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Optical soliton solutions to the Fokas–Lenells equation via sine-Gordon expansion method and $(m + (G'/G))$-expansion method

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Abstract. The purpose of this study is to find some novel soliton solutions of Fokas–Lenells (FL) equation where the perturbation terms are taken into account with nonlinearity. The sine-Gordon expansion method (SGEM) and the $(m + (G'/G))$-expansion method are used in this context. The dark, bright, dark–bright and singular optical soliton solutions are successfully obtained. Moreover, the constraint conditions for guaranteeing the existence of solutions are also given.

Keywords. Optical soliton solutions; Fokas–Lenells equation; sine-Gordon expansion method; $(m + (G'/G))$ method.

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1. Introduction

The development of soliton waves and its application have been favourable and interesting in recent decades. Formulating and discussing models of optical solitary energy propagating along a chain of other biological molecules have attracted a lot of interest. New achievements and their applications in the study of topological solitons and transformation phenomena in polyacetylene chains under the influence of electrical field are given in [1]. The nonlinear complex physical phenomena associated with nonlinear partial differential equations (NLPDEs) are interesting in different fields of science such as physics, plasma, fibres, nonlinear optics, biology, chemistry, geochemistry, and engineering with their related fields like fluid mechanics [2].

In monomode optical fibres, the Fokas–Lenells (FL) equation has been proposed when considering certain non-linear effects of higher order [3]. The FL equation is a model to describe the propagation of femtosecond pulse through optical silica fibre in single mode. In addition, the FL equation appears as a model defining nonlinear pulse propagation in optical fibres. The FL equation is a fully integrable equation derived by using two Hamilton operators linked to the NLS equation [4]. Another significant feature of the FL equation is the first adverse flow of the derivative NLS equation's integrable structure [5]. The FL equation includes a wealth of physical characteristics in the theory of solitary waves and phenomena of optical fibres. Besides its integrability, this equation can derive more solitary and rogue wave alternatives. Because the FL equation is a significant physical model of ultrashort pulse in media, very helpful work has been done.

Different analytical approaches have been used to find the soliton solutions of FL equation as well as the coupled FL equation. The $\exp(-\phi(\xi))$ function approach has been utilised for obtaining novel soliton solutions of FL equation [6]. The extended trial function approach has been used to find bright, dark and singular soliton solutions of FL equation [7]. The extended trial function method was utilised to seek optical soliton solutions to the FL equation [8]. Darboux transformation using a limiting operator was used to study all types of 1-soliton solutions, which contain the bright–dark, the dark–antidark as well as the breather-like soliton

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solutions [9]. Triki and Wazwaz [10] investigated the FL equation that represents the propagation of ultra-short pulses in optical fibres. Zhang *et al* [11] studied, via Darboux transformation, the general coupled nonlinear FL system. Biswas *et al* [12] studied the optical soliton solutions of the FL equation via three exotic and efficient integration schemes. Also, optical soliton solution of the FL equation was constructed by the mapping method [13].

Different methods, such as shooting method [14–17], finite element method [18], Haar wavelet method [19], finite forward difference method [20–22], homotopy perturbation method [23], Adomian decomposition method [24,25], functional variable method [26], exponential function method [27], Laplace transformation method [28], improved Bernoulli sub-ODE method [29], wavelets method [30], Lie symmetry analysis along with the (G'/G)-expansion method [31], modified exponential function method [32], decomposition-Sumudu-like-integral-transform method [33], self-similar method [34], extended simple equation method, and $\exp(-\Phi(\xi))$-expansion method [35], unified method [36] and $\tan(\phi(\xi)/2)$ method [37], have been utilised to find numerical and analytical solutions of nonlinear differential equation.

The study of dark–bright soliton interactions is a particularly intriguing topic. In the case of dark solitons, the interaction effect for (local) cubic nonlinearities is one of repulsion [38]. On the other hand, for bright solitons, the effect is crucially dependent on their relative period, as has recently been shown experimentally in the atomic realm for attractive condensates [39]. Dark–bright solitons display both these characteristics and in addition, show the interactions of the dark solitons of one component with the bright ones of the other, an interaction that is less explored.

