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Finite-time synchronisation of uncertain delay spatiotemporal networks via unidirectional coupling technology

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Abstract. In this paper, the problem of finite-time synchronisation of uncertain delay spatiotemporal networks via unidirectional coupling technology is investigated. Based on Lyapunov theorem and finite-time stability theory, an effective finite-time synchronisation scheme is designed to achieve finite-time synchronisation between uncertain delay spatiotemporal networks, and adaptive estimations of coupling coefficient, unknown parameter and uncertain network topology are realised. Then, the Fisher–Kolmogorov spatiotemporal model is used as the state equation of the network node for numerical simulation. The simulation results show that the finite-time synchronisation scheme is effective.

Keywords. Network synchronisation; finite-time; Lyapunov theorem; unidirectional coupling.

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1. Introduction

In recent years, network synchronisation has always been a hot topic of network dynamics research, and has attracted extensive attention from scholars. At present, the synchronisation of complex networks has applications in information communication, biological science and transportation fields [1–4]. Many types of network synchronisation have also been reported, including finite-time synchronisation [5–7], exponential synchronisation [8,9], projection synchronisation [10,11], cluster synchronisation [12,13], etc.

Among all types of synchronisations, network finite-time synchronisation shows better robustness and anti-jamming properties, which makes the cost of network synchronisation small and the synchronisation effect stable. Especially the time for achieving network synchronisation can be estimated according to finite-time theory. People always hope that complex networks can undergo synchronisation in finite time under many circumstances. Therefore, the study of network finite-time synchronisation has become a hot topic. With intensive research on finite-time synchronisation, many results have been reported successively. Typically, Saravanan *et al* [14] studied the finite-time non-fragile dissipativity issue of time-delayed neural networks by constructing appropriate Lyapunov–Krasovskii functional. Using the Lyapunov–Krasovskii theorem, Zhou *et al* [15] realised finite-time synchronisation between the drive and the response networks by designing a simple feedback controller. Aghababa and Aghababa constructed a new controller using sliding mode control method, and studied the finite-time synchronisation of the network composed of nonlinear chaotic systems [16]. What is more, Qiu *et al* [17] used Lyapunov function method to realise finite-time synchronisation for complex dynamic networks. Wang *et al* [18] constructed a complex network and studied its finite-time synchronisation problem. Duan *et al* [19] realised the finite-time synchronisation of cellular neural networks. Zhang *et al* [20] further studied the finite-time synchronisation of complex dynamic networks by designing linear feedback controllers based on the finite-time synchronisation theory and Lyapunov principle. The above schemes have laid a solid foundation for scholars to further study the network finite-time synchronisation.

However, the aforementioned researches mainly focus on the finite-time synchronisation of time networks. In the real world, the structure of the network is relatively complex, the state variables not only evolve with time, but also are inseparable with the spatial evolution. To our knowledge, there are few reports on the finite-time synchronisation of spatiotemporal networks now. Meanwhile, the actual network has many uncertainties due to

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the effect of external environment or human factor, and the dynamic characteristics of the network are inevitably perturbed by these factors. Therefore, it is very important to study the impact on unknown parameters of network synchronisation. At present, there are some research results of finite-time synchronisation of uncertain networks. For example, Jing *et al* [21] studied the parameter identification of uncertain neural networks according to the finite-time theory. Li *et al* [22] analysed uncertain complex networks by discrete control method based on finite-time theory. Li *et al* studied uncertain neural networks by using adaptive control method [23]. In fact, the network topology is often unknown due to the different number of nodes and the different connection modes between nodes. This means that in practical applications, the network topology has some differences and also has some influence on the network synchronisation process. Therefore, the problem of identifying the topological structure of uncertain spatiotemporal networks and realising finite-time synchronisation deserves discussion.

So far, there are many ways to achieve the network finite-time synchronisation for theoretical research and practical application, such as coupling technology [24,25], adaptive technology [26,27], sliding mode control technology [28,29] and so on. The coupling technology can be divided into unidirectional coupling and bidirectional coupling. The unidirectional coupling technology only applies coupling to the response network, the form is simple, and the synchronisation time is also faster than other technologies. Therefore, scholars often choose this technology to study network synchronisation. In addition, the interference with delay on network synchronisation is inevitable, especially, time delay often plays a great role in the case of long-distance communication and traffic congestion. In order to simulate a more real network, the effect of delay on network synchronisation should also be taken into account. Until now, it is not perfect enough to do research on delay finite-time synchronisation of uncertain spatiotemporal networks using unidirectional coupling technology.

Based on the above analysis, this paper addresses the problem of finite-time synchronisation between delay spatiotemporal networks with uncertain parameters by using unidirectional coupled control technology. First, the finite-time synchronisation scheme, the adaptive laws of coupling coefficient and unknown parameter and network topology are designed according to Lyapunov theorem and finite-time stability theory. This scheme not only effectively realises finite-time synchronisation between time-delay spatiotemporal networks with uncertain parameters, but also accurately identifies coupling coefficient, unknown parameter and uncertain network topology. Finally, the Fisher–Kolmogorov spatiotemporal model is used as the state equation of the network node to verify the feasibility of this scheme.

