Pramana – J. Phys. (2020) 94:24
https://doi.org/10.1007/s12043-019-1892-2

Magnetohydrodynamic creeping flow around a weakly permeable spherical particle in cell models

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MS received 15 February 2019; revised 23 September 2019; accepted 10 October 2019

Abstract. The present paper studies the impact of applied uniform transverse magnetic field on the flow of incompressible conducting fluid around a weakly permeable spherical particle bounded by a spherical container. Analytical solution of the problem is obtained using Happel and Kuwabara cell models. The concerned flow is parted in two regions, bounded fluid region and internal porous region, to be governed by Stokes and Darcy's law respectively. At the interface between the fluid and the permeable region, the boundary conditions used are continuity of normal component of velocity, Saffman's boundary condition and continuity of pressure. For the cell surface, Happel and Kuwabara models together with continuity in radial component of the velocity has been used. Expressions for drag force, hydrodynamic permeability and Kozeny constant acting on the spherical particle under magnetic effect are presented. Representation of hydrodynamic permeability for varying permeability parameters, particle volume fraction, slip parameter and Hartmann numbers are represented graphically. Also, the magnitude of Kozeny constant for weakly permeable and semipermeable sphere under a magnetic effect has been presented. In limiting cases many important results are obtained.

Keywords. Weakly permeable spherical particle; Stokes flow; Darcy's law; Hartmann number; drag force; cell model.

PACS Nos **47.15.G−; 47.56.+r; 47.65.−d**

1. Introduction

Area of flow through porous medium is of great importance and so has attracted the interest of many researchers over the years. Its application is in diverging fields of science including chemical, biomedical, industrial, meteorology, environmental engineering and many more. Weakly permeable formulations are generally of low permeability. Some examples include flow through porous fluidisation, flow of a fluid mixture, flow in packed beds, sedimentation, fall of snow-flakes etc. The type of low permeability porous membrane which permits only specific types of particles to move through it is known as semipermeable membrane. This type of membrane allows only small particles like ions and molecules of water to pass through it. Osmosis is a phenomenon in which the diffusion of water takes place through a semipermeable membrane from an area of high concentration to an area of low concentration. It is seen in plants and animals.

Study of creeping flow through porous particles of varying geometry is generally modelled using Stokes equation for the flow of fluid and Darcy law [1] or Brinkman's equation [2] for the flow in porous region. Darcy's law is applicable for the flow through media having low porosity, whereas Brinkman's equation is used for the flow through media having higher porosity. Earlier, while studying Darcy flow problems, slip boundary conditions have not been extensively used. Later, Beavers and Joseph [3] and Saffman [4] proposed slip boundary conditions. Joseph and Tao [5] discussed the motion of viscous fluid past a porous body with spherical structure by using Darcy's law for flow in porous region. Subsequently, Singh and Gupta [6] have showed the dependence of coefficient of drag on the permeability. Shapovalov [7] has solved the problem of flow around a semipermeable particle using the same boundary conditions as in [5] and showed that the drag on a semipermeable sphere is lower than the drag on a nonpermeable sphere.

