}$$

Equation (23) gives the following solutions:

When $m_1>0$,

$$\Phi_1(\eta)=\left(\frac{-m_1m_2\,\mathrm{sech}^2(\sqrt{m_1}\eta)}{m_2^2-m_1m_3(1+\varepsilon\,\tanh(\sqrt{m_1}\eta))^2}\right)^{1/2}. \tag{25}$$

When $m_1>0$,

$$\Phi_2(\eta)=\left(\frac{m_1m_2\,\mathrm{csch}^2(\sqrt{m_1}\eta)}{m_2^2-m_1m_3(1+\varepsilon\coth(\sqrt{m_1}\eta))^2}\right)^{1/2}. \tag{26}$$

When $m_1>0$, $\Delta>0$,

$$\Phi_3(\eta)=\left(\frac{2m_1}{\varepsilon\sqrt{\Delta}\,\cosh(2\sqrt{m_1}\eta)-m_2}\right)^{1/2}. \tag{27}$$

When $m_1<0$, $\Delta>0$,

$$\Phi_4(\eta)=\left(\frac{2m_1}{\varepsilon\sqrt{\Delta}\,\cos(2\sqrt{-m_1}\eta)-m_2}\right)^{1/2}. \tag{28}$$

When $m_1>0$, $\Delta<0$,

$$\Phi_5(\eta)=\left(\frac{2m_1}{\varepsilon\sqrt{-\Delta}\,\sinh(2\sqrt{m_1}\eta)-m_2}\right)^{1/2}. \tag{29}$$

When $m_1<0$, $\Delta>0$,

$$\Phi_6(\eta)=\left(\frac{2m_1}{\varepsilon\sqrt{\Delta}\,\sin(2\sqrt{-m_1}\eta)-m_2}\right)^{1/2}. \tag{30}$$

When $m_1>0, m_3>0$,

$$\Phi_7(\eta)=\left(\frac{-m_1\,\mathrm{sech}^2(\sqrt{m_1}\eta)}{m_2-2\varepsilon\sqrt{m_1m_3}\,\tanh(\sqrt{m_1}\eta)}\right)^{1/2}. \tag{31}$$

When $m_1<0, m_3>0$,

$$\Phi_8(\eta)=\left(\frac{-m_1\,\sec^2(\sqrt{-m_1}\eta)}{m_2+2\varepsilon\sqrt{-m_1m_3}\,\tan(\sqrt{-m_1}\eta)}\right)^{1/2}. \tag{32}$$

When $m_1>0, m_3>0$,

$$\Phi_9(\eta)=\left(\frac{m_1\,\mathrm{csch}^2(\sqrt{m_1}\eta)}{m_2+2\varepsilon\sqrt{m_1m_3}\,\coth(\sqrt{m_1}\eta)}\right)^{1/2}. \tag{33}$$

When $m_1<0, m_3>0$,

$$\Phi_{10}(\eta)=\left(\frac{-m_1\,\csc(\sqrt{-m_1}\eta)}{m_2^2+2\varepsilon\sqrt{-m_1m_3}\,\tanh(\sqrt{-m_1}\eta)}\right)^{1/2}. \tag{34}$$

When $m_1>0$, $\Delta=0$,

$$\Phi_{11}(\eta)=\left(-\frac{m_1}{m_2}\left(1+\varepsilon\,\tanh\left(\frac{\sqrt{m_1}}{2}\eta\right)\right)\right)^{1/2}. \tag{35}$$

When $m_1>0$, $\Delta=0$,

$$\Phi_{12}(\eta)=\left(-\frac{m_1}{m_2}\left(1+\varepsilon\,\coth\left(\frac{\sqrt{m_1}}{2}\eta\right)\right)\right)^{1/2}. \tag{36}$$

When $m_1>0$,

$$\Phi_{13}(\eta)=4\left(\frac{m_1\mathrm{e}^{2\varepsilon\sqrt{m_1}\eta}}{(\mathrm{e}^{2\varepsilon\sqrt{m_1}\eta}-4m_2)^2-64m_1m_3}\right)^{1/2}. \tag{37}$$

When $m_1>0, m_2=0$,

$$\Phi_{14}(\eta)=4\left(\frac{\pm m_1\mathrm{e}^{2\varepsilon\sqrt{m_1}\eta}}{(1-64m_1m_3\mathrm{e}^{4\varepsilon\sqrt{m_1}\eta}}\right)^{1/2}, \tag{38}$$

where $\Delta=m_2^2-4m_1m_3$ and $\varepsilon=\pm1$.