2. Analysis of finite-time synchronisation principle for uncertain delay networks

Consider a spatiotemporal network consisting of N identical nodes. The state equation of the ith node in the network is described by

$$\begin{aligned}\frac{\partial x_i(r,t)}{\partial t} &= f(x_i(r,t),\alpha_i) + P\sum_{j=1}^{N} c_{ij}x_j(r,t-\tau)\\ &= F(x_i(r,t)) + S(x_i(r,t))\alpha_i\\ &\quad + P\sum_{j=1}^{N} c_{ij}x_j(r,t-\tau), \end{aligned} \tag{1}$$

where $x_i(r,t) = [x_{i1}(r,t), x_{i2}(r,t), \ldots, x_{in}(r,t)]^T \in R^n$ are the state variables of node i, $f\colon R\times R^n \to R^n$ is a nonlinear node state equation, τ is a coupled delay and $\tau \geq 0$. $C = [c_{ij}]_{N\times N}$ is the coupling matrix of the network, and it represents the topological structure of the network. c_{ij} is defined as follows: if there exists a connection from node j to node $i (i \neq j)$, then $c_{ij} > 0$. Otherwise, $c_{ij} = 0$. The diagonal element of matrix C is defined by

$$c_{ii} = -\sum_{j=1,j\neq i}^{N} c_{ij}, \quad i = 1,2,\ldots,N. \tag{2}$$

Network (1) is used as the drive network and the state equation of the response network is constructed as follows:

$$\begin{aligned}\frac{\partial y_i(r,t)}{\partial t} &= f(y_i(r,t),\hat{\alpha}_i)\\ &\quad + P\sum_{j=1}^{N} \hat{c}_{ij}y_j(r,t-\tau)\\ &\quad + \hat{b}_i(y_i(r,t) - x_i(r,t))\\ &= F(y_i(r,t)) + S(y_i(r,t))\hat{\alpha}_i\\ &\quad + P\sum_{j=1}^{N} \hat{c}_{ij}y_j(r,t-\tau)\\ &\quad + \hat{b}_i(y_i(r,t) - x_i(r,t)), \end{aligned} \tag{3}$$

where $y_i(r,t) = (y_{i1}(r,t), y_{i2}(r,t), \ldots, y_{in}(r,t))^T \in R^n$ are the state variables of node I, $\hat{C} = (\hat{c}_{ij})_{N\times N}$ is the identification quantity of the response network topology structure, $\hat{b}_i(y_i(r,t) - x_i(r,t))$ is the external coupling term of the network and $\hat{b}_i$ is the identification quantity of the coupling strength coefficient.

Define the error function of the drive network and the response network:

$$e_i(r,t) = y_i(r,t) - x_i(r,t). \quad (4)$$

Then, we get the time derivative of the error function as

$$\begin{aligned}\frac{\partial e_i(r,t)}{\partial t} &= \frac{\partial y_i(r,t)}{\partial t} - \frac{\partial x_i(r,t)}{\partial t} \\ &= f(y_i(r,t),\hat{\alpha}_i) + P\sum_{j=1}^{N}\hat{c}_{ij}y_j(r,t-\tau) \\ &\quad + \hat{b}_i(y_i(r,t) - x_i(r,t)) \\ &\quad - f(x_i(r,t),\alpha_i) - P\sum_{j=1}^{N}c_{ij}x_j(r,t-\tau) \\ &= \sum_{i=1}^{N}[F(y_i(r,t)) - F(x_i(r,t)) \\ &\quad + S(y_i(r,t))\hat{\alpha}_i - S(x_i(r,t))\alpha_i] \\ &\quad + P\sum_{j=1}^{N}\hat{c}_{ij}y_j(r,t-\tau) \\ &\quad - P\sum_{j=1}^{N}c_{ij}x_j(r,t-\tau) \\ &\quad + \hat{b}_i(y_i(r,t) - x_i(r,t)). \end{aligned} \quad (5)$$

DEFINITION 1

For the drive network (1) and the response network (3), if there is a certain time $t_1 > 0$ for any $i = 1,2,\ldots,N$, $\lim_{t\to t_1}|e(r,t)| \to 0$ and $\lim_{t\to t_1}(\tilde{\alpha}_i) = \lim_{t\to t_1}(\tilde{c}_{ij}) = \lim_{t\to t_1}(\tilde{b}_i) = 0$ always established when $t \geq t_1$. It means that the two networks can synchronise in a finite time, and the identification of unknown parameter and uncertain topological structure can be realised. The parameter errors to be identified are defined as $\tilde{\alpha}_i = \hat{\alpha}_i - \alpha_i$, $\tilde{c}_{ij} = \hat{c}_{ij} - c_{ij}$ and $\tilde{b}_i = \hat{b}_i - b_i$.

Assumption 1 [30]. For any $x_i(r,t), y_i(r,t) \in R^n$, suppose that there exists a real number $l > 0$ satisfying the following inequalities:

$$\begin{aligned}&|f(y_i(r,t),\alpha_i) - f(x_i(r,t),\alpha_i)| \\ &\quad \leq l\,|y_i(r,t) - x_i(r,t)|,\end{aligned} \quad (6)$$

where the norm $|\cdot|$ of variable x is defined as $|x| = (x^Tx)^{1/2}$.

Lemma 1 [30]. *For any $x_i(r,t), y_i(r,t) \in R^n$, there is a positive definite matrix $H \in R^{n\times n}$, and the following inequality relations are satisfied*:

$$\begin{aligned}x_i(r,t)^T y_i(r,t) \leq \frac{1}{2}(&x_i(r,t)^T Hx_i(r,t) \\ &+ y_i(r,t)^T H^{-1}y_i(r,t)).\end{aligned} \quad (7)$$

Lemma 2 [31]. *Positive definite functions satisfy the following differential equations*:

$$\frac{\partial V(r,t)}{\partial t} + pV^{\eta}(r,t) \leq 0, \quad \forall t \geq t_0, \quad (8)$$

where constant $0 < \eta < 1$ and $p > 0$, for any t_0, such that

$$V^{1-\eta}(r,t) \leq V^{1-\eta}(r,t_0) - p(1-\eta)(t-t_0),$$

$$t_0 \leq t \leq t_1$$

and

$$V(r,t_0) \equiv 0, \quad \forall t \geq t_1. \quad (9)$$

Then, we get

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{p(1-\eta)}. \quad (10)$$

Lemma 3 [32]. *Suppose $\theta_1,\theta_2,\ldots,\theta_n \in R^n$, and $0 < q < 2$ are all positive real numbers. Then the following inequality holds*:

$$\begin{aligned}&|\theta_1|^q + |\theta_2|^q + \cdots + |\theta_n|^q \\ &\quad \geq (\theta_1^2 + \theta_2^2 + \cdots + \theta_n^2)^{q/2}.\end{aligned} \quad (11)$$