Published online: 03 January 2020

Flow past a cluster of particles occurs in diverse circumstances from sedimentation problem to filtration of membrane and also in the rheology of suspensions. Nevertheless, it is very difficult to formulate these types of problems mathematically as we have to analyse the complex interaction of numerous particles. For this purpose, unit cell model technique was introduced [8]. It is among the most effective technique for exploring the behaviour of concentrated disperse system and porous media. The concept involved in cell model technique is that a complete assemblage is partitioned into various similar cells and each cell carries a particle covered by a fluid wrap. By this technique, the problem of large number of particles is now reduced to the problem of single particle with a fluid wrap. It is one of the most important advantage of using cell model. In cell model technique we can neglect the effect of container walls. Cells of varying shapes can be used, but the use of spherically shaped cell is more convenient. On the cell surface, the boundary condition must consider the impact of neighbouring particles, and so it has to be discussed in details. In order to report the influence of neighbouring particles on the particle entered at the cell different boundary conditions are proposed for cell surface. This concept was initially put forward by Happel [9] and Kuwabara [10] which leads to almost similar results. However, they followed different approaches in terms of boundary conditions. The model proposed by Happel assumes zero shear stress condition on the external cell surface, whereas model proposed by Kuwabara assumes zero vorticity on the outer cell surface. Kuwabara model considers the interchange of mechanical energy within the cell and the surrounding but in the case of Happel model there is no such interchange of energy between the cell and the environment. Cell model technique is extensively used for solving the boundary value problems for particles in dense system. Previously, Dassios *et al* [11] investigated flow past a spheroidal particle in bounded medium using Happel and Kuwabara cell model. Vainshtein *et al* [12] handled the problem of creeping flow past and within a permeable spheroidal particle. Ramkissoon and Rahaman [13] analysed the motion of a solid spherical particle in a spheroidal vessel and obtained the drag force acting on the inner sphere. Srinivasacharya [14] focussed on the motion of a porous sphere in a spherical vessel by using Brinkman's equation for the porous region. Senchenko and Keh [15] studied flow past a slight deformed sphere. The study of hydrodynamic permeability of membranes comprising spherical porous shell was considered by Yadav *et al* [16]. Prakash *et al* [17] examined Stokes flow past a cluster of porous particle for both Brinkamn's and Darcy's law separately and compared the overall bed permeability. Saad [18] considered the axisymmetric Stokes flow around a porous spheroidal particle in a concentric spheroidal container and evaluated the drag force acted on the bounded spheroid. Prasad and Kaur [19] described the flow of spheroidal droplet containing micropolar fluid. Thus, cell technique is presently very popular among scientific community and researchers. By using Beavers–Joseph–Saffman–Jones condition, Prasad and Bucha [20] demonstrated the viscous flow past a permeable particle of spheroidal geometry. Flow of micropolar fluid past a spherical particle implanted in a porous media was discussed by Krishnan and Shukla [21]. Recently, Khanukaeva *et al* [22] examined the motion of micropolar fluid in a direction parallel to porous cylindrical cells by using two different boundary conditions on the surface of the cell. They concluded that the hydrodynamic permeability on the cells weakly depends on the boundary conditions.

Wide range of problems including flow past different particles are solved and important results are obtained. Recently, research has been carried out considering the magnetohydrodynamics (MHD) flow. Researchers are concentrated on observing the impact of applied magnetic field on the motion of fluid past different bodies. The significance of introducing magnetic field on the nature of flow can clearly be seen in biological fluid. These types of fluids in action with MHD are known as biomagnetic fluids. Application of such fluids can be seen in the transport of drugs to marked zone, treatment of tumours, cancer etc. During the literature survey, we observed that Globe [23] and Gold [24] found the influence of magnetic field on the flow past channels and pipes. Later, Rudraiah *et al* [25] discussed the Hartmann flow past a permeable bed under the action of magnetic forces. For determining flow rate with respect to Hartmann number, Mazumdar *et al* [26] considered uniform magnetic field of constant intensity. Haldar and Ghosh [27] solved fluid flow past a cylindrical tube in order to find the effect of Hartmann number. Cramer and Pai [28], Davidson [29], Geindreau and Aurialt [30], Verma and Datta [31], Jayalakshmamma *et al* [32], Srivastava and Deo [33], Verma and Singh [34] etc. have studied the effect of magnetic field on fluid flow rate. Influence of MHD on hydrodynamic permeability of a porous membrane consisting of spherical particles was studied by Srivastava *et al* [35]. Yadav *et al* [36] studied the effect of MHD on the hydrodynamic permeability of a membrane comprising spherical particles having porous shell. Saad [37] has examined the effect of magnetic field on the flow past a porous sphere and cylinders surrounded by a cell. Recently, Kumar *et al* [38] examined the analysis of entropy generation concept for unsteady MHD Jeffrey fluid flow over a semi-infinite vertical flat plate. Kashyap *et al* [39] studied the effects of Soret and variable porosity on an unsteady MHD flow of an upper convected Maxwell fluid through an expanding or contracting channel. Prasad and Bucha [40] investigated the MHD effect on the motion of fluid through a semipermeable spherical particle and have obtained an explicit expression of drag. Subsequently, MHD flow past a cylindrical shell using Brinkman's model for the parallel flow problem has been analysed by Prasad and Bucha [41]. The aforementioned studies motivated us to study flow past a weakly permeable sphere in the cell model under the influence of magnetic field.

In this paper, we aim to observe the influence of applied uniform magnetic field on a weakly permeable sphere in a unit cell model. Applicable boundary conditions are the continuity of normal component of velocity, Saffman's slip boundary condition, continuity of pressure and for the cell surface Happel and Kuwabara model along with continuity of radial component of velocity. Region I denotes the porous structure and region II denotes the external flow region bounded by cell surface. Stokes equation and Darcy's law are used for flow in regions I and II, respectively. Stream functions and pressures for both the regions are calculated. Explicit expression for drag on the external surface of the weakly permeable sphere is evaluated. Magnitude of hydrodynamic permeability for different parameters including permeability, volume fraction, slip parameter and Hartmann numbers are shown by graphs. The variation of Kozeny constant for varying geometry is shown in the table.

2. Problem formulation

Consider the conducting axisymmetric fluid flow past a weakly permeable sphere having radius $r = b$ held in a spherical vessel of radius $r = a$ moving with a uniform velocity (U) as shown in figure 1. The outer fluid and inner porous regions are symbolised by i, where $i = 1, 2$ respectively.

We assume the following conditions:

1. Flow is slow and steady.
2. Reynolds number $\mathrm{Re} = Ub/\nu$ is considered to be sufficiently small so that the flow is governed by Stoke's law. Here ν is the kinematic viscosity of the fluid.
3. A uniform magnetic field is considered in transverse direction of the flow, i.e., $\vec{H}^{(i)} = H_o\vec{e}_\phi$, $i = 1, 2$.
4. Magnetic Reynolds $\mathrm{Re_m} = Ub\mu_h\sigma$ is assumed to be very small. Here, σ is the electric conductivity of the fluid and μ_h is the magnetic permeability of the fluid.
5. The magnetic permeability (μ_h) is assumed to be the same for liquid and porous region.

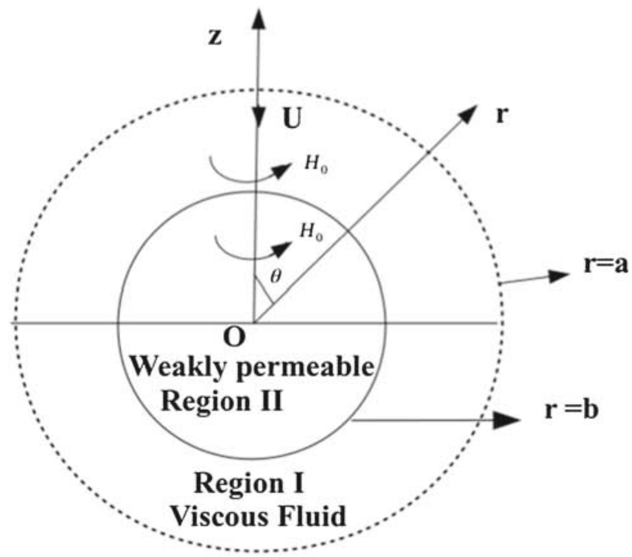

Figure 1. Geometry of weakly permeable sphere in bounded medium under the MHD effect.

6. External electric field is absent.
7. Induced magnetic field is neglected.
8. Magnetic force (called Lorentz force) $\vec{F}$ which is defined as

$$\vec{F} = \mu_h\vec{J} \times \vec{H}, \tag{1}$$

where $\vec{J}$ and $\vec{H}$ are the electric current density and the magnetic field intensity respectively. Thus,

$$\vec{F} = \mu_h^2\sigma(\vec{q} \times \vec{H}) \times \vec{H}. \tag{2}$$

Lorentz force balances pressure and viscous stresses in order to obtain modified Stokes and Brinkman's equation. Hence, the flow in region I is governed by modified Stokes equation [8,32,35,37,40,41] and flow in region II is governed by the modified Darcy's law [1,25,30,40].

$$\nabla \cdot \vec{q}^{\,(1)} = 0, \tag{3}$$

$$\nabla p^{(1)} + \mu\nabla \times \nabla \times \vec{q}^{\,(1)} - \mu_h^2\sigma_1(\vec{q}^{\,(1)} \times \vec{H}^{(1)}) \times \vec{H}^{(1)} = 0 \tag{4}$$

and

$$\nabla \cdot \vec{q}^{\,(2)} = 0, \tag{5}$$

$$\nabla p^{(2)} + \frac{\mu}{k}\vec{q}^{\,(2)} - \frac{\mu_h^2\sigma_2}{\varepsilon}(\vec{q}^{\,(2)} \times \vec{H}^{(2)}) \times \vec{H}^{(2)} = 0, \tag{6}$$

where $\vec{q}^{\,(i)}$ is the velocity vector, $p^{(i)}$ is the pressure, μ is the coefficient of viscosity, $\vec{H}^{(i)}$ is the magnetic field intensity, σ_i is the electric conductivity in both the regions and k, ε are the permeability and porosity of the porous region respectively.

Non-dimensionalising variables are introduced in order to convert the governing equations into dimensionless form. So, consider

$$r = b\tilde{r}, \quad \vec{q}^{(i)} = U\tilde{\vec{q}}^{(i)},$$

$$\nabla = \frac{\tilde{\nabla}}{b}, \quad p^{(i)} = \frac{\mu U}{b}\tilde{p}^{(i)}, \quad \vec{H}^{(i)} = H_0\tilde{\vec{H}}^{(i)}. \tag{7}$$

Substituting them in eqs (3)–(6) and then dropping the tildes, we get

$$\nabla \cdot \vec{q}^{(1)} = 0, \tag{8}$$

$$\nabla p^{(1)} + \nabla \times \nabla \times \vec{q}^{(1)} - \alpha^2(\vec{q}^{(1)} \times \vec{H}^{(1)}) \times \vec{H}^{(1)} = 0, \tag{9}$$

$$\nabla \cdot \vec{q}^{(2)} = 0, \tag{10}$$

$$\nabla p^{(2)} + \xi_1^2\,\vec{q}^{(2)} - \xi_2^2(\vec{q}^{(2)} \times \vec{H}^{(2)}) \times \vec{H}^{(2)} = 0, \tag{11}$$

where

$$\alpha = \sqrt{\frac{\mu_h^2 H_0^2 \sigma_1 b^2}{\mu}}, \quad \xi_2 = \sqrt{\frac{\mu_h^2 H_0^2 \sigma_2 b^2}{\varepsilon\mu}}$$

are the Hartmann numbers for regions I and II respectively. $\beta^2 = \xi_1^2 + \xi_2^2$ where $\xi_1^2 = b^2/k$ and $k_1 = 1/\xi_1^2$ are the permeability parameters in dimensionless form.

Consider (r, θ, ϕ) as a spherical polar coordinate system. Since the motion of the fluid flow is axially symmetric, all the quantities engaged in fluid flow are independent of ϕ. Thus, the velocity vectors are denoted as

$$\vec{q}^{(i)} = q_r^{(i)}(r,\theta)\,\vec{e}_r + q_\theta^{(i)}(r,\theta)\,\vec{e}_\theta, \quad i = 1, 2. \tag{12}$$

Introduce stream functions $\psi^{(i)}$, $i = 1, 2$ for the flow in external and porous region respectively. The velocity components in terms of stream function are given as

$$q_r^{(i)} = -\frac{1}{r^2\sin\theta}\frac{\partial\psi^{(i)}}{\partial\theta},$$

$$q_\theta^{(i)} = \frac{1}{r\sin\theta}\frac{\partial\psi^{(i)}}{\partial r}, \quad i = 1, 2. \tag{13}$$

Using eq. (13) in eqs (9) and (11) and eliminating pressure, we get the equations

$$E^2(E^2 - \alpha^2)\psi^{(1)} = 0 \tag{14}$$

and

$$E^2\psi^{(2)} = 0 \tag{15}$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}$$

is the Stokes stream function operator.

3. Boundary conditions

In order to identify the flow velocity component outside and inside the permeable sphere we use appropriate boundary conditions. Boundary conditions used for the present problem are the continuity of normal component of velocity, Saffman's slip boundary condition [4,42] along with continuity of pressure. At the cell surface of the fluid envelope, two different boundary conditions, Happel and Kuwabara, together with the continuity of radial component of velocity are used.

Mathematically, boundary conditions at the fluid–porous interface of the sphere $r = b$ are

$$q_r^{(1)} = q_r^{(2)}, \tag{16}$$

$$\frac{\partial q_\theta^{(1)}}{\partial r} = \frac{1}{\lambda\sqrt{k}}\,q_\theta^{(1)}, \tag{17}$$

$$p^{(1)} = p^{(2)}, \tag{18}$$

where λ is the non-dimensional Beaver–Joseph–Saffman slip coefficient and it depends on the nature of the porous medium. As the range of $1/\lambda$ is chosen to be $0.1 < 1/\lambda < 4$ [3,4,43], λ lies between 0.25 and 10 [20,44]. The case $\lambda = 0$ for extremely low permeability indicates the problem of flow past a semipermeable particle.

Boundary condition at the cell surface $r = a$ are

$$q_r^{(1)} + U\cos\theta = 0. \tag{19}$$

Happel model:

$$t_{r\theta}^{(1)} = 0. \tag{20}$$

Kuwabara model:

$$\nabla \times \vec{q}^{(1)} = 0. \tag{21}$$

Here, $t_{r\theta}^{(1)}$ is the tangential stress for region I.

The boundary conditions on the spherical surface $r = 1$ in terms of stream functions $\psi^{(i)}$, where $i = 1, 2$ are

$$\frac{\partial\psi^{(1)}}{\partial\theta} = \frac{\partial\psi^{(2)}}{\partial\theta}, \tag{22}$$

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial \psi^{(1)}}{\partial r}\right] = \frac{\xi_1}{\lambda}\left[\frac{1}{r}\frac{\partial \psi^{(1)}}{\partial r}\right], \tag{23}$$

$$p^{(1)} = p^{(2)}, \tag{24}$$

and the boundary condition at the surface of the cell $r = \eta^{-1}$ with $\eta = b/a$ are

$$\frac{\partial \psi^{(1)}}{\partial \theta} - r^2 \cos\theta \sin\theta = 0. \tag{25}$$

Happel model:

$$2r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi^{(1)}}{\partial r}\right) - E^2\psi^{(1)} = 0. \tag{26}$$

Kuwabara model:

$$E^2\psi^{(1)} = 0. \tag{27}$$

4. Solution of the problem

Solutions for regions I and II obtained by solving eqs (14) and (15) are given as

$$\psi^{(1)} = \frac{1}{2}\left[A r^2 + \frac{B}{r} + C\sqrt{r}\, K_{3/2}(\alpha r) + D\sqrt{r}\, I_{3/2}(\alpha r)\right]\sin^2\theta, \tag{28}$$

$$\psi^{(2)} = \frac{1}{2}E r^2 \sin^2\theta. \tag{29}$$

The pressure acting on both the flow regions are

$$p^{(1)} = \alpha^2\left(A r - \frac{B}{2r^2}\right)\cos\theta, \tag{30}$$

$$p^{(2)} = \beta^2 E r \cos\theta, \tag{31}$$

where $I_{3/2}(\alpha r)$ and $K_{3/2}(\alpha r)$ represent the modified Bessel function of the first and second kinds of order 3/2 respectively.

5. Drag force acting on the body

The drag force acted on the weakly permeable sphere under the magnetic effect can be obtained by using the formula [8,37]

$$F_D = \pi\mu U b\int_0^{\pi}\left[r^4\sin^3\theta\frac{\partial}{\partial r}\left(\frac{E^2\psi^{(1)}}{r^2\sin^2\theta}\right) - \alpha^2 r^2\sin\theta\frac{\partial\psi^{(1)}}{\partial r}\right]_{r=1}\mathrm{d}\theta.$$

Using eq. (28) in the above integral, we obtain

$$F_D = \frac{2}{3}\pi\mu b U\alpha^2(B - 2A - 2CS_2 - 2DU_2). \tag{32}$$

Happel model:

$$F_D = 4\pi\mu b U\left[\frac{\alpha((4W_5\alpha^3\eta^4 - \Delta_4\beta^2\eta^{0.5})\xi_1 - (\Delta_5\eta^{0.5} - 4W_5\alpha^3\eta^4)\lambda)}{(\Delta_3\eta^{0.5} - 12W_{10}\beta^2\eta^3)\xi_1 + (\Delta_6\eta^{0.5} + \Delta_7)\lambda}\right]. \tag{33}$$

Kuwabara model:

$$F_D = -4\pi\mu b U\left[\frac{\alpha^2(W_4\beta^2\xi_1 + \Delta_1\lambda)}{(2W_{13}\eta^3 + \alpha W_{14})\xi_1 - \Delta_2\lambda}\right], \tag{34}$$

where the expressions for $S_i, U_i, i = 1$–4; $W_i, i = 1$–24; $\Delta_i, i = 1$–8 are given in Appendix.

5.1 *Specific results*

Bounded medium:

Case I. If $\lambda = 0$ in eqs (33) and (34), the case of flow past a semipermeable sphere is obtained. The drag reduces to

Happel model:

$$F_D = 4\pi\mu b U\left[\frac{\alpha(4W_5\alpha^3\eta^4 - \Delta_4\beta^2\eta^{1/2})}{\Delta_3\eta^{1/2} - 12W_{10}\beta^2\eta^3}\right]. \tag{35}$$

Kuwabara model:

$$F_D = -4\pi\mu b U\left[\frac{W_4\alpha^2\beta^2}{2W_{13}\eta^3 + \alpha W_{14}}\right]. \tag{36}$$

Case II. If $\alpha = 0$ and $\xi_2 = 0$, i.e., there is no magnetic effect in both the regions, in eqs (35) and (36), the drag in the absence of magnetic effect is given as

Happel model:

$$F_D = 4\pi\mu U b \times\left[\frac{2(\xi_1^2 - 10)\eta^5 + 3\xi_1^2}{2(\xi_1^2-10)\eta^6+3(2-\xi_1^2)\eta^5+\xi_1^2(3\eta-2)-1}\right]. \tag{37}$$

Kuwabara model:

$$F_D = 4\pi\mu U b \times \left[\frac{15\xi_1^2}{2(\xi_1^2-10)\eta^6-10(\xi_1^2+2)\eta^3+2\xi_1^2(9\eta-5)-5}\right]. \tag{38}$$

Case III. If $\xi_1 \to \infty$, i.e., permeability $k \to 0$, in eqs (33) and (34), the drag on the solid sphere under the magnetic effect is given as

Happel model:

$$F_D = -2\pi\mu bU\left[\frac{(2\alpha\eta W_9 + W_4 Z)\alpha}{\Delta_8}\right]. \tag{39}$$

Kuwabara model:

$$F_D = 2\pi\mu bU\left[\frac{W_4\alpha^2}{W_4\eta^3 - W_1\alpha}\right] \tag{40}$$

which is similar to the results obtained by Saad [37].

Case IV. If $\xi_1 \to \infty$ in eqs (37) and (38), the drag on the solid sphere without magnetic effect is given as

Happel model:

$$F_D = 4\pi\mu\, bU\left[\frac{2\eta^5+3}{2\eta^6-3\eta^5+3\eta-2}\right]. \tag{41}$$

Kuwabara model:

$$F_D = 6\pi\mu bU\left[\frac{5}{\eta^6-5\eta^3+9\eta-5}\right] \tag{42}$$

which agrees with the result of Saad [18].

Unbounded medium: As $a \to \infty$ or $\eta = 0$, we have the drag force acting on the weakly permeable sphere in an unbounded medium.

Case I. In the presence of magnetic field

$$F_{\text{inf}} = -\frac{4\pi\mu bU[(\alpha^2+3\alpha+3)\,\beta^2\,\xi_1 + \delta_1\lambda]}{(2\beta^2+\alpha^2+\alpha+1)\xi_1 + \delta_2\lambda}, \tag{43}$$

where

$$\delta_1 = (\alpha^3+2\alpha^2+3\alpha+3)\beta^2 - 2\alpha^2(\alpha+1),$$
$$\delta_2 = 2(\alpha+2)\beta^2 + \alpha^3 + 2\alpha^2 + 3\alpha + 3.$$

Case II. If $\lambda = 0$ in eq. (43), the drag force acting on a semipermeable sphere under the magnetic field

$$F_{\text{inf}} = -\frac{4\pi\mu bU(\alpha^2+3\alpha+3)\beta^2}{2\beta^2+\alpha^2+\alpha+1} \tag{44}$$

which is similar to the results obtained by Prasad and Bucha [40].

Case III. If the magnetic field is absent, i.e., $\alpha = 0$ and $\xi_2 = 0$ in eq. (43), the drag reduces to

$$F_{\text{inf}} = -6\pi\mu Ub\left[\frac{2\xi_1^2(\xi_1+\lambda)}{(2\xi_1^2+1)\xi_1 + (4\xi_1^2+3)\lambda}\right]. \tag{45}$$

Case IV. If $\lambda = 0$ in eq. (45), the drag reduces to

$$F_{\text{inf}} = -6\pi\mu Ub\left[\frac{2\xi_1^2}{2\xi_1^2+1}\right] \tag{46}$$

which is identical to the result obtained by Shapovalov [7].

Case V. If $\xi_1 \to \infty$, i.e., permeability $k = 0$ in eq. (44), it behaves as solid sphere in the presence of the magnetic field and the drag is

$$F_{\text{inf}} = -2\pi\mu Ub(\alpha^2+3\alpha+3) \tag{47}$$

which is identical to the results of Prasad and Bucha [40].

Case VI. If $\xi_1 \to \infty$ in (46), the drag is

$$F_{\text{inf}} = -6\pi\mu Ub \tag{48}$$

which is the famous expression known as Stokes drag past a solid sphere [8].

6. Graphical representation and tabulation

6.1 *Hydrodynamic permeability*

Hydrodynamic permeability (L_{11}) that represents one of the element of the Onsager matrix [45], is an important quantity while studying membrane filtration process. The ratio of the uniform flow rate (U) and the cell gradient pressure (F_D/V) is defined as the hydrodynamic permeability of a membrane under the magnetic effect [8,16,35,36].

$$L_{11} = \frac{U}{F_D/V}, \tag{49}$$

where $V = (4/3)\pi a^3$ is the cell volume. Also, the cell radius (a) is so selected that the volume fraction of the particle inside the cell (γ^3) is identical to the volume fraction in the real membrane, and so

$$\gamma^3 = \left(\frac{b}{a}\right)^3 = 1 - \varepsilon \tag{50}$$

with ε as the outer porosity or fractional void volume of the membrane [17,42,46,47] and $1 - \varepsilon$ is known as the volume fraction of the solid particle [17,43,46,47].

Using eq. (32) in the above equation, we have

$$L_{11} = \left[\frac{2b^2}{\mu\gamma^3\alpha^2(B - 2A - 2CS_2 - 2DU_2)}\right]. \tag{51}$$

In non-dimensional form, we have

$$L_{11} = \left[\frac{2}{\gamma^3\alpha^2(B - 2A - 2CS_2 - 2DU_2)}\right]. \tag{52}$$

Hydrodynamic permeability is a function of eight arguments, i.e., $L_{11}(\gamma, \alpha, \beta, \lambda, \eta, k_1, \xi_1, \xi_2)$, in which the parameters β, η, ξ_1 are dependent function as $\xi_1^2 = b^2/k$, $\beta^2 = \xi_1^2 + \xi_2^2$, $k_1 = 1/\xi_1^2$ and $\eta = \gamma$.

Representation of the natural logarithm of hydrodynamic permeability (L_{11}) against different parameters are shown in figures 2–6 for both Happel and Kuwabara models. Various parameters involved in the flow of the weakly permeable spherical particle are: permeability parameter (k_1), particle volume fraction (γ), slip parameter (λ), Hartmann numbers (α) and ξ_2.

Figure 2 shows the plot of natural logarithm of L_{11} against k_1 for varying values of α. It shows that hydrodynamic permeability is an increasing function of permeability which indicates that increasing permeability ease the fluid flow through the membrane. However, with increasing α, ln L_{11} decreases and the difference between both the models also decreases.

Figure 3 depicts the variation of ln L_{11} against γ for varying values of α for the fluid region. We observe that for a fixed value of α, hydrodynamic permeability decreases with increasing values of γ. Similar nature of flow is shown by Srivastava *et al* [35] and Yadav *et al* [36].

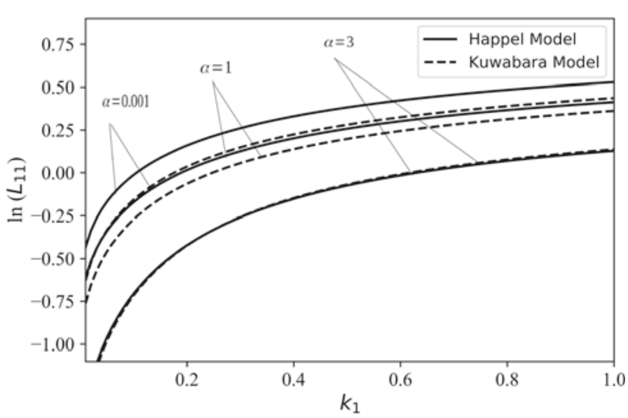

Figure 2. Natural logarithm of L_{11} against k_1 when $\gamma = 0.5$, $\xi_2 = 2$, $\lambda = 1$ for different values of α.

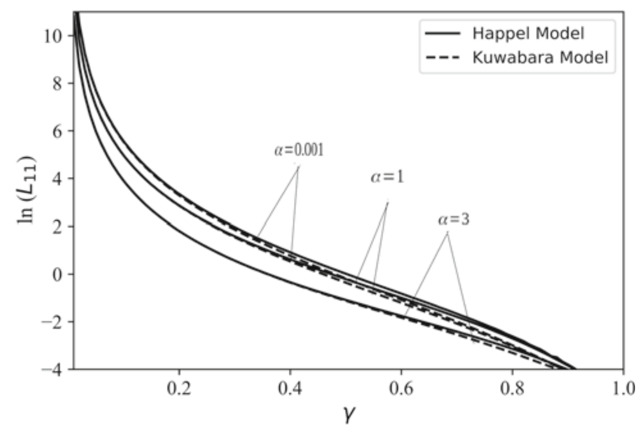

Figