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Optical travelling wave solutions for the Biswas–Arshed model in Kerr and non-Kerr law media

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Abstract. This paper scrutinises the newly proposed Biswas–Arshed model for soliton propagation through optical fibres, with small group velocity dispersion and in the absence of self-phase modulation. Spatio-temporal dispersions of higher order are considered to balance with group velocity dispersion. First integral and functional variable methods are employed to recover solitary wave, shock wave, singular wave and singular periodic wave solutions for the two nonlinear forms of the model through Kerr law and power-law nonlinearity. The constraint relations are also figured for the manifestation of these optical solutions.

Keywords. Biswas–Arshed model; first integral method; functional variable method; travelling wave solutions.

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1. Introduction

The channel of optical solitons has been an important issue in several kinds of nonlinear waveguides and nonlinear optics. Solitons are vibrant in numerous fields and are significant in optics by virtue of their growth and control in optical fibres. Other than their various interesting properties, solitons are important in optical continuum generation and can be used to transfer information over very large distances. Optical solitons are important in nonlinear optics and they provide stimulus to work in diverse areas of photonics. These are the basic pulses that dictate the governing dynamics of social media, internet industry and numerous other areas. The investigation of optical solitons is fundamentally vital in nonlinear waveguides and fibre optics. The dynamics of soliton transmission is effectively addressed by different types of nonlinear evolution equations (NLEEs) [1–14]. Recently, Biswas and Arshed projected a very innovative model [15] which deals with low group velocity dispersion (GVD) and small nonlinearity and this model is called as Biswas–Arshed model (BAM). This model is fundamentally important in examining the dynamics of solitons in optical fibres and metamaterials in situations where both GVD and nonlinearity are small. Multiple results have been stated for this model [16–23].

The spotlight of this article is the integrability facet of BAM with power law and Kerr law of nonlinearity. The first integral and functional variable methods are utilised to recover solitary wave, shock wave, singular wave and singular periodic wave solutions for this model. The constraint relations are also listed in the course of the procedure. The details are given in subsequent sections.

2. Review of the methods

In this section, we briefly describe the integration schemes that will be implemented on the governing model to obtain a variety of travelling wave solutions.

2.1 *First integral method*

This method is introduced by Feng [24] and has been effectively applied to formulate exact solutions to a variety of NLEEs [25–29]. We summarise the first integral method in the following steps:

Consider a nonlinear partial differential equation (PDE) of the form

$$G(v, v_t, v_x, v_{tt}, v_{xt}, v_{xx}, \ldots) = 0, \tag{1}$$

where G is a polynomial in $v(x, t)$ and its partial derivatives.

By using the travelling wave transformation $v(x, t) = U(\zeta)$, where $\zeta = x - ct$, eq. (1) is reduced to an ordinary differential equation (ODE)

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$$H(U, -cU', U', c^2U'', -cU'', U'', \ldots) = 0, \tag{2}$$

where

$$U' = \frac{\mathrm{d}U}{\mathrm{d}\zeta}, \quad U'' = \frac{\mathrm{d}^2U}{\mathrm{d}\zeta^2}$$

and so on.

Next, we assume that

$$S(\zeta) = U(\zeta), \qquad T(\zeta) = U'(\zeta), \tag{3}$$

which leads to a system of nonlinear ODEs

$$\begin{cases} S'(\zeta) = T(\zeta), \\ T'(\zeta) = J(S(\zeta), T(\zeta)). \end{cases} \tag{4}$$

By applying Division Theorem for two variables in the complex domain C, we can find the first integral to eq. (4) which condenses eq. (2) into a first-order integrable ODE. By resolving this equation, an exact solution to (1) can be determined.

Division Theorem. *Assume that $P_1(y, z)$ and $P_2(y, z)$ are polynomials in $C[y, z]$, and $P_1(y, z)$ is irreducible in $C[y, z]$. If $P_2(y, z)$ vanishes at all zeros of $P_1(y, z)$, then there exists a polynomial $P_3(y, z)$ in $C[y, z]$ such that*

$$P_2(y, z) = P_1(y, z)P_3(y, z).$$

2.2 *Functional variable method*

This is an effective and direct algebraic method for the generation of solitons and other periodic solutions. It was first introduced by Zerarka *et al* [30] and has been further utilised by many researchers to construct exact solutions to a class of nonlinear wave equations [31–35].

In ODE (2), we make a transformation such that $U(\zeta)$ is considered as a functional variable of the form

$$U_\zeta = F(U). \tag{5}$$

Using eq. (5) and it successive derivatives, ODE (2) can be converted in terms of U, F and its derivatives as

$$L(U, F, F', F'', \ldots) = 0, \tag{6}$$

where

$$F' = \frac{\mathrm{d}F}{\mathrm{d}U}, \quad F'' = \frac{\mathrm{d}^2F}{\mathrm{d}U^2}$$

and so on.

By integrating eq. (6), we obtain an expression for F, which together with eq. (5) give exact solutions to eq. (1).

3. Biswas–Arshed model

In this section, we investigate the BAM through Kerr law and power-law nonlinearities.

3.1 *Form-I (Kerr law nonlinearity)*

The BAM with Kerr law nonlinearity is given by (Form-I [15])

$$\begin{aligned} &iv_t + a_1v_{xx} + a_2v_{xt} + i(b_1v_{xxx} + b_2v_{xxt}) \\ &\quad = i[\phi(|v|^2v)_x + \sigma(|v|^2)_xv + \theta|v|^2v_x]. \end{aligned} \tag{7}$$

Here, $v(x, t)$ is designated for the wave profile of the soliton, a_2 and a_1 are respectively the coefficients of spatio-temporal dispersion (STD) and GVD while b_1 and b_2 are respectively the coefficients of third-order dispersion and STD. On the right side of eq. (7), σ and θ provide the effect of nonlinear dispersion while ϕ provides the effect of self-steepening.

3.1.1 *Mathematical prefaces.*

To begin with the integration process, we consider the transformation

$$v(x, t) = U(\zeta)\mathrm{e}^{i\psi(x,t)}, \tag{8}$$

where $U(\zeta)$ denotes the amplitude portion and

$$\zeta = x - ct, \tag{9}$$

where c is the soliton velocity and the phase component $\psi(x, t)$ is given by

$$\psi(x, t) = -\kappa x + \omega t + \varepsilon. \tag{10}$$

ε is the phase constant, ω is the wave number and κ denotes the frequency of the soliton.

Substituting eqs (8)–(10) into eq. (7) and splitting into real and imaginary parts, we obtain

$$\begin{aligned} &(a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega)U'' \\ &\quad - (a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega)U \\ &\quad = \kappa(\phi + \theta)U^3 \end{aligned} \tag{11}$$

and

$$\begin{aligned} &(b_1 - b_2c)U''' + (b_2c\kappa^2 + 2b_2\omega\kappa - 3b_1\kappa^2 \\ &\quad - c - 2a_1\kappa + a_2c\kappa + a_2\omega)U' \\ &\quad = (3\phi + 2\sigma + \theta)U^2U', \end{aligned} \tag{12}$$

where $' = \mathrm{d}/\mathrm{d}\zeta$.

Integrating eq. (12), we attain

$$3(b_1 - b_2c)U'' + 3(b_2c\kappa^2 + 2b_2\omega\kappa - 3b_1\kappa^2 - c - 2a_1\kappa + a_2c\kappa + a_2\omega)U = (3\phi + 2\sigma + \theta)U^3, \quad (13)$$

where the integration constant is taken to be zero. As the same function $U(\zeta)$ satisfies both (11) and (13), the following constraint relation is retrieved:

$$\frac{3\kappa(\phi+\theta)}{3\phi+2\sigma+\theta} = \frac{-(a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega)}{b_2c\kappa^2+2b_2\omega\kappa-3b_1\kappa^2-c-2a_1\kappa+a_2c\kappa+a_2\omega} = \frac{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}{b_1-b_2c},$$

where

$$a_1 = \frac{-1}{4\kappa^2(\sigma-\theta)^2}[4b_2c\kappa^3\sigma^2-8b_2c\kappa^3\sigma+4b_2c\kappa^3\theta^2 + 6a_2c\kappa^2\sigma\phi-6a_2c\kappa^2\phi\theta-4b_2\omega\kappa^2\sigma^2 + 2a_2c\kappa^2\sigma^2+8b_2\omega\kappa^2\theta\sigma+2a_2c\kappa^2\theta\sigma - 4b_2\omega\kappa^2\theta^2-4a_2c\kappa^2\theta^2-6a_2\omega\kappa\sigma\phi+6a_2\omega\kappa\theta\phi - 6a_2\omega\kappa\sigma^2+6a_2\omega\kappa\theta\sigma-9c\kappa\phi^2-9c\kappa\sigma\phi-9c\kappa\theta\phi + 9\omega\phi^2+15\omega\sigma\phi+3\omega\theta\phi+6\omega\sigma^2+3\omega\theta\sigma],$$

$$b_1 = \frac{1}{8\kappa^3(\sigma-\theta)^2}[8b_2c\kappa^3\sigma^2-16b_2c\kappa^3\sigma\theta+8b_2c\kappa^3\theta^2 + 6a_2c\kappa^2\sigma\phi-6a_2c\kappa^2\phi\theta+4a_2c\kappa^2\sigma^2 - 2a_2c\kappa^2\sigma\theta-2a_2c\kappa^2\theta^2-6a_2\omega\kappa\sigma\phi+6a_2\omega\kappa\theta\phi - 4a_2\omega\kappa\sigma^2+2a_2\omega\kappa\sigma\theta+2a_2\omega\kappa\theta^2 - 9c\kappa\phi^2-6c\kappa\sigma\phi-12c\kappa\theta\phi-6c\kappa\sigma\theta-3c\kappa\theta^2 + 9\omega\phi^2+12\omega\sigma\phi+6\omega\theta\phi+4\omega\sigma^2+4\omega\theta\sigma+\omega\theta^2]. \quad (14)$$

Now, we investigate eq. (11) using first integral and functional variable methods as long as eq. (14).

3.1.2 *First integral approach.* Setting $A = a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega$ and $B = a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega$ in eq. (11), we have

$$AU'' - BU - \kappa(\phi+\theta)U^3 = 0. \quad (15)$$

Using (3) in eq. (15), we have the following two-dimensional autonomous system:

$$\begin{cases} S'(\zeta) = T(\zeta), \\ T'(\zeta) = \left(\frac{B}{A}\right)S(\zeta) + \left(\frac{\kappa(\phi+\theta)}{A}\right)(S(\zeta))^3. \end{cases} \quad (16)$$

Assume that $S(\zeta)$ and $T(\zeta)$ are nontrivial solutions to system (16) and $Q(S,T) = \sum_{i=0}^{M}\alpha_i(S)T^i$ is an irreducible polynomial in the complex domain C such that

$$Q(S(\zeta),T(\zeta)) = \sum_{i=0}^{M}\alpha_i(S(\zeta))T^i(\zeta) = 0, \quad (17)$$

where $\alpha_i(S)$ $(i = 0, 1, ..., M)$ are polynomials of S and $\alpha_M(S) \neq 0$. Equation (17) is the first integral to system (16). Note that $(\mathrm{d}Q/\mathrm{d}\zeta)$ is a polynomial of S and T, and $Q(S(\zeta),T(\zeta)) = 0$ infers that $(\mathrm{d}Q/\mathrm{d}\zeta)|_{(16)} = 0$. By Division Theorem, there exists a polynomial $g(S) + h(S)T$ in the complex domain C such that

$$\frac{\mathrm{d}Q}{\mathrm{d}\zeta} = \frac{\mathrm{d}Q}{\mathrm{d}S}\frac{\mathrm{d}S}{\mathrm{d}\zeta} + \frac{\mathrm{d}Q}{\mathrm{d}T}\frac{\mathrm{d}T}{\mathrm{d}\zeta} = [g(S)+h(S)T]\sum_{i=0}^{M}\alpha_i(S)T^i. \quad (18)$$

Taking $M = 1$ in (17) and using eqs (16) and (18), we get

$$\alpha_0'(S)T + \alpha_1'(S)T^2 + \alpha_1(S)\left(\frac{B}{A}S + \frac{\kappa(\phi+\theta)}{A}S^3\right) = [g(S)+h(S)T][\alpha_0(S)+\alpha_1(S)T]. \quad (19)$$

Comparing the coefficients of T^i $(i = 2, 1, 0)$ in eq. (19), we have

T^2 coeff:

$$\alpha_1'(S) = \alpha_1(S)h(S). \quad (20)$$

T^1 coeff:

$$\alpha_0'(S) = \alpha_1(S)g(S) + \alpha_0(S)h(S). \quad (21)$$

T^0 coeff:

$$\alpha_1(S)\left(\frac{B}{A}S + \frac{\kappa(\phi+\theta)}{A}S^3\right) = \alpha_0(S)g(S). \quad (22)$$

Since $\alpha_i(S)$ $(i = 0, 1)$ are polynomials, from eq. (20) we conclude that $\alpha_1(S)$ is constant and $h(S) = 0$. Let us take $\alpha_1(S) = 1$. Balancing the degrees of $g(S)$ and $\alpha_0(S)$, we have $\deg(g(S)) = 1$ only. Assume that

$$g(S) = A_0 + A_1S, \quad \text{where} \quad A_1 \neq 0. \quad (23)$$

Then, from eq. (21), we obtain

$$\alpha_0(S) = B_0 + A_0 S + \frac{1}{2} A_1 S^2, \tag{24}$$

where B_0 is the constant of integration.

Inserting eqs (23) and (24) into (22) and equating all the coefficients of powers of S to zero, we attain a nonlinear system of equations, which on solving by Maple retrieves

$$A_0 = 0,\ A_1 = \pm\frac{\sqrt{2\kappa\phi + 2\kappa\theta}}{\sqrt{A}},$$
$$B_0 = \mp\frac{B}{\sqrt{A}\sqrt{2\kappa\phi + 2\kappa\theta}}. \tag{25}$$

Using (25) in (17), we get

$$T(\zeta) = \pm\frac{B}{\sqrt{A}\sqrt{2\kappa\phi + 2\kappa\theta}} \mp \frac{\sqrt{\kappa\phi + \kappa\theta}}{\sqrt{2A}} S^2(\zeta). \tag{26}$$

Combining (26) with (16), we attain the exact solutions to eq. (15). The shock wave and singular wave solutions to eq. (7) can then be written as

$$v(x,t) = \pm\sqrt{\frac{B}{\kappa\phi + \kappa\theta}} \tanh\left(\sqrt{\frac{B}{2A}}(x - ct)\right) \times \mathrm{e}^{i(-\kappa x + \omega t + \epsilon)}, \tag{27}$$

$$v(x,t) = \pm\sqrt{\frac{B}{\kappa\phi + \kappa\theta}} \coth\left(\sqrt{\frac{B}{2A}}(x - ct)\right) \times \mathrm{e}^{i(-\kappa x + \omega t + \epsilon)}. \tag{28}$$

These solutions are valid for $AB > 0$.

The solutions (27) and (28) can be reduced to the following singular periodic solutions:

$$v(x,t) = \pm\sqrt{\frac{-B}{\kappa\phi + \kappa\theta}} \tan\left(\sqrt{\frac{-B}{2A}}(x - ct)\right) \times \mathrm{e}^{i(-\kappa x + \omega t + \epsilon)}, \tag{29}$$

$$v(x,t) = \pm\sqrt{\frac{-B}{\kappa\phi + \kappa\theta}} \cot\left(\sqrt{\frac{-B}{2A}}(x - ct)\right) \times \mathrm{e}^{i(-\kappa x + \omega t + \epsilon)}, \tag{30}$$

provided $AB < 0$.

3.1.3 *Functional variable approach.* In this subsection, we utilise the functional variable technique to solve eq. (11).

Assume that

$$U_\zeta = F(U). \tag{31}$$

This will convert eq. (11) to

$$[F^2(U)]' = \frac{2(a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega)}{(a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega)} U + \frac{2\kappa(\phi + \theta)}{(a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega)} U^3. \tag{32}$$

From eq. (32), we obtain the expression for $F(U)$ as

$$F(U) = \sqrt{\frac{a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega}{a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega} U} \times \sqrt{1 - \frac{\kappa(\phi + \theta)}{2(a_2\omega\kappa + b_2\omega\kappa^2 - a_1\kappa^2 - b_1\kappa^3 - \omega)} U^2}. \tag{33}$$

Using eq. (31) in relation (33), the exact solution of eq. (11) can be written in the following form:

$$U(\zeta) = \pm\sqrt{\frac{2(a_2\omega\kappa + b_2\omega\kappa^2 - a_1\kappa^2 - b_1\kappa^3 - \omega)}{\kappa(\phi + \theta)}} \times \mathrm{sech}\left[\sqrt{\frac{a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega}{a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega}}\zeta\right]. \tag{34}$$

Using the travelling wave transformation (8), the following solitary wave and singular wave solutions of eq. (7) can be derived:

$$v(x,t) = \pm\sqrt{\frac{2(a_2\omega\kappa + b_2\omega\kappa^2 - a_1\kappa^2 - b_1\kappa^3 - \omega)}{\kappa(\phi + \theta)}} \mathrm{sech} \times \left(\sqrt{\frac{a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega}{a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega}}(x - ct)\right) \times \mathrm{e}^{i(-\kappa x + \omega t + \epsilon)}, \tag{35}$$

$$v(x,t) = \pm\sqrt{\frac{2(a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega)}{\kappa(\phi + \theta)}} \mathrm{csch} \times \left(\sqrt{\frac{a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega}{a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega}}(x - ct)\right) \times \mathrm{e}^{i(-\kappa x + \omega t + \epsilon)}. \tag{36}$$

The validity condition for the existence of these solutions is given by

$$(a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega)\times(a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega) > 0.$$

Solutions (35) and (36) can be reduced to the following singular periodic solutions:

$$v(x,t)=\pm\sqrt{\frac{2(a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega)}{\kappa(\phi+\theta)}}\times\sec\left(\sqrt{\frac{a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega}{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}}(x-ct)\right)\times e^{i(-\kappa x+\omega t+\epsilon)}, \tag{37}$$

$$v(x,t)=\pm\sqrt{\frac{2(a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega)}{\kappa(\phi+\theta)}}\times\csc\left(\sqrt{\frac{a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega}{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}}(x-ct)\right)\times e^{i(-\kappa x+\omega t+\epsilon)}, \tag{38}$$

provided

$$(a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega)(a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega) < 0.$$

3.2 *Form-II (Power-law nonlinearity)*

Model (7) with power law nonlinearity is given by (Form-II [15])

$$iv_t + a_1v_{xx} + a_2v_{xt} + i(b_1v_{xxx} + b_2v_{xxt}) = i[\phi(|v|^{2n}v)_x + \sigma(|v|^{2n})_xv + \theta|v|^{2n}v_x]. \tag{39}$$

3.2.1 *Mathematical prefaces.* Plugging (8)–(10) into eq. (39) and decomposing into real and imaginary components, we obtain

$$(a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega)U'' - (a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega)U = \kappa(\phi + \theta)U^{2n+1} \tag{40}$$

and

$$(b_1 - b_2c)U''' + (b_2c\kappa^2 - 3b_1\kappa^2 + 2b_2\omega\kappa - 2a_1\kappa + a_2c\kappa - c + a_2\omega)U' = [(2n+1)\phi + 2n\sigma + \theta]U^{2n}U'. \tag{41}$$

Integrating eq. (41) and setting the integration constant to zero, we attain

$$(2n+1)(b_1 - b_2c)U'' + (2n+1)(b_2c\kappa^2 - 3b_1\kappa^2 + 2b_2\omega\kappa - 2a_1\kappa + a_2c\kappa - c + a_2\omega)U = [(2n+1)\phi + 2n\sigma + \theta]U^{2n+1}. \tag{42}$$