Substituting eq. (25) into (24) and setting the coefficients of the power of $\Phi^i(\eta)$ to zero, we obtain an overdetermined nonlinear algebraic system in $n_0,\ n_1,\ c$ and ω. Solving the nonlinear algebraic system yields the following explicit expressions for the parameters:

Case 1.

$$n_1=\pm\frac{1}{2}\sqrt{\frac{3Lm_1}{\delta}},\quad n_0=\pm\frac{1}{2}\sqrt{-\frac{3Lm_1}{\delta}},$$

$$\omega=\frac{1}{4}\frac{4a\kappa^2+5Lm_1}{b\kappa-1},\quad c=4\gamma\kappa^2\pm\frac{4}{3}\sqrt{-3L\delta m_1},$$

Case 2.

$$n_1=\pm\frac{1}{2}\sqrt{\frac{3Lm_2}{\delta}},\quad n_0=0,$$

$$\omega = -\frac{1}{4}\frac{Lm_1 - 4a\kappa^2}{b\kappa - 1}, \quad c = 4\gamma\kappa^2,$$

where

$$L = (a - bv).$$

Now, we shall choose only Case (1) of the exact solution (1). In view of Case (1), we obtain new types of exact solutions of eq. (1) as follows:

When $m_1 > 0$,

$$q_{12}(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{-3Lm_1}{\delta}}\left[1 + \operatorname{sech}\left(\sqrt{m_1}(x - vt)\right)\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x + \left(\frac{1}{4}\frac{4a\kappa^2 + 5Lm_1}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_{13}(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{3Lm_1}{\delta}}\left[1 + \operatorname{csch}\left(\sqrt{m_1}(x - vt)\right)\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x + \left(\frac{1}{4}\frac{4a\kappa^2 + 5Lm_1}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_{14}(x,t) = \left(\pm\frac{1}{2}\frac{\sqrt{-3L\delta m_1}}{\delta} \times \left[1 + \sqrt{-\frac{2}{\varepsilon\cosh\left(2\sqrt{m_1}(x - vt)\right) - m_2}}\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x + \left(\frac{1}{4}\frac{4a\kappa^2 + 5Lm_1}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_{15}(x,t) = \left(\pm\frac{1}{2}\frac{\sqrt{-3L\delta m_1}}{\delta} \times \left[1 + \sqrt{-\frac{2}{\varepsilon i\sinh\left(2\sqrt{m_1}(x - vt)\right) - m_2}}\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x + \left(\frac{1}{4}\frac{4a\kappa^2 + 5Lm_1}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_{16}(x,t) = \left(\pm\frac{1}{2}\frac{\sqrt{-3L\delta m_1}}{\delta} \times \left[1 + \sqrt{-\frac{2m_2 e^{2\varepsilon\sqrt{m_1}(x-vt)}}{\left(e^{2\varepsilon\sqrt{m_1}(x-vt)} - 4m_2\right)^2}}\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x + \left(\frac{1}{4}\frac{4a\kappa^2 + 5Lm_1}{b\kappa - 1}\right)t + \theta_0\right)\right].$$

When $m_1 < 0$,

$$q_{17}(x,t) = \left(\pm\frac{1}{2}\frac{\sqrt{-3L\delta m_1}}{\delta} \times \left[1 + \sqrt{-\frac{2}{\varepsilon\cos\left(2\sqrt{-m_1}(x - vt)\right) - m_2}}\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x + \left(\frac{1}{4}\frac{4a\kappa^2 + 5Lm_1}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_{18}(x,t) = \left(\pm\frac{1}{2}\frac{\sqrt{-3L\delta m_1}}{\delta} \times \left[1 + \sqrt{-\frac{2}{\varepsilon\sin\left(2\sqrt{-m_1}(x - vt)\right) - m_2}}\right]\right)^{1/2} \times \exp\left[i\left(-\kappa x + \left(\frac{1}{4}\frac{4a\kappa^2 + 5Lm_1}{b\kappa - 1}\right)t + \theta_0\right)\right].$$

When $m_1 > 0$,

$$q_{19}(x,t) = \left(\pm\frac{1}{2}\sqrt{-\frac{3Lm_1}{\delta}}\operatorname{sech}\left(\sqrt{m_1}(x - vt)\right)\right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{Lm_1 - 4a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_{20}(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{3Lm_1}{\delta}}\operatorname{csch}\left(\sqrt{m_1}(x - vt)\right)\right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{Lm_1 - 4a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_{21}(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{6Lm_1}{\delta}}\sqrt{\frac{m_1}{\varepsilon\cosh\left(2\sqrt{m_1}(x - vt)\right) - m_2}}\right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{Lm_1 - 4a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

$$q_{22}(x,t) = \left(\pm\frac{1}{2}\sqrt{\frac{6Lm_1}{\delta}}\sqrt{\frac{m_1}{\varepsilon i\sinh\left(2\sqrt{m_1}(x - vt)\right) - m_2}}\right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{Lm_1 - 4a\kappa^2}{b\kappa - 1}\right)t + \theta_0\right)\right],$$