Then we further get

$$\sum_{i=1}^{n}|\theta_n|^q \geq \left(\sum_{i=1}^{n}|\theta_n|^2\right)^{q/2}. \quad (12)$$

When $q = 1$,

$$|\theta_1| + |\theta_2| + \cdots + |\theta_n| \geq \sqrt{\theta_1^2 + \theta_2^2 + \cdots + \theta_n^2}.$$

Theorem 1. *If the drive network (1) and the response network (3) are synchronised, the following adaptive laws of unknown parameter, uncertain topology and coupling coefficient need to be designed*:

$$\begin{aligned}\frac{\partial \hat{\alpha}_i^T}{\partial t} = -\lambda_i\Bigg[&e_i^T(r,t)S(y_i(r,t)) \\ &+ \frac{k}{\sqrt{\lambda_i}^{\omega+1}}|\tilde{\alpha}_i|^{\omega}\operatorname{sign}(\tilde{\alpha}_i)\Bigg],\end{aligned} \quad (13)$$

$$\frac{\partial\hat{c}_{ij}}{\partial t} = -\xi_{ij}\left[Pe_i^T(r,t)y_j(r,t-\tau) + \frac{k}{\sqrt{\xi_{ij}}^{\omega+1}}\left|\tilde{c}_{ij}\right|^{\omega}\operatorname{sign}(\tilde{c}_{ij})\right], \tag{14}$$

$$\frac{\partial\hat{b}_i}{\partial t} = -\delta_i\left[e_i^T(r,t)e_i(r,t) + \frac{k}{\sqrt{\delta_i}^{\omega+1}}|\tilde{b}_i|^{\omega}\operatorname{sign}(\tilde{b}_i) + \frac{\tilde{b}_i k}{\tilde{b}_i^2}|e_i(r,t)|^{\omega+1} + \sqrt{2}^{\omega+1}\frac{\tilde{b}_i k}{\tilde{b}_i^2}\left(\int_{t-\tau}^{t} e_i^T(r,\psi)e_i(r,\psi)\,\mathrm{d}\psi\right)^{(\omega+1)/2} + \frac{\tilde{b}_i H}{\tilde{b}_i^2}e_i^T(r,t)e_i(r,t)\right], \tag{15}$$

where $k, \lambda_i, \xi_i, \delta_i$ are any positive real numbers and sign$(\cdot)$ is a symbolic function. At this time, the error function is asymptotically stable, the trajectory is approaching $e_i(r,t) = 0$ and time t_1 satisfies

$$t_1 \geq \frac{2V^{(1-\omega)/2}(r,0)}{\sqrt{2}^{\omega+1}k(1-\omega)}. \tag{16}$$

Proof. Choose the Lyapunov function

$$V(r,t) = \frac{1}{2}\sum_{i=1}^{N}e_i^T(r,t)e_i(r,t) + \frac{1}{2}\sum_{i=1}^{N}\frac{1}{\lambda_i}\tilde{\alpha}_i^T\tilde{\alpha}_i + \frac{1}{2}\sum_{i=1}^{N}\frac{1}{\delta_i}\tilde{b}_i^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\xi_{ij}}\tilde{c}_{ij}^2 + \int_{t-\tau}^{t}\sum_{i=1}^{N}e_i^T(r,\psi)e_i(r,\psi)\,\mathrm{d}\psi. \tag{17}$$

Taking the time derivative of $V(r,t)$ is

$$\frac{\partial V(r,t)}{\partial t} = \sum_{i=1}^{N}e_i^T(r,t)\frac{\partial e_i(r,t)}{\partial t} + \sum_{i=1}^{N}\frac{1}{\lambda_i}\frac{\partial\tilde{\alpha}_i^T}{\partial t}\tilde{\alpha}_i + \sum_{i=1}^{N}\frac{1}{\delta_i}\tilde{b}_i\frac{\partial\tilde{b}_i}{\partial t} + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\xi_{ij}}\tilde{c}_{ij}\frac{\partial\tilde{c}_{ij}}{\partial t} + \sum_{i=1}^{N}e_i^T(r,t)e_i(r,t) - \sum_{i=1}^{N}e_i^T(r,t-\tau)e_i(r,t-\tau). \tag{18}$$

Substitute error function (5) into the above equation to obtain

$$\frac{\partial V(r,t)}{\partial t} = \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\xi_{ij}}\tilde{c}_{ij}\frac{\partial\tilde{c}_{ij}}{\partial t} + \sum_{i=1}^{N}\frac{1}{\lambda_i}\frac{\partial\tilde{\alpha}_i^T}{\partial t}\tilde{\alpha}_i + \sum_{i=1}^{N}\frac{1}{\delta_i}\tilde{b}_i\frac{\partial\tilde{b}_i}{\partial t} + \sum_{i=1}^{N}e_i^T(r,t)e_i(r,t) - \sum_{i=1}^{N}e_i^T(r,t-\tau)e_i(r,t-\tau) + \sum_{i=1}^{N}\hat{b}_ie_i^T(r,t)e_i(r,t) + \sum_{i=1}^{N}e_i^T(r,t)[F(y_i(r,t)) - F(x_i(r,t)) + S(y_i(r,t))\hat{\alpha}_i - S(x_i(r,t))\alpha_i] + \sum_{i=1}^{N}\sum_{j=1}^{N}Pe_i^T(r,t)\hat{c}_{ij}y_j(r,t-\tau) - \sum_{i=1}^{N}\sum_{j=1}^{N}Pe_i^T(r,t)c_{ij}x_j(r,t-\tau). \tag{19}$$