The same function $U(\zeta)$ satisfies both (40) and (42), and the following constraint condition is recovered:

$$\frac{(2n+1)\kappa(\phi+\theta)}{(2n+1)\phi+2n\sigma+\theta} = \frac{-(a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega)}{b_2c\kappa^2+2b_2\omega\kappa-3b_1\kappa^2-c-2a_1\kappa+a_2c\kappa+a_2\omega} = \frac{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}{b_1-b_2c},$$

where

$$a_1=\frac{-1}{4n^2\kappa^2(\sigma+\phi)^2}[4b_2c\kappa^3\phi^2n^2+8b_2c\kappa^3\sigma\phi n^2 +4n^2b_2c\kappa^3\sigma^2-4n^2b_2\omega\kappa^2\phi^2+2n^2a_2c\kappa^2\phi^2 -8n^2b_2\omega\kappa^2\sigma\phi+4n^2a_2c\kappa^2\sigma\phi-4n^2b_2\omega\kappa^2\sigma^2 +2n^2a_2c\kappa^2\sigma^2+2na_2c\kappa^2\phi^2+2na_2c\kappa^2\sigma\phi +2na_2c\kappa^2\theta\phi+2na_2c\kappa^2\sigma\theta-6n^2a_2\omega\kappa\phi^2 -12n^2a_2\omega\kappa\sigma\phi-6n^2a_2\omega\kappa\sigma^2-2na_2\omega\kappa\phi^2+\omega\theta^2 -2na_2\omega\kappa\sigma\phi-2na_2\omega\kappa\theta\phi-2na_2\omega\kappa\theta\sigma -3nc\kappa\phi^2-3nc\kappa\sigma\phi-3nc\kappa\theta\sigma+6n^2\omega\phi^2+2\omega\theta\phi +12n^2\omega\sigma\phi+6n^2\omega\sigma^2-c\kappa\phi^2-2c\kappa\theta\phi-c\theta^2\kappa +5n\omega\phi^2+5n\omega\sigma\phi+5n\omega\theta\phi+5n\omega\sigma\theta\omega\phi^2],$$

$$b_1=\frac{1}{8n^2\kappa^3(\sigma+\phi)^2}[8n^2b_2c\kappa^3\phi^2+16n^2b_2c\kappa^3\sigma\phi +8n^2b_2c\kappa^3\sigma^2+4n^2a_2c\kappa^2\phi^2+8n^2a_2c\kappa^2\sigma\phi +4n^2a_2c\kappa^2\sigma^2+2na_2c\kappa^2\phi^2+2na_2c\kappa^2\sigma\phi +2na_2c\kappa^2\theta\phi+2na_2c\kappa^2\sigma\theta-4n^2a_2\omega\kappa\phi^2+\omega\theta^2 -8n^2a_2\omega\kappa\sigma\phi-4n^2a_2\omega\kappa\sigma^2-2na_2\omega\kappa\phi^2 -2na_2\omega\kappa\sigma\phi-2na_2\omega\kappa\theta\phi-2na_2\omega\kappa\sigma\theta+\omega\phi^2 -2nc\kappa\theta\phi-2nc\kappa\theta\sigma+4n^2\omega\phi^2+8n^2\omega\sigma\phi +4n^2\omega\sigma^2-c\kappa\phi^2-2c\kappa\theta\phi-c\kappa\theta^2+4n\omega\phi^2$$

$$+4n\omega\sigma\phi+4n\omega\theta\phi+4n\omega\sigma\theta-2nc\kappa\sigma\phi+2\omega\theta\phi-2nc\kappa\phi^2]. \tag{43}$$

Equation (40) is going to be analysed using functional variable and first integral methods as long as eq. (43) holds.

3.2.2 *Functional variable approach.* In this subsection, we would like to extend the functional variable method to solve the BAM with power-law nonlinearity.

The functional variable transformation given in eq. (31) converts eq. (40) to

$$[F^2(U)]' = \frac{2(a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega)}{(a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega)}U+\frac{2\kappa(\phi+\theta)}{(a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega)}U^{2n+1}. \tag{44}$$

From eq. (44) we obtain the expression for $F(U)$ as

$$F(U)=\sqrt{\frac{a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega}{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}}\times U\sqrt{1-\frac{\kappa(\phi+\theta)}{(n+1)(a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega)}U^{2n}}. \tag{45}$$

Using eq. (31) in relation (45), the exact solution of eq. (40) can be written in the following form:

$$U(\zeta)=\pm\left[\sqrt{\frac{(n+1)(a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega)}{\kappa(\phi+\theta)}}\right]^{1/n}\times\operatorname{sech}^{1/n}\left(n\sqrt{\frac{a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega}{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}}\zeta\right). \tag{46}$$

Using travelling wave transformation (8), the following solitary wave and singular wave solutions of eq. (39) can be derived:

$$v(x,t)=\pm\left[\sqrt{\frac{(n+1)(a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega)}{\kappa(\phi+\theta)}}\right]^{1/n}\times\operatorname{sech}^{1/n}\times\left(n\sqrt{\frac{a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega}{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}}(x-ct)\right)\times e^{i(-\kappa x+\omega t+\epsilon)}, \tag{47}$$

$$v(x,t)=\pm\left[\sqrt{\frac{(n+1)(a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega)}{\kappa(\phi+\theta)}}\right]^{1/n}\times\operatorname{csch}^{1/n}\times\left(n\sqrt{\frac{a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega}{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}}(x-ct)\right)\times e^{i(-\kappa x+\omega t+\epsilon)}. \tag{48}$$

Solutions (47) and (48) are valid for

$$(a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega)(a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega)>0.$$

Solutions (47) and (48) can be reduced to the following singular periodic solutions:

$$v(x,t)=\pm\left[\sqrt{\frac{(n+1)(a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega)}{\kappa(\phi+\theta)}}\right]^{1/n}\times\sec^{1/n}\times\left(n\sqrt{\frac{a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega}{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}}(x-ct)\right)\times e^{i(-\kappa x+\omega t+\epsilon)}, \tag{49}$$

$$v(x,t)=\pm\left[\sqrt{\frac{(n+1)(a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega)}{\kappa(\phi+\theta)}}\right]^{1/n}\times\csc^{1/n}\times\left(n\sqrt{\frac{a_2\omega\kappa+b_2\omega\kappa^2-a_1\kappa^2-b_1\kappa^3-\omega}{a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega}}(x-ct)\right)\times e^{i(-\kappa x+\omega t+\epsilon)}, \tag{50}$$

provided

$$(a_1\kappa^2-a_2\omega\kappa+b_1\kappa^3-b_2\omega\kappa^2+\omega)\times(a_1-a_2c+3b_1\kappa-2b_2c\kappa-b_2\omega)<0.$$

3.2.3 *First integral approach.* To retrieve solutions in closed form, we substitute

$$U = V^{1/2n} \tag{51}$$

in eq. (40). This gives

$$\begin{aligned}&(a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega)\\&\quad\times[(1-2n)(V')^2 + 2nVV'']\\&\quad-4n^2(a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega)V^2\\&\quad-4n^2\kappa(\phi+\theta)V^3 = 0.\end{aligned} \tag{52}$$

Setting

$$A = a_1 - a_2c + 3b_1\kappa - 2b_2c\kappa - b_2\omega$$

and

$$B = a_1\kappa^2 - a_2\omega\kappa + b_1\kappa^3 - b_2\omega\kappa^2 + \omega$$

in eq. (52), we have

$$\begin{aligned}&A[(1-2n)(V')^2 + 2nVV''] - 4n^2BV^2\\&\quad-4n^2\kappa(\phi+\theta)V^3 = 0.\end{aligned} \tag{53}$$

Substituting $S(\zeta) = V(\zeta)$, $T(\zeta) = V'(\zeta)$ in eq. (53), we have the following two-dimensional autonomous system:

$$\begin{cases} S'(\zeta)=T(\zeta),\\ T'(\zeta)=\left(1-\dfrac{1}{2n}\right)\dfrac{T^2}{S} + \dfrac{2n}{AS}(BS^2 + \kappa(\phi+\theta)S^3).\end{cases} \tag{54}$$

By applying the transformation $\mathrm{d}\zeta/\mathrm{d}\eta = S$, system (54) can be rewritten as

$$\begin{cases} \dfrac{\mathrm{d}S}{\mathrm{d}\eta} = ST,\\ [12pt]\dfrac{\mathrm{d}T}{\mathrm{d}\eta}=\left(1-\dfrac{1}{2n}\right)T^2+\dfrac{2n}{A}(BS^2+\kappa(\phi+\theta)S^3).\end{cases} \tag{55}$$

Suppose that $M = 1$ in eq. (17). Using eqs (18) and (55), we obtain

$$\begin{aligned}&\alpha_0'(S)(ST)+\alpha_1'(S)(ST^2)\\&\quad+\alpha_1(S)\left[\left(1-\frac{1}{2n}\right)T^2+\frac{2n}{A}\{BS^2+\kappa(\phi+\theta)S^3\}\right]\\&\quad=[g(S)+h(S)T][\alpha_0(S)+\alpha_1(S)T].\end{aligned} \tag{56}$$

Comparing the coefficients of T^i $(i = 2, 1, 0)$ in eq. (56), we have

T^2 coeff:

$$S\alpha_1'(S) = \alpha_1(S)\left\{h(S) - \left(1-\frac{1}{2n}\right)\right\}. \tag{57}$$

T^1 coeff:

$$S\alpha_0'(S) = \alpha_1(S)g(S) + \alpha_0(S)h(S). \tag{58}$$

T^0 coeff:

$$\alpha_1(S)\left[\frac{2n}{A}\{BS^2 + \kappa(\phi+\theta)S^3\}\right] = \alpha_0(S)g(S). \tag{59}$$

As $\alpha_i(S)$ $(i = 0, 1)$ are polynomials, from eq. (57) we conclude that $\alpha_1(S)$ is constant and

$$h(S) = \left(1-\frac{1}{2n}\right).$$

Let us take $\alpha_1(S)=1$. Balancing the degrees of $g(S)$ and $\alpha_0(S)$, we deduce that $\deg(g(S)) = 1$ and $\deg(\alpha_0(S)) = 2$. Assume that

$$g(S) = A_0 + A_1S \tag{60}$$

and

$$\alpha_0(S) = B_0 + B_1S + B_2S^2, \tag{61}$$

where $A_1 \neq 0$ and $B_2 \neq 0$.

Replacing eqs (60) and (61) into (58), we attain

$$A_0 = \left(\frac{1}{2n} - 1\right)B_0 \quad\text{and}\quad A_1 = \frac{B_1}{2n}. \tag{62}$$

Inserting $\alpha_0(S)$, $\alpha_1(S)$ and $g(S)$ into (59) and equating all the coefficients of powers of S to zero, we attain a nonlinear system of equations, which on solving by Maple retrieves

$$B_0=0,\quad B_1=\pm 2n\sqrt{\frac{B}{A}},\quad B_2=\mp\frac{2n\kappa(\phi+\theta)}{\sqrt{AB}}. \tag{63}$$

Using (63) in (17), we attain

$$T(\zeta)=\mp 2n\sqrt{\frac{B}{A}}S(\zeta) \pm \frac{2n\kappa(\phi+\theta)}{\sqrt{AB}}S^2(\zeta). \tag{64}$$

Combining (64) with (54), we attain exact solution to eq. (53). The shock wave and singular wave solutions to eq. (39) can then be written as

$$v(x,t) = \left[\pm\frac{B}{2\kappa(\phi+\theta)}\left\{1\pm\tanh\left(\frac{n\sqrt{B}}{\sqrt{A}}(x-ct)\right)\right\}\right]^{1/2n} \times e^{i(-\kappa x+\omega t+\epsilon)}, \tag{65}$$

$$v(x,t) = \left[\pm\frac{B}{2\kappa(\phi+\theta)}\left\{1\pm\coth\left(\frac{n\sqrt{B}}{\sqrt{A}}(x-ct)\right)\right\}\right]^{1/2n} \times e^{i(-\kappa x+\omega t+\epsilon)}. \tag{66}$$

The validity condition for the existence of these solutions is $AB > 0$.

4. Conclusion

In this work, with the aid of first integral and functional variable methods, we successfully retrieved solitary wave, shock wave, singular wave and singular periodic wave solutions in the presence of restraint conditions, to the newly proposed BAM in the absence of self-phase modulation. Two nonlinear forms, Kerr and power-law forms, of this model are investigated using these integration techniques. By comparing our solutions with the solutions given in [15–18,21–23], we conclude that apart from a few, all the other solutions are new and valuable to this subject. These solutions will be exceptionally useful for future studies.

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