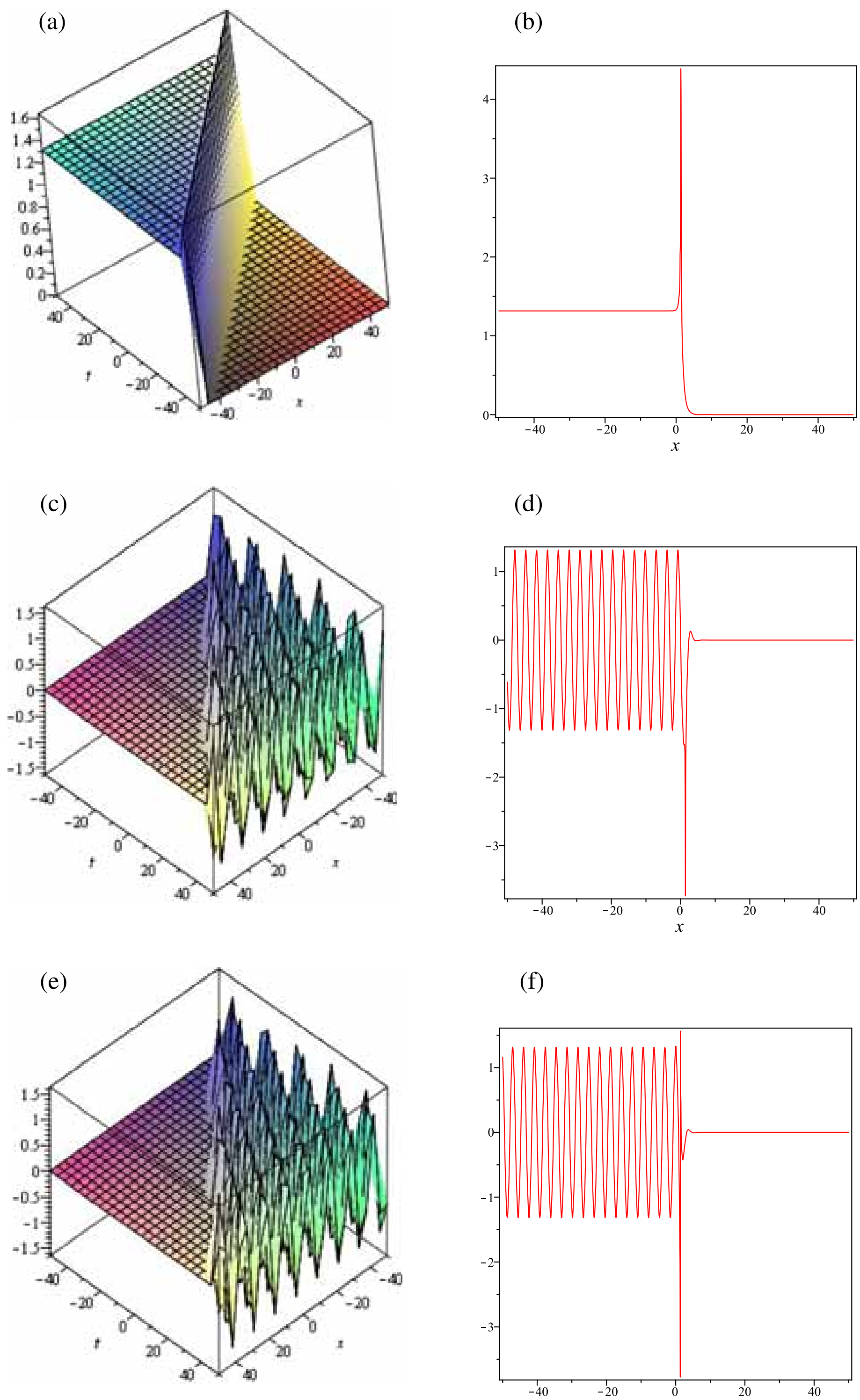

Figure 1. Graphs of (**a**) and (**b**) absolute values, (**c**) and (**d**) real values and (**e**) and (**f**) imaginary values of $q_1(x,t)$ are shown at $B = 3, D = 2, \sigma = -1, a = 3, b = 2, v = 1, \delta = -2, \gamma = 1, \theta_0 = 2$ and by considering the values $-50 < x < 50, -50 < t < 50$ for (**a**), (**c**), (**e**) and $50 < x < 50, t = 1$ for (**b**), (**d**), (**f**).

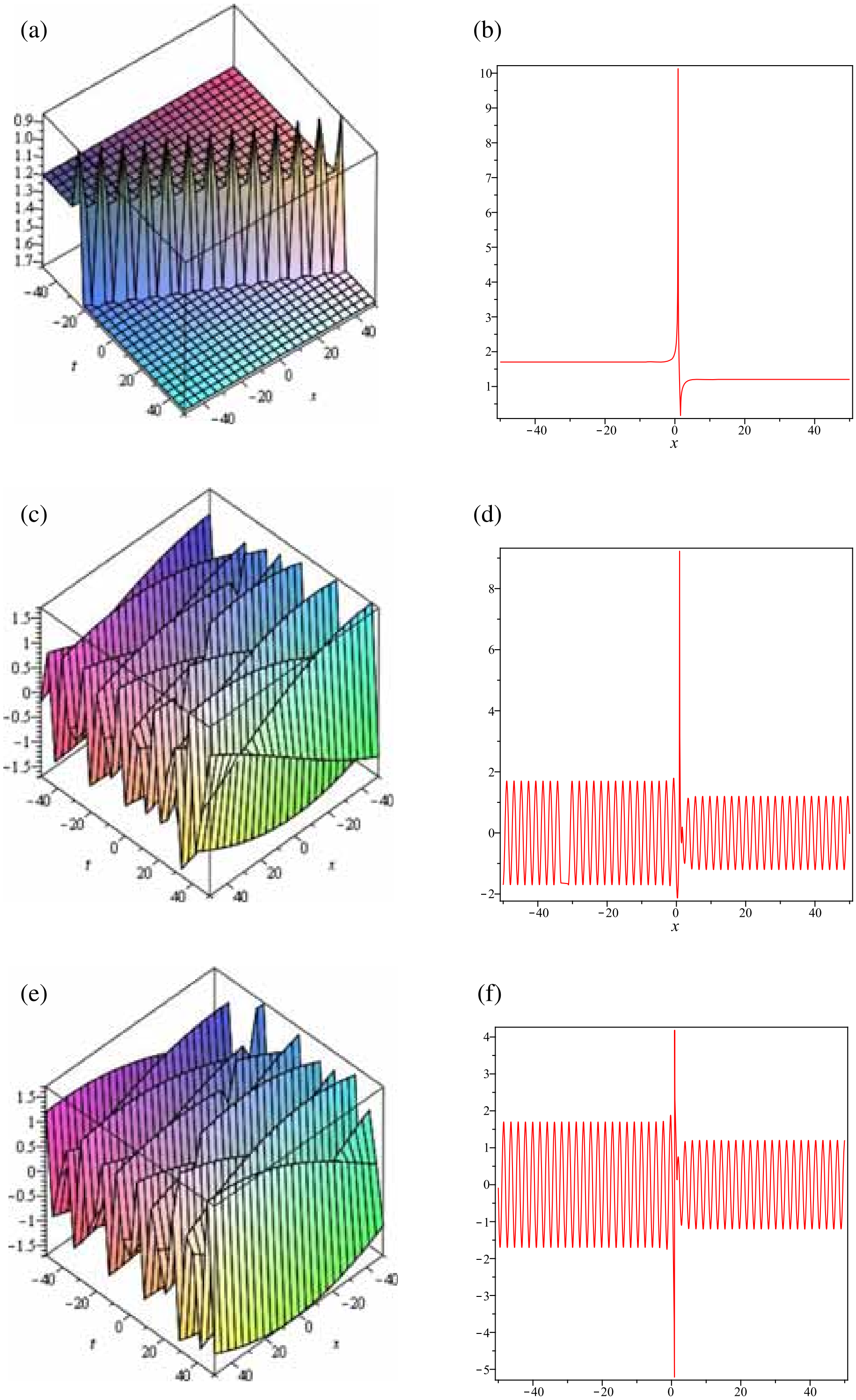

Figure 2. Graphs of (**a**) and (**b**) absolute values, (**c**) and (**d**) real values and (**e**) and (**f**) imaginary values of $q_6(x, t)$ are shown at $K = -3, a = 3, b = -2, v = 2, \delta = -2.5, \kappa = 3, \gamma = -2, \theta_0 = -2$ and by considering the values $-50 < x < 50$, $-50 < t < 50$ for (**a**), (**c**), (**e**) and $50 < x < 50$, $t = 1$ for (**b**), (**d**), (**f**).

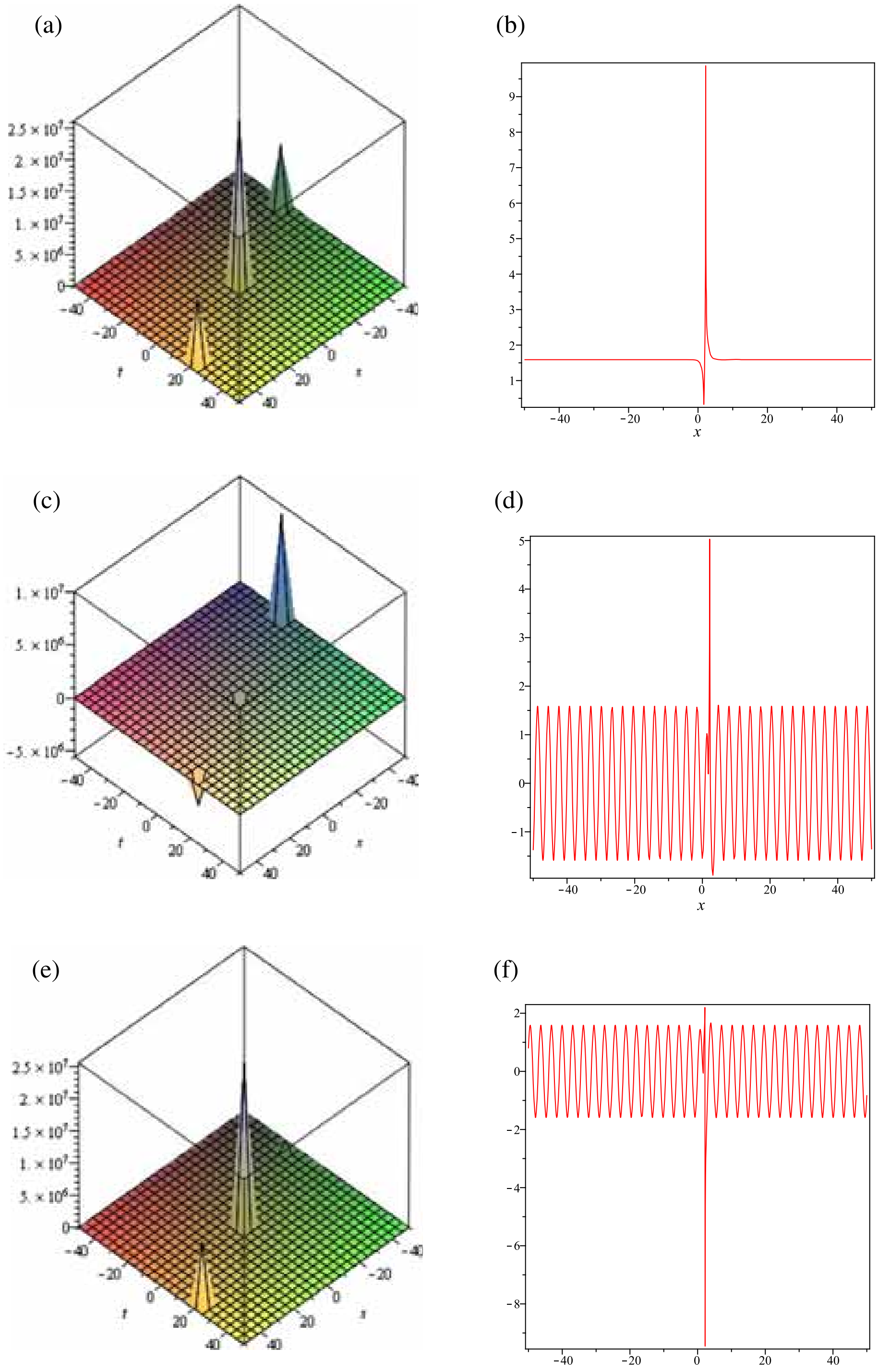

Figure 3. Graphs of (**a**) and (**b**) absolute values, (**c**) and (**d**) real values and (**e**) and (**f**) imaginary values of $q_{13}(x, t)$ are shown at $m-1 = 3$, $a = -1$, $b = -2$, $v = 2.2$, $\delta = -1.2$, $\gamma = 1$, $\theta_0 = 1$ and by considering the values $-50 < x < 50$, $-50 < t < 50$ for (**a**), (**c**), (**e**) and $50 < x < 50$, $t = 1$ for (**b**), (**d**), (**f**).

$q_{23}(x,t)$

$$= \left(\pm 2\sqrt{\frac{3Lm_1m_2}{\delta}} \sqrt{\frac{e^{2\sqrt{m_1}(x-vt)}}{\left(e^{2\sqrt{m_1}(x-vt)} - 4m_2\right)^2}} \right)^{1/2} \times \exp\left[i\left(-\kappa x - \left(\frac{1}{4}\frac{Lm_1 - 4a\kappa^2}{b\kappa - 1} \right) t + \theta_0 \right) \right].$$

In figures 1–3, we draw three-dimensional and two-dimensional graphics of absolute, imaginary and real values of $q_1(x,t)$, $q_6(x,t)$ and $q_{13}(x,t)$ respectively, which denote the dynamics of solutions with appropriate parametric selections. We draw three-dimensional graphs of figures 1–3 when $-50 < x < 50$, $-50 < t < 50$ and two-dimensional graphs of figures 1–3 when $-50 < x < 50$, $t = 1$. To the best of our knowledge, these optical soliton solutions have not been published in the literature. In figure 1, $B = 3, D = 2, \sigma = -1, a = 3, b = 2, v = 1, \delta = -2, \gamma = 1, \theta_0 = 2$, in figure 2, $q_6(x,t)$ with $K = -3, a = 3, b = -2, v = 2, \delta = -2.5, \kappa = 3, \gamma = -2, \theta_0 = -2$ and in figure 3, $q_{13}(x,t)$ with $m - 1 = 3, a = -1, b = -2, v = 2.2, \delta = -1.2, \gamma = 1, \theta_0 = 1$ represent the exact travelling wave solutions of the the LPD model with Kerr law nonlinearity.

4. Conclusions

This paper reports some new travelling wave solutions to the LPD model with Kerr nonlinearity by adopting three methods, namely Bäcklund transformation method, Kudryashov method and a new auxiliary ODE. By choosing appropriate parameters from the obtained solutions, the travelling wave profile seems to soliton profile method. These solitons appear with the corresponding integrability conditions that are also known as constraint conditions which are necessary for these solitons to exist. The results of this paper confirm that the aforementioned methods are powerful algorithms for the analytic treatment of a wide range of nonlinear systems of partial differential equations which arise in optical fibres.

References

[1] J V Guzman, R T Alqahtani, Q Zhou, M F Mahmood, S P Moshokoa, M Z Ullah, A Biswas and M Belic, *Optik* **144**, 115 (2017)
[2] J Manafian, M Foroutan and A Guzali, *Eur. Phys. J. Plus* **132**(**11**), 494 (2017)
[3] R T Alqahtani, M M Babatin and A Biswas, *Optik* **154**, 109 (2018)
[4] A Biswas, M Ekici, A Sonmezoglu, H Triki, F B Majid, Q Zhou, S P Moshokoa, M Mirzazadeh and M Belic, *Optik* **158**, 705 (2018)
[5] A Biswas, Y Yildirim, E Yasar, Q Zhou, S P Moshokoa and M Belic, *Optik* **160**, 24 (2018)
[6] A Bansal, A Biswas, H Triki, Q Zhou, S P Moshokoa and M Belic, *Optik* **160**, 86 (2018)
[7] H Rezazadeh, M Mirzazadeh, S M Mirhosseini-Alizamini, A Neirameh, M Eslami and Q Zhou, *Optik* **164**, 414 (2018)
[8] H Rezazadeh, *Optik* **167**, 218 (2018)
[9] A Biswas, H Rezazadeh, M Mirzazadeh, M Eslami, M Ekici, Q Zhou and M Belic, *Optik* **165**, 288 (2018)
[10] A Biswas, M O Al-Amr, H Rezazadeh, M Mirzazadeh, M Eslami, Q Zhou, S P Moshokoa and M Belic, *Optik* **165**, 233 (2018)
[11] C Yang, W Li, W Yu, M Liu, Y Zhang G Ma and W Liu, *Nonlinear Dynam.* **92**(**2**), 203 (2018)
[12] W Li, G Ma, W Yu, Y Zhang, M Liu, C Yang and W Liu, *Chin. Phys. B* **27**(**3**), 030504 (2018)
[13] M Liu, W Liu, L Pang, H Teng, S Fang and Z Wei, *Opt. Commun.* **406**, 72 (2018)
[14] W Liu, M Liu, Y OuYang, H Hou, G Ma, M Lei and Z Wei, *Nanotechnology* **29**(**17**), 174002 (2018)
[15] Q Zhou, *Proc. Rom. Acad. Ser. A* **18**, 223 (2017)
[16] Q Zhou and A Biswas, *Superlattice Microst.* **109**, 588 (2017)
[17] Q Zhou, A Sonmezoglu, M Ekici and M Mirzazadeh, *J. Mod. Opt.* **64**(**21**), 2345 (2017)
[18] Q Zhou, M Ekici, M Mirzazadeh and A Sonmezoglu, *J. Mod. Opt.* **64**(**16**), 1677 (2017)
[19] A Sonmezoglu, M Yao, M Ekici, M Mirzazadeh and Q Zhou, *Nonlinear Dynam.* **88**(**1**), 595 (2017)
[20] H Triki, R T Alqahtani, Q Zhou and A Biswas, *Superlattice Microst.* **111**, 326 (2017)
[21] A Messouber, H Triki, F Azzouzi, Q Zhou, A Biswas, S P Moshokoa and M Belic, *Opt. Commun.* **425**, 64 (2018)
[22] A Biswas, Y Yildirim, E Yasar, Q Zhou, S P Moshokoa and M Belic, *Optik* **16**, 170 (2018)
[23] A R Seadawy, D Lu and M Iqbal, *Pramana – J. Phys.* **93**: 10 (2019)
[24] M S Osman, *Pramana – J. Phys.* **93**: 26 (2019)
[25] H F Ahmed, M S M Bahgat and M Zaki, *Pramana – J. Phys.* **92**: 38 (2019)
[26] S Banerjee and B Ghosh, *Pramana – J. Phys.* **90**: 42 (2018)
[27] M Lakshmanan, K Porsezian and M Daniel, *Phys. Lett. A* **133**(**9**), 483 (1998)
[28] K Porsezian, M Daniel and M Lakshmanan, *J. Math. Phys.* **33**, 1807 (1992)
[29] H Aminikhah, A H Refahi Sheikhani and H Rezazadeh, *Sci. Iran. Trans. B* **23**(**3**), 1048 (2016)
[30] M Eslami and H Rezazadeh, *Calcolo* **53**(**3**), 475 (2016)
[31] M Eslami, F S Khodadad, F Nazari and H Rezazadeh, *Opt. Quantum Electron.* **49**(**12**), 391 (2017)

[32] M Eslami, H Rezazadeh, M Rezazadeh and S S Mosavi, *Opt. Quantum Electron.* **49**(**8**), 279 (2017)
[33] A Biswas, M Mirzazadeh, M Eslami, D Milovic and M Belic, *Frequenz* **68**(**11–12**), 525 (2014)
[34] F S Khodadad, F Nazari, M Eslami and H Rezazadeh, *Opt. Quantum Electron.* **49**(**11**), 384 (2017)
[35] M Eslami, *Appl. Math. Comput.* **285**, 141 (2016)
[36] M Eslami, *Nonlinear Dynam.* **85**(**2**), 813 (2016)
[37] M Eslami and M Mirzazadeh, *Nonlinear Dynam.* **83**(**1–2**), 731 (2016)
[38] M Ekici, M Mirzazadeh and M Eslami, *Nonlinear Dynam.* **84**(**2**), 669 (2016)
[39] K Hosseini, F Samadani, D Kumar and M Faridi, *Optik* **157**, 1101 (2018)
[40] D Kumar, A R Seadawy and A K Joardar, *Chin. J. Phys.* **56**(**1**), 75 (2017)
[41] K Hosseini, D Kumar, M Kaplan and E Y Bejarbaneh, *Commun. Theor. Phys.* **68**(**6**), 761 (2017)
[42] D Kumar, A R Seadawy and R Chowdhury, *Opt. Quantum Electron.* **50**(**2**), 108 (2018)
[43] D Kumar, K Hosseini and F Samadani, *Optik* **149**, 439 (2017)
[44] H Bulut, T A Sulaiman and H M Baskonus, *Opt. Quantum Electron.* **50**(**2**), 87 (2018)
[45] D Kumar, J Manafian, F Hawlader and A Ranjbaran, *Optik* **160**, 159 (2018)
[46] B Lu, *Phys. Lett. A* **376**(**28–29**), 2045 (2012)
[47] A Sirendaoreji, *Phys. Lett. A* **356**(**2**) 124 (2018)