According to adaptive laws (13)–(15), eq. (19) can be rewritten as

$$\frac{\partial V(r,t)}{\partial t} = \sum_{i=1}^{N}e_i^T(r,t)[F(y_i(r,t)) - F(x_i(r,t)) + S(y_i(r,t))\hat{\alpha}_i - S(x_i(r,t))\alpha_i] + \sum_{i=1}^{N}\sum_{j=1}^{N}Pe_i^T(r,t)\hat{c}_{ij}y_j(r,t-\tau) - \sum_{i=1}^{N}\sum_{j=1}^{N}Pe_i^T(r,t)c_{ij}x_j(r,t-\tau) + \sum_{i=1}^{N}\hat{b}_ie_i^T(r,t)e_i(r,t) + \sum_{i=1}^{N}\frac{1}{\lambda_i}\Big\{-\lambda_i\Big[e_i^T(r,t)S(y_i(r,t)) + \frac{k}{\sqrt{\lambda_i}^{\omega+1}}|\tilde{\alpha}_i|^{\omega}\operatorname{sign}(\tilde{\alpha}_i)\Big]\Big\}\tilde{\alpha}_i$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\xi_{ij}}\tilde{c}_{ij}\left\{-\xi_{ij}\left[(Pe_i^T(r,t)y_j(r,t-\tau)+\frac{k}{\sqrt{\xi_{ij}}^{\omega+1}}|\tilde{c}_{ij}|^{\omega}\mathrm{sign}(\tilde{c}_{ij}))\right]\right\}$$
$$+\sum_{i=1}^{N}\frac{1}{\delta_i}\tilde{b}_i\left\{-\delta_i\left[(e_i^T(r,t)e_i(r,t)+\frac{k}{\sqrt{\delta_i}^{\omega+1}}|\tilde{b}_i|^{\omega}\mathrm{sign}(\tilde{b}_i)+\frac{\tilde{b}_ik}{\tilde{b}_i^2}|e_i(r,t)|^{\omega+1}+\sqrt{2}^{\omega+1}\frac{\tilde{b}_ik}{\tilde{b}_i^2}\left(\int_{t-\tau}^{t}e_i^T(r,\psi)e_i(r,\psi)\,\mathrm{d}\psi\right)^{(\omega+1)/2}+\frac{\tilde{b}_iH}{\tilde{b}_i^2}e_i^T(r,t)e_i(r,t))\right]\right\}+\sum_{i=1}^{N}e_i^T(r,t)e_i(r,t)$$
$$-\sum_{i=1}^{N}e_i^T(r,t-\tau)e_i(r,t-\tau). \qquad (20)$$

According to Lemma 1 and Assumption 1,

$$\frac{\partial V(r,t)}{\partial t}\le l\sum_{i=1}^{N}e_i^T(r,t)e_i(r,t)+\sum_{i=1}^{N}P\mu_ie_i^T(r,t)e_i(r,t-\tau)-\sum_{i=1}^{N}|\tilde{\alpha}_i|^{\omega+1}\frac{k}{\sqrt{\lambda_i}^{\omega+1}}-\sum_{i=1}^{N}\sum_{j=1}^{N}|\tilde{c}_{ij}|^{\omega+1}\frac{k}{\sqrt{\xi_{ij}}^{\omega+1}}-\sum_{i=1}^{N}|\tilde{b}_i|^{\omega+1}\frac{k}{\sqrt{\delta_i}^{\omega+1}}-k|e_i(r,t)|^{\omega+1}$$
$$-\sqrt{2}^{\omega+1}k\sum_{i=1}^{N}\left(\int_{t-\tau}^{t}e_i^T(r,\psi)e_i(r,\psi)\,\mathrm{d}\psi\right)^{(\omega+1)/2}-H\sum_{i=1}^{N}e_i^T(r,t)e_i(r,t)+\sum_{i=1}^{N}e_i^T(r,t)e_i(r,t)-\sum_{i=1}^{N}e_i^T(r,t-\tau)e_i(r,t-\tau)+b_ie_i^T(r,t)e_i(r,t), \qquad (21)$$

where μ_i is the eigenvalue of the coupling matrix C.

According to Lemmas 1–3, eq. (21) can be arranged as follows:

$$\frac{\partial V(r,t)}{\partial t}\le\sum_{i=1}^{N}\left(l+\frac{1}{2}P\mu_i-H+b_i+1\right)e_i^T(r,t)e_i(r,t)+\sum_{i=1}^{N}\left(\frac{1}{2}P\mu_i-1\right)e_i^T(r,t-\tau)e_i(r,t-\tau)-k\sum_{i=1}^{N}|e_i(r,t)|^{\omega+1}-k\sum_{i=1}^{N}\frac{1}{\sqrt{\lambda_i}^{\omega+1}}|\tilde{\alpha}_i|^{\omega+1}$$
$$-k\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\sqrt{\xi_{ij}}^{\omega+1}}|\tilde{c}_{ij}|^{\omega+1}-k\sum_{i=1}^{N}\frac{1}{\sqrt{\delta_i}^{\omega+1}}|\tilde{b}_i|^{\omega+1}-\sqrt{2}^{\omega+1}k\sum_{i=1}^{N}\left(\int_{t-\tau}^{t}e_i^T(r,\psi)e_i(r,\psi)\,\mathrm{d}\psi\right)^{(\omega+1)/2}, \qquad (22)$$