We consider the nonlinear Schrödinger FL equation [40,41] defined as

$$iu_t + a_1 u_{xx} + a_2 u_{xt} + |u|^2(bu + i\sigma u_x) = i(\alpha u_x + \gamma(|u|^{2m}u)_x + \delta(|u|^{2m})_x u). \tag{1}$$

Equation (1) with its full nonlinearity has been solved via exp-function approach by considering $\sigma = b = 0$, and some new singular and combo-soliton solutions are presented [6]. In ref. [7], bright, dark and singular soliton solutions have been constructed by using extended trial function approach by choosing $m = 1$. In ref. [12], bright, dark and singular soliton solutions have been retrieved via three integration schemes by choosing $m = 1$. Equation (1) has also been solved by neglecting right-hand side of eq. (1) and some novel class of exact combined solitary wave solution has been reported [10]. In this paper, via the sine-Gordon expansion method (SGEM) and the $(m + (G'/G))$-expansion method, we construct some novel optical soliton solutions of FL equation. The variable approach of the travelling wave will convert the NLPDEs into nonlinear ordinary differential equations (NLODE) and the resulting equations are solved analytically for different values of nonzero-constants.

2. General form of the SGEM

Consider the sine-Gordon equation [42–45]:

$$\phi_{xx} - \phi_{tt} = -\delta^2 \sin(\phi), \tag{2}$$

where $\phi = \phi(x,t)$ and δ is a non-zero real number. Using the wave transform

$$\phi = \phi(x,t) = u(\xi), \quad \xi = x - \upsilon t, \tag{3}$$

on eq. (2) and simplifying the results, we get

$$u'' = \frac{\delta^2}{1-\upsilon^2}\sin(u), \tag{4}$$

where $u = u(\xi)$, ξ and υ are the amplitude and speed of the travelling wave. Integrating eq. (4) and simplifying it, one can get

$$\left(\left(\frac{u}{2}\right)'\right)^2 = \frac{\delta^2}{1-\upsilon^2}\sin^2\left(\frac{u}{2}\right) + c, \tag{5}$$

where c is the constant of integration. Suppose

$$\frac{u}{2} = w(\xi), \quad \frac{\delta^2}{1-\upsilon^2} = \beta^2,$$

and putting them into eq. (4), the result is

$$w'(\xi) = \sqrt{\beta^2 \sin^2(w(\xi)) + c}. \tag{6}$$

Taking $c = 0$ and $\beta = 1$, the solution of eq. (5) becomes

$$w'(\xi) = \sin(w(\xi)). \tag{7}$$

Using the separation method, eq. (6) possesses the solutions as follows:

$$\sin(w) = \sin(w(\xi)) = \left.\frac{2pe^{\xi}}{p^2 e^{2\xi} + 1}\right|_{p=1} = \operatorname{sech}(\xi), \tag{8}$$

$$\cos(w) = \cos(w(\xi)) = \left.\frac{p^2 e^{\xi} - 1}{p^2 e^{2\xi} + 1}\right|_{p=1} = \tanh(\xi), \tag{9}$$

where P is a constant of integral. These two significant solutions achieve the definition of the SGEM to get the solution of the NPDE of the form

$$P(\phi, \phi_x, \phi_t, \phi_{tt}, \phi_{xx}, \phi_{xt}, \ldots) = 0. \quad (10)$$

Now, we consider

$$u(\xi) = \sum_{i=1}^{n} \tanh^{i-1}(\xi)\,[B_i\,\mathrm{sech}(w) \pm A_i\,\tanh(w)] + A_0. \quad (11)$$

Now, due to eqs (8) and (9), eq. (11) can be rewritten as follows:

$$u(w) = \sum_{i=1}^{n} \cos^{i-1}(w)\,[B_i \sin(w) + A_i \cos(w)] + A_0. \quad (12)$$

The value of n is obtained by balancing the highest power of nonlinear term and the highest derivative appearing in the transformed NODE.

3. Governing model and mathematical analysis

Consider eq. (1) and setting $m = 1$, we get

$$iu_t + a_1 u_{xx} + a_2 u_{xt} + |u|^2 (bu + i\sigma u_x) = i(\alpha u_x + \gamma(|u|^2 u)_x + \delta(|u|^2)_x u). \quad (13)$$

Here, $u = u(x,t)$ symbolises a complex field envelope, x, t symbolise the spatial and temporal variables. The coefficients a_1, b represent velocity dispersion and a_2 is the spatiotemporal dispersion while σ and δ are the nonlinear dispersion terms that provide additional dispersive effect. Moreover, α provides the intermodal dispersion and γ is the self-steepening term that prevents the formation of shock waves. The complexity terms with γ and δ come with full nonlinearity where m provides the full nonlinearity parameter. To start and apply SGEM, let the wave transform be

$$u(x,t) = U(\xi)\mathrm{e}^{i\theta(x,t)}, \quad \xi = x - \upsilon t, \quad (14)$$

$$\theta(x,t) = -kx + \omega t + \theta_0, \quad (15)$$

where $U(\xi)$ and $\theta(x,t)$ symbolise the shape of the pulse and the phase component, respectively. υ, k, ω and θ_0 denote the velocity of the soliton, the frequency, the soliton wave number and phase constant, respectively. Using eqs (14) and (15) with eq. (13) and separating the results into imaginary and real parts, the imaginary part can be written as

$$(\upsilon a_2 k + a_2\omega - \upsilon - \alpha - 2a_1 k)\,U' + (\sigma - 3\gamma - 2\delta)U^2 U' = 0. \quad (16)$$