If the conditions

$$l+\frac{1}{2}P\mu_i-H+b_i+1<0, \qquad (23)$$
$$\frac{1}{2}P\mu_i-1<0, \qquad (24)$$

are satisfied, we have

$$\frac{\partial V(r,t)}{\partial t}\le-\sqrt{2}^{\omega+1}k\left[\frac{1}{2}\sum_{i=1}^{N}e_i^T(r,t)e_i(r,t)+\frac{1}{2}\sum_{i=1}^{N}\frac{1}{\lambda_i}\tilde{\alpha}_i^T\tilde{\alpha}_i+\frac{1}{2}\sum_{i=1}^{N}\frac{1}{\delta_i}\tilde{b}_i^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\xi_{ij}}\tilde{c}_{ij}^2+\int_{t-\tau}^{t}\sum_{i=1}^{N}e_i^T(r,\psi)e_i(r,\psi)\,\mathrm{d}\psi\right]^{(\omega+1)/2}$$
$$<-\sqrt{2}^{\omega+1}kV^{(\omega+1)/2}. \qquad (25)$$

According to finite-time stability theory and Lemma 2, $V(r,t)$ is semi-negative definite and the error function $e_i(r,t)$ converges to zero in the finite time

$$t_1 \geq \frac{2V^{(1-\omega)/2}(r,0)}{\sqrt{2}^{\omega+1}k(1-\omega)}.$$

It is proved that the adaptive estimations of unknown parameter, topological structure and coupling coefficient are feasible. □

3. Simulation and discussion

Numerical simulations are performed in this section. One-dimensional Fisher–Kolmogorov system with spatiotemporal chaotic behaviour is selected as the state equation of the drive and the response network nodes. The form of the Fisher–Kolmogorov system is as follows:

$$\frac{\partial x_d(r,t)}{\partial t} = \varepsilon x_d(r,t)\left(1-\frac{x_d(r,t)}{\rho}\right) + D\nabla^2 x_d(r,t), \tag{26}$$

where ρ, ε and D are system parameters, and $D = 5, \rho = 1, \varepsilon = 0.5$. Periodic boundary conditions are selected for the system, and its initial values are selected randomly.

In the simulation process, $N = 11$ is chosen as the number of nodes in the two networks. Assuming that D is an uncertain parameter, its recognition amount is $\hat{D}_i$. The state equation of the drive and the response networks is constructed as follows:

$$\frac{\partial x_i(r,t)}{\partial t} = \varepsilon x_d(r,t)\left(1-\frac{x_d(r,t)}{\rho}\right) + D\nabla^2 x_d(r,t) + P\sum_{j=1}^{N} c_{ij}x_j(r,t-\tau), \tag{27}$$

$$\frac{\partial y_i(r,t)}{\partial t} = \varepsilon y_d(r,t)\left(1-\frac{y_d(r,t)}{\rho}\right) + \hat{D}_i\nabla^2 y_d(r,t) + P\sum_{j=1}^{N} \hat{c}_{ij}y_j(r,t-\tau) + \hat{b}_i(y_i(r,t)-x_i(r,t)). \tag{28}$$

Since the topology of the network can be arbitrary, the connection between 11 nodes of the drive network is selected as follows:

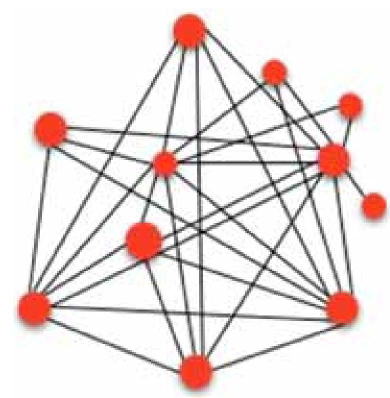

Figure 1. Scaleless network connection diagram.

$$C = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -4 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -5 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -5 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -6 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & -7 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -8 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -10 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -9 \end{bmatrix}_{11\times 11}. \tag{29}$$

The topology of the network is shown in figure 1.

Concurrently, it is assumed that the coupling coefficient b_i and the topology c_{ij} of the response network are unknown. The specific expressions of the adaptive laws of identifying parameters and network topology and coupling coefficients according to eqs (13)–(15) are as follows:

$$\frac{\partial \hat{D}_i}{\partial t} = -\lambda_i\left[e_i^T(r,t)S(y_i(r,t)) + \frac{k}{\sqrt{\lambda_i}^{\omega+1}}|\tilde{D}_i|^{\omega}\mathrm{sign}(\tilde{D}_i)\right], \tag{30}$$

$$\frac{\partial \hat{c}_{ij}}{\partial t} = -\xi_{ij}\left[Pe_i^T(r,t)y_j(r,t-\tau) + \frac{k}{\sqrt{\xi_{ij}}^{\omega+1}}|\tilde{c}_{ij}|^{\omega}\mathrm{sign}(\tilde{c}_{ij})\right], \tag{31}$$

$$\frac{\partial \hat{b}_i}{\partial t} = -\delta_i\left[e_i^T(r,t)e_i(r,t) + \frac{k}{\sqrt{\delta_i}^{\omega+1}}|\tilde{b}_i|^{\omega}\mathrm{sign}(\tilde{b}_i) + \frac{\tilde{b}_i k}{\tilde{b}_i^2}|e_i(r,t)|^{\omega+1}\right.$$

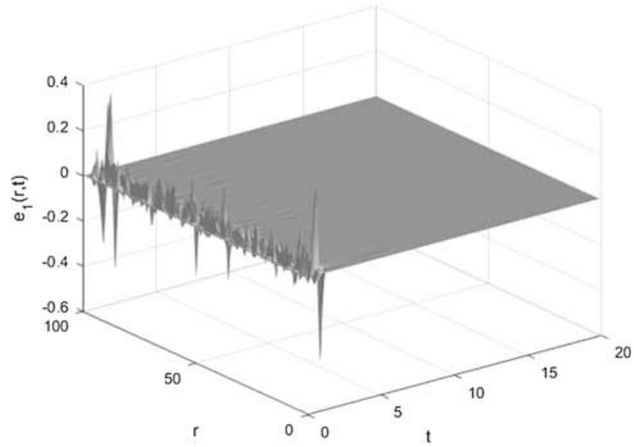

Figure 2. Spatiotemporal evolution of error function $e_1(r,t)$.