Equaling the linear coefficients of independent functions to zero gives

$$\upsilon = \frac{a_2\omega - \alpha - 2a_1 k}{a_2 k - 1}, \quad (17)$$

$$\sigma = 3\gamma + 2\delta. \quad (18)$$

These symbolise the speed of the wave, a couple of constraint relations on the parameters, as well. On the other hand, the real part yields

$$(a_1 - \upsilon a_2)\,U'' - (w + a_1 k^2 - a_2 k\omega + \alpha k)U + (b + \sigma k - \gamma k)\,U^3 = 0. \quad (19)$$

Evaluate the balance between U'' and U^3, and we obtain $n = 1$. Using the value of balance and putting it into eq. (12), we get

$$u(w) = B_1 \sin(w) + A_1 \cos(w) + A_0 \quad (20)$$

and

$$w' = \sin(w). \quad (21)$$

Using eq. (20) along with eq. (21), and implement it to eq. (19), we get a trigonometric equation. Solving the resultant equation, we discuss the following cases (figures 1–4):

Case 1. When

$$A_0 = 0, \quad A_1 = \frac{\sqrt{2a_1 k - 2a_2\omega}}{\sqrt{\left(-k + a_2(-2 + k^2)\right)(b + 2k(\gamma + \delta))}},$$

$$B_1 = 0, \quad \alpha = \frac{a_1(2 + k^2 - a_2 k(-2 + k^2)) + \omega + a_2(-2k + a_2(-2 + k^2))\omega}{-k + a_2(-2 + k^2)},$$

we get

$$u_1(x,t) = \frac{\mathrm{e}^{\mathrm{i}(-kx + t\omega + \theta_0)}\sqrt{2a_1 k - 2a_2\omega}\,\tanh(x - t\nu)}{\sqrt{(-k + a_2(-2 + k^2))(b + 2k(\gamma + \delta))}}. \quad (22)$$

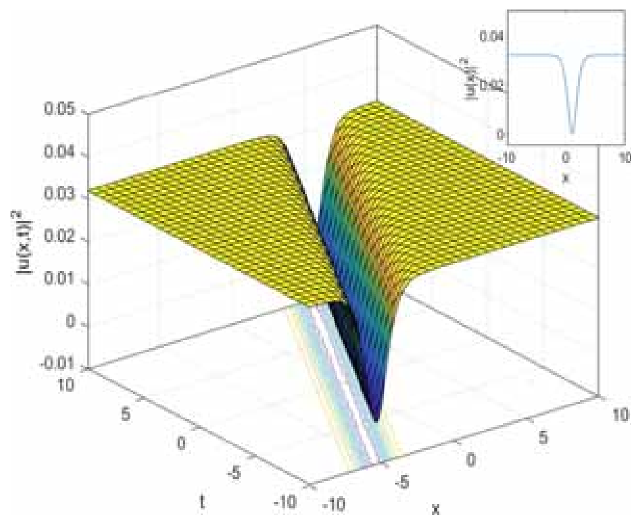

Figure 1. 3D and 2D surfaces of eq. (22) when $a_1 = 2$, $a_2 = 4$, $\omega = -0.1$, $k = 4$, $\delta = 0.01$, $\theta_0 = 0.1$, $\alpha = 0.2$, $b = 2$, $\gamma = 1$ and $t = 2$ for 2D.