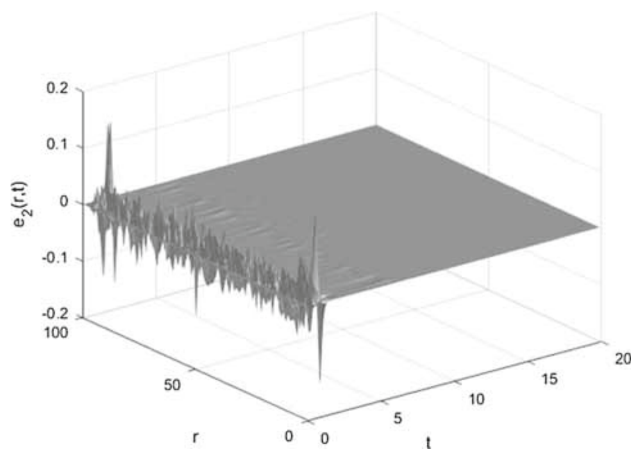

Figure 3. Spatiotemporal evolution of error function $e_2(r,t)$.

$$+\sqrt{2}^{\omega+1}\frac{\tilde{b}_i k}{\tilde{b}_i^2} \times \left(\int_{t-\tau}^{t} e_i^T(r,\psi)e_i(r,\psi)\,\mathrm{d}\psi\right)^{(\omega+1)/2} + \frac{\tilde{b}_i H}{\tilde{b}_i^2} e_i^T(r,t)e_i(r,t)\Bigg]. \quad (32)$$

The values of parameters are $k = 1.6$, $\omega = 0.5$, $H = 1.2$, $\delta_i = 0.01$, $\xi_i = 0.05$ and $\lambda_i = 0.05$. Coupling strength $P = 1.7$, coupled delay selection $\tau = 0.02$. The initial values of the state variables of the two complex networks are taken as random numbers.

In numerical simulation, the space coordinates of the state equations of two nodes are divided into 100 lattices, and periodic boundary conditions are selected. The evolutions of the error function with time between two networks are shown in figures 2–12.

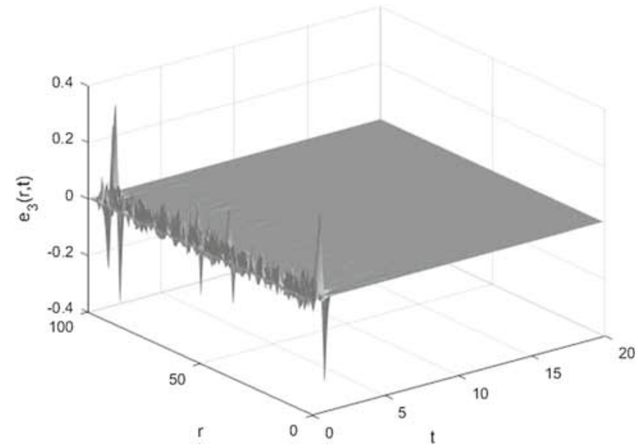

Figure 4. Spatiotemporal evolution of error function $e_3(r,t)$.

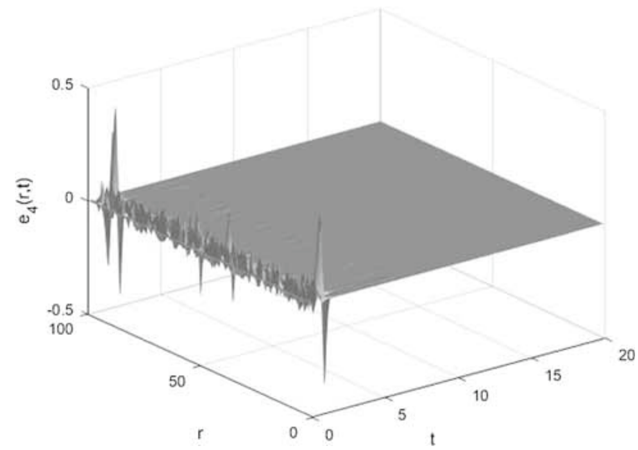

Figure 5. Spatiotemporal evolution of error function $e_4(r,t)$.

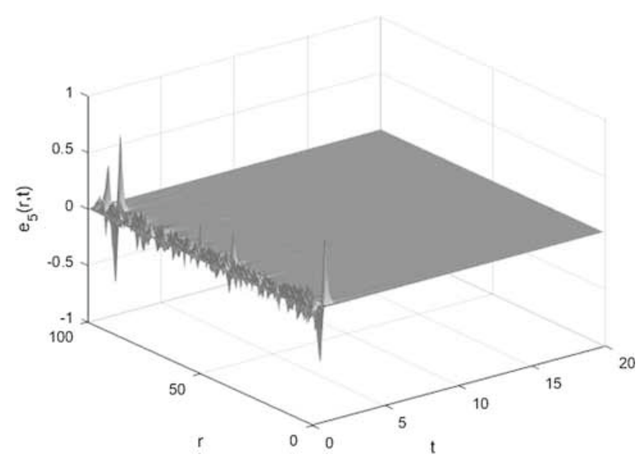

Figure 6. Spatiotemporal evolution of error function $e_5(r,t)$.

As can be seen from the figures, the initial values of the state variables of the two networks are different, and

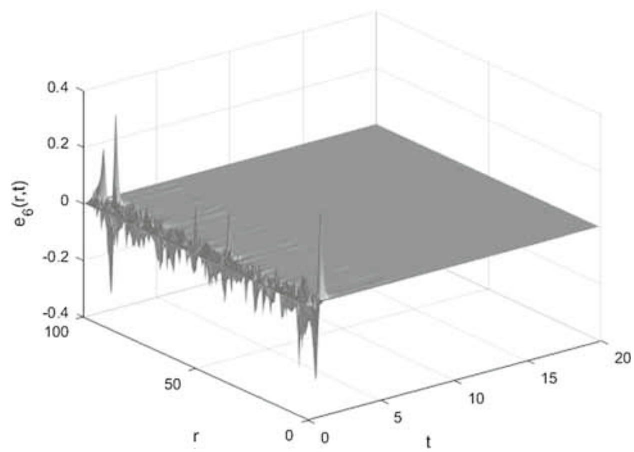

Figure 7. Spatiotemporal evolution of error function $e_6(r, t)$.

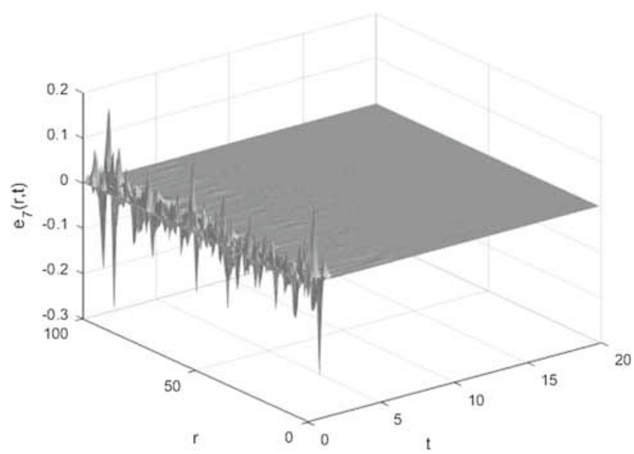

Figure 8. Spatiotemporal evolution of error function $e_7(r, t)$.

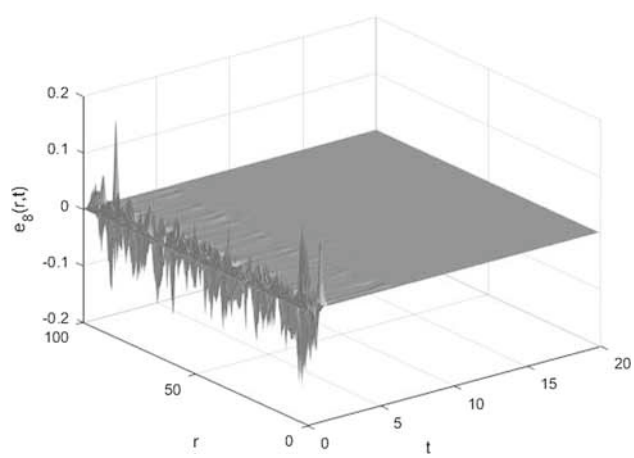

Figure 9. Spatiotemporal evolution of error function $e_8(r, t)$.

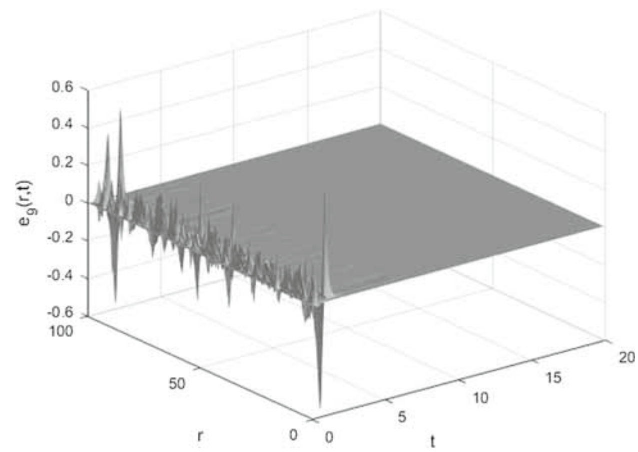

Figure 10. Spatiotemporal evolution of error function $e_9(r, t)$.

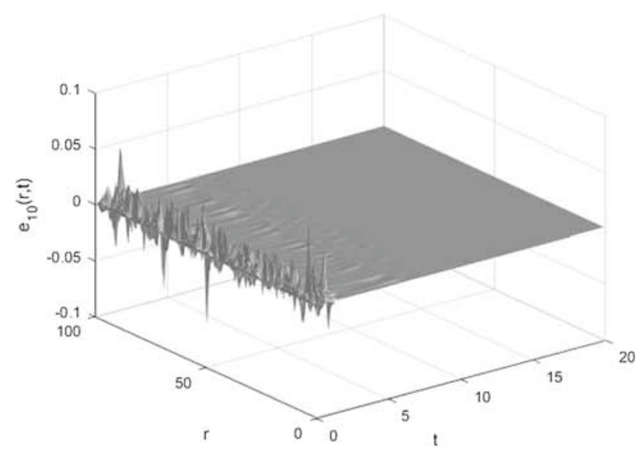

Figure 11. Spatiotemporal evolution of error function $e_{10}(r, t)$.

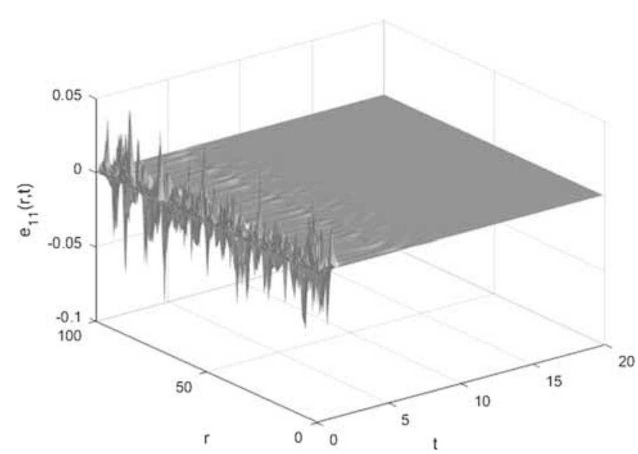

Figure 12. Spatiotemporal evolution of error function $e_{11}(r, t)$.

the error trajectories fluctuate obviously at the beginning stage. However, the error trajectories gradually stabilise and tend to zero under the coupling effect in a finite time, implying that finite-time synchronisation between two spatiotemporal networks is achieved.

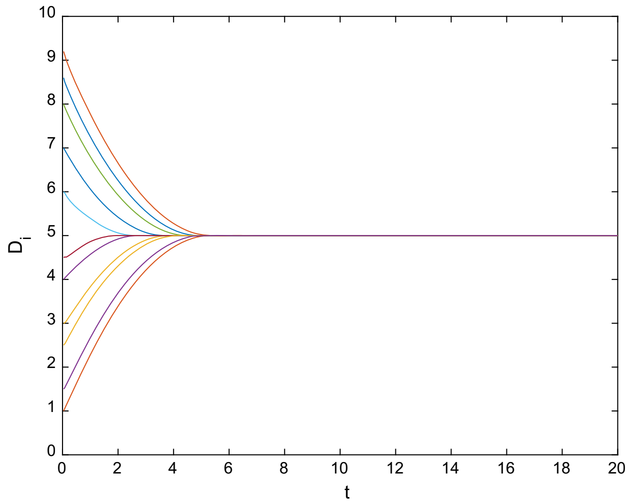

Figure 13. Identification of unknown parameters D_i.

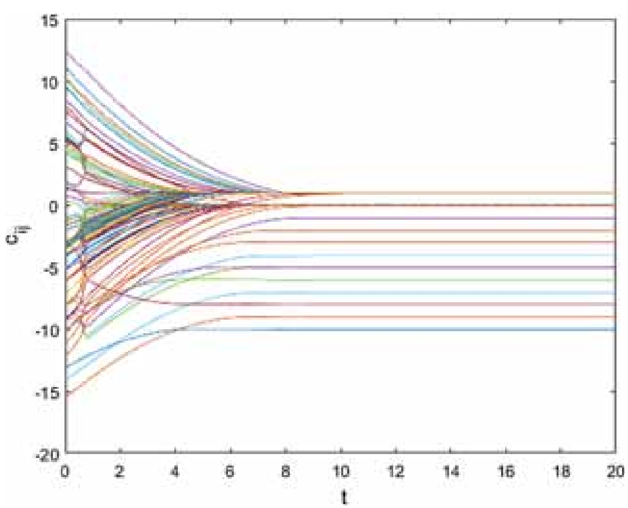

Figure 14. Identification of the topological structure.

The identification process of unknown parameters at any lattice point in space is shown in figure 13. The trajectories of the unknown parameters in the graph coincide rapidly after a period of evolution and tend to true values. Figure 14 shows the identification of unknown topological structures. We can see that the corresponding topological structure can be accurately identified by comparing the identification results of the network connection matrix formula (25). The identification process of unknown coupling coefficient is shown in figure 15. It can be found from the graph that the trajectories of the coupling coefficient gradually stabilise and tend to a certain value after a short oscillation.

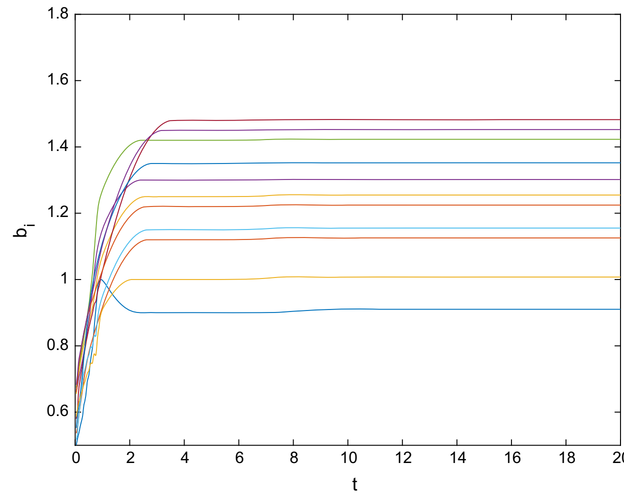

Figure 15. Identification of the coupling coefficient b_i.

4. Conclusion

In this paper, the problem of finite-time synchronisation for uncertain delay spatiotemporal networks via unidirectional coupling technology was studied. The simulation results were given to verify the feasibility of this synchronisation scheme from the aspect of implementation, which indicated that the two spatiotemporal networks can achieve synchronisation in a finite time. Meanwhile, the adaptive estimations of the uncertain coupling coefficient, unknown parameters and uncertain network topology structure were realised. In addition, the designed finite-time synchronisation scheme can be applied to the network of arbitrary topology. In other words, the network synchronisation effect is not affected by the number of nodes and the connection mode of the nodes. Even if the topology is changed, the network can achieve synchronisation in a finite time. This reflects the good synchronisation performance of the network, and this scheme has a certain universality and practicability. In the meantime, the advantages of finite-time synchronisation with strong antijamming and robustness are verified.

The unidirectional coupling technology has many advantages. It not only can be applied to two spatiotemporal networks of the same number of nodes, but also to spatiotemporal networks of different number of nodes. The main research work of this paper is to analyse the finite-time synchronisation of the former, not the latter. Our future work will focus on the construction of spatiotemporal networks with different number of nodes, which has important practical significance for exploring finite-time synchronisation. In addition, this work mainly studied continuous uncertain spatiotemporal networks, but not the finite-time synchronisation of discrete networks. The focus of our future work will be to use this technology to study such networks.

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