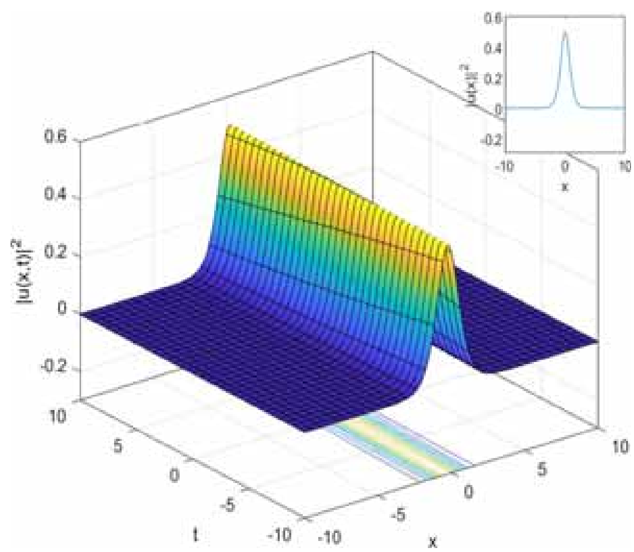

Figure 2. 3D and 2D surfaces of eq. (23) when $a_1 = 0.2$, $a_2 = 2$, $\omega = 0.07$, $k = -2$, $\delta = 0.1$, $\theta_0 = 0.1$, $\alpha = 0.2$, $b = 5$, $\gamma = 1$ and $t = 2$ for 2D.

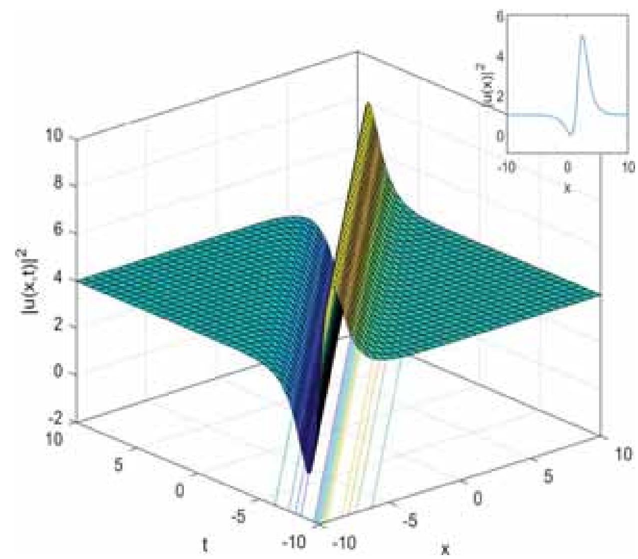

Figure 3. 3D and 2D surfaces of eq. (24) when $a_1 = 0.2$, $a_2 = 0.2$, $\omega = 0.07$, $A_1 = 1$, $B_1 = 2$ and $t = 2$ for 2D.

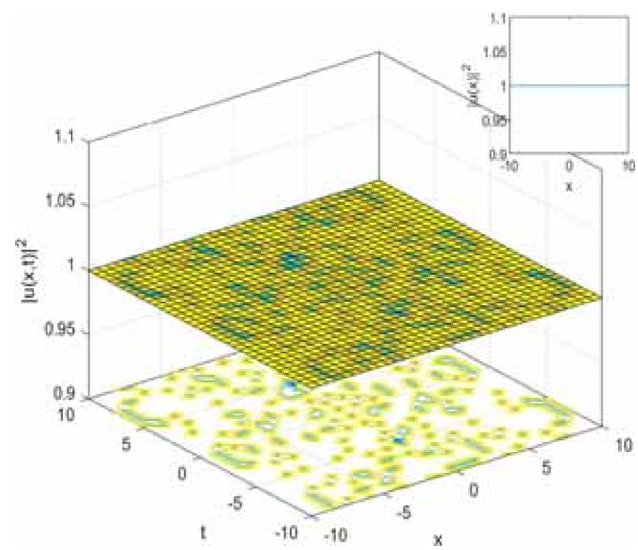

Figure 4. 3D and 2D surfaces of eq. (25) when $a_1 = 0.2$, $a_2 = 2$, $k = -2$, $\delta = 0.1$, $\theta_0 = 4$, $\alpha = 4$, $b = 5$, $A_1 = 1$ and $t = 2$ for 2D.

Case 2. When

$$B_1 = -\frac{\sqrt{-2a_1k + 2a_2\omega}}{\sqrt{\left(a_2 - k + a_2k^2\right)(b + 2k(\gamma + \delta))}}, \quad A_0 = 0, \quad A_1 = 0,$$

$$\alpha = \frac{-a_1(1 - k^2 + a_2(k + k^3)) + \omega + a_2(a_2 - 2k + a_2k^2)\omega}{a_2 - k + a_2k^2},$$

we get

$$u_2(x,t) = \frac{e^{i(-kx + t\omega + \theta_0)}\sqrt{-2a_1k + 2a_2\omega}\,\mathrm{sech}\,(x - t\nu)}{\sqrt{(a_2 - k + a_2k^2)(b + 2k(\gamma + \delta))}}. \quad (23)$$

Case 3. When

$$A_0 = 0, \quad \alpha = -\frac{a_1}{a_2}, \quad k = \frac{a_2\omega}{a_1}, \quad \gamma = -\delta - \frac{a_1b}{2a_2\omega},$$

we get

$$u_3(x,t) = \mathrm{e}^{\mathrm{i}((t-\frac{a_2 x}{a_1})\omega+\theta_0)} \times \operatorname{sech}\left(\frac{a_1 t}{a_2} - x\right)\left(B_1 - A_1 \sinh\left(\frac{a_1 t}{a_2} - x\right)\right). \quad (24)$$

Case 4. When

$$\omega = \frac{a_1 + a_1 k(a_2+2k-2a_2k^2) + (a_2+2k-2a_2k^2)\alpha}{-2+a_2(a_2+4k-2a_2k^2)},$$

$$\gamma = -\frac{a_1+a_2\alpha+A_1^2(2+a_2(-a_2-4k+2a_2k^2))\,(b+2k\delta)}{2A_1^2 k(2-4a_2k+a_2^2\left(-1+2k^2\right))},$$

$$A_0 = 0, \quad B_1 = -\mathrm{i}A_1,$$

we get

$$u_4(x,t) = \mathrm{e}^{\mathrm{i}(-kx+t\omega+\theta_0)} \times \left(\begin{array}{l} -\mathrm{i}A_1 \operatorname{sech}\left(x - \dfrac{t\,(2a_1k+\alpha-a_2\omega)}{-1+a_2k}\right) \\ +A_1 \tanh\left(x - \dfrac{t\,(2a_1k+\alpha-a_2\omega)}{-1+a_2k}\right) \end{array}\right). \quad (25)$$

4. General form of the $(m+(G'/G))$ method

In this section, the basic concept of the $(m+(G'/G))$ method are illustrated as bellow:

Step 1: Assume that the general form of NLPDE is expressed as

$$P(u, u_x, u_t, u_{xx}, \ldots) = 0. \quad (26)$$

Suppose the wave transformation takes the following form:

$$u(x,t) = U(\xi), \quad \xi = x + \nu t. \quad (27)$$

Inserting eq. (27) into eq. (26), we obtain

$$P(U, U', U'', \ldots) = 0. \quad (28)$$

Step 2: Suppose that the trial solution of eq. (28) is given by

$$U(\xi) = \sum_{i=-n}^{n} c_i (m+F)^i = c_{-n}(m+F)^{-n} + \cdots + c_0 + c_1(m+F) + \cdots + c_n(m+F)^n, \quad (29)$$

where c_n, $n = 0, \ldots, n$ and m are nonzero constants. According to the principles of balance, we find the value of n. In this article, we define F as

$$F = \frac{G'(\xi)}{G(\xi)}, \quad (30)$$

where $G(\xi)$ satisfies $G'' + (\lambda + 2m)G' + \mu G = 0$.

Step 3: Putting eq. (29) into eq. (28) and using (30), then collecting all terms with the same order of $(m+F)^n$, we get the system of algebraic equations for ν, a_n, $n = 0, \ldots, n, \lambda$ and μ. As a result, solving the obtained system, we get the explicit and exact solution of eq. (26).

5. Application of the $(m+(G'/G))$ method

Consider eq. (19) and using the value of balance $n = 1$, we can rewrite eq. (29) as follows:

$$U(\xi) = c_{-1}(m+F)^{-1} + c_0 + c_1(m+F). \quad (31)$$

Putting eq. (31) and its derivative into eq. (19), we get the following cases and solutions of eq. (1).

Case 1. When

$$c_{-1} = 0, \quad \alpha = -\frac{a_1}{a_2},$$

$$\omega = -\frac{a_1 b}{2a_2(\gamma+\delta)}, \quad k = -\frac{b}{2(\gamma+\delta)},$$

we get the following solutions:

Set 1. When $\Delta > 0$, we get the hyperbolic function solution (figure 5)

$$u(x,t) = \mathrm{e}^{\frac{1}{2}\mathrm{i}\left(\frac{b(-a_1t+a_2x)}{a_2(\gamma+\delta)}+2\theta_0\right)}\left(\frac{c_{-1}}{m+\mathrm{e}^{\frac{(a_1t-a_2x)(2m+\lambda)}{2a_2}}(A_2\cosh(\theta_1)+A_1\sinh(\theta_2))}+c_0\right), \quad (32)$$

where

$$\theta_1 = \frac{(-a_1t+a_2x)\sqrt{\Delta}}{2a_2}.$$

Set 2. When $\Delta < 0$, we get the trigonometric function solution (figure 6)

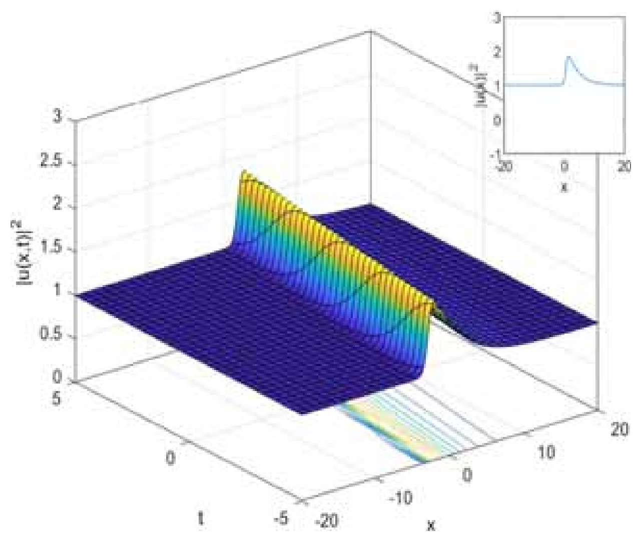

Figure 5. 3D and 2D surfaces of eq. (32) when $A_1 = 1$, $A_2 = 2$, $a_1 = 1$, $a_2 = 2$, $\gamma = 1$, $\delta = 2$, $b = 1$, $c_0 = 1$, $c_{-1} = 1$, $\lambda = 1$, $\mu = -1$, $m = 1$ and $t = 2$ for 2D.

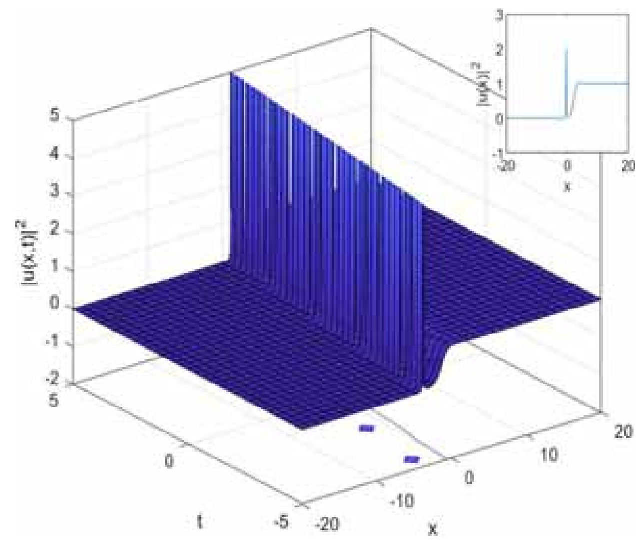

Figure 6. 3D and 2D surfaces of eq. (33) when $A_1 = 1$, $A_2 = 2$, $a_1 = 1$, $a_2 = 2$, $\gamma = 1$, $\delta = 2$, $b = 1$, $c_0 = 1$, $c_{-1} = 1$, $\lambda = 1$, $\mu = 3$, $m = 1$ and $t = 2$ for 2D.

$$u(x,t)=\mathrm{e}^{\frac{1}{2}\mathrm{i}\left(\frac{b(-a_1t+a_2x)}{a_2(\gamma+\delta)}+2\theta_0\right)}\left(\frac{c_{-1}}{m+\mathrm{e}^{\frac{(a_1t-a_2x)(2m+\lambda)}{2a_2}}(A_2\cos(\theta_2)+A_1\sin(\theta_2))}+c_0\right), \tag{33}$$

where

$$\theta_2 = \frac{(-a_1t+a_2x)\sqrt{-\Delta}}{2a_2}.$$

Set 3. When $\Delta = 0$, we get exponential function solution (figure 7)

$$u(x,t)=\mathrm{e}^{\frac{1}{2}\mathrm{i}\left(\frac{b(-a_1t+a_2x)}{a_2(\gamma+\delta)}+2\theta_0\right)}\left(\frac{a_2\mathrm{e}^{\sqrt{\mu}x}c_{-1}}{a_2\mathrm{e}^{\sqrt{\mu}x}m+\mathrm{e}^{\frac{a_1\sqrt{\mu}t}{a_2}}(A_1a_2-a_1A_2t+a_2A_2x)}+c_0\right). \tag{34}$$

Case 2. When

$$\alpha = -\frac{a_1}{a_2},\quad \omega = \frac{a_1k}{a_2},\quad b=-2k(\gamma+\delta),$$

we get the following solutions:

Set 1. When $\Delta > 0$, we get the hyperbolic function solution (figure 8)

$$u(x,t)=\mathrm{e}^{\mathrm{i}\left(\frac{a_1kt}{a_2}-kx+\theta_0\right)}\left(\frac{c_{-1}}{m+\mathrm{e}^{\frac{(a_1t-a_2x)(2m+\lambda)}{2a_2}}(A_2\cosh(\theta_1)+A_1\sinh(\theta_1))}+c_0 + \left(m+\mathrm{e}^{\frac{(a_1t-a_2x)(2m+\lambda)}{2a_2}}(A_2\cosh(\theta_1)+A_1\sinh(\theta_1))\right)c_1\right), \tag{35}$$

where

$$\theta_1 = \frac{(-a_1t+a_2x)\sqrt{\Delta}}{2a_2}.$$

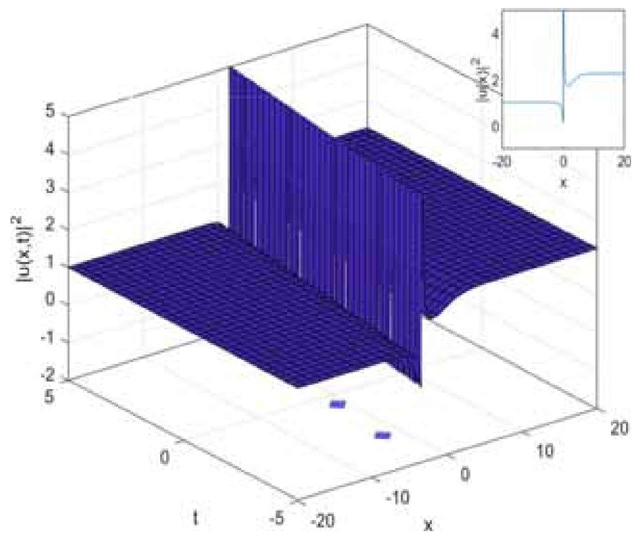

Figure 7. 3D and 2D surfaces of eq. (34) when $A_1 = 1$, $A_2 = 2$, $a_1 = 1$, $a_2 = 2$, $\gamma = 1$, $\delta = 2$, $b = 1$, $c_0 = 1$, $c_{-1} = 1$, $\mu = 1$, $m = 2$ and $t = 2$ for 2D.

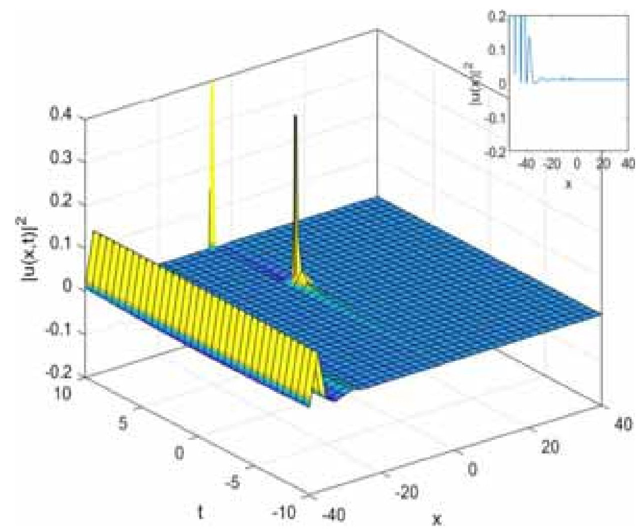

Figure 9. 3D and 2D surfaces of eq. (36) when $A_1 = 0.1$, $A_2 = 0.2$, $a_1 = 0.1$, $a_2 = 4$, $c_0 = 0.1$, $c_1 = 0.0001$, $c_{-1} = 0.0005$, $\lambda = 0.1$, $\mu = -0.5$, $m = 0.2$ and $t = 2$ for 2D.

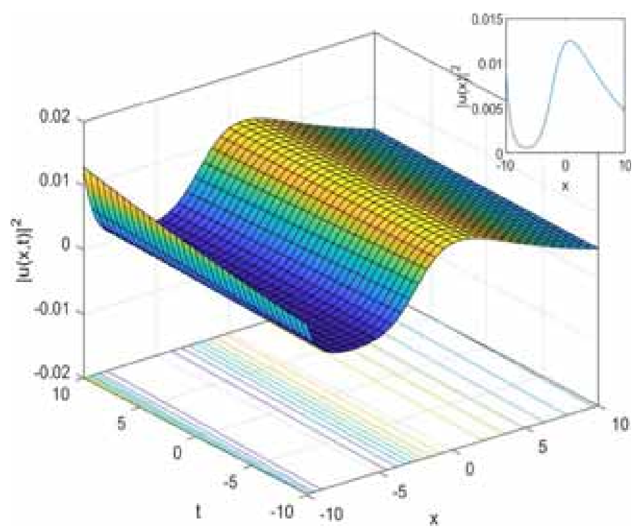

Figure 8. 3D and 2D surfaces of eq. (35) when $A_1 = 0.1$, $A_2=0.2$, $a_1=0.1$, $a_2=4$, $c_0=0.01$, $c_1=0.001$, $c_{-1}=-0.05$, $\lambda=0.01$, $\mu=-0.1$, $m=0.3$ and $t = 2$ for 2D.

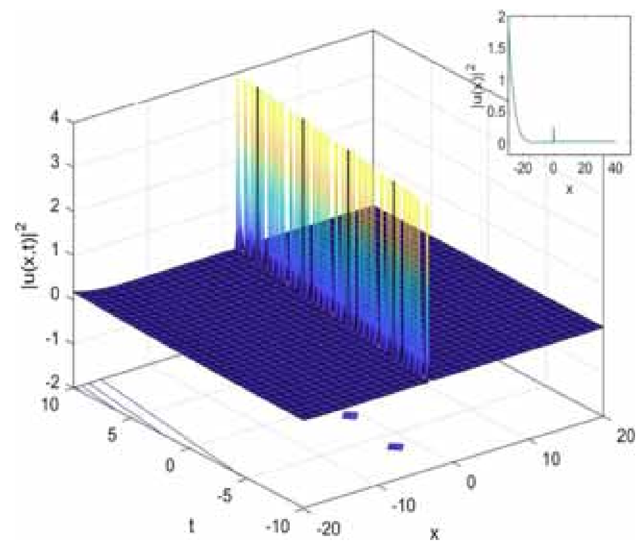

Figure 10. 3D and 2D surfaces of eq. (37) when $A_1 = 0.1$, $A_2 = 0.02$, $a_1 = 0.1$, $a_2 = 0.4$, $c_0 = 0.1$, $c_1 = 0.01$, $c_{-1} = 0.005$, $\lambda = -2$, $\mu = 1$, $m = 2$ and $t = 2$ for 2D.

Set 2. When $\Delta < 0$, we get the trigonometric function solution (figure 9)

$$u(x,t) = \mathrm{e}^{\mathrm{i}\left(\frac{a_1 k t}{a_2} - kx + \theta_0\right)} \left(\frac{c_{-1}}{m + \mathrm{e}^{\frac{(a_1 t - a_2 x)(2m+\lambda)}{2a_2}} (A_2 \cos(\theta_2) + A_1 \sin(\theta_2))} + c_0 + \left(m + \mathrm{e}^{\frac{(a_1 t - a_2 x)(2m+\lambda)}{2a_2}} (A_2 \cos(\theta_2) + A_1 \sin(\theta_2))\right) c_1 \right), \tag{36}$$

where

$$\theta_2 = \frac{(-a_1 t + a_2 x)\sqrt{-\Delta}}{2a_2}.$$

Set 3. When $\Delta = 0$, we get the exponential function solution (figure 10)

$$u(x,t)=\mathrm{e}^{\mathrm{i}\left(\frac{a_1kt}{a_2}-kx+\theta_0\right)} \times\left(\frac{a_2c_{-1}}{a_2m+\mathrm{e}^{\frac{(a_1t-a_2x)\sqrt{\mu}}{a_2}}(A_1a_2-a_1A_2t+a_2A_2x)}+c_0 +\left(m+\frac{\mathrm{e}^{\frac{(a_1t-a_2x)\sqrt{\mu}}{a_2}}(A_1a_2-a_1A_2t+a_2A_2x)}{a_2}\right)c_1\right). \quad (37)$$

6. Conclusion

In this manuscript, the SGEM and $(m + (G'/G))$-expansion methods were used to find optical soliton solutions of the perturbed FL equation with Kerr law nonlinearity. The dark, bright, dark-bright and singular optical soliton solutions are obtained. It is understood that dark solitary waves identify solitary waves with intensity lower than the background, bright solitary waves whose maximum intensity is greater than the background and singular solitary solutions which are solitary waves with discontinuous derivatives. Examples of such solitary waves are compactions,which have finite (compact) support, and peakons, whose peaks have a discontinuous first derivative. All results are new when compared to other soliton solutions reported in refs [6,7,10,12] and also all solutions satisfy the main FL equation. The results obtained in this research may have a great impact on various fields of nonlinear sciences such as applied mathematics, physics, engineering, etc. With the results presented, we can see that the suggested methods are useful, efficient and replicable for taking out the soliton solutions of strong NLPDEs in a wide range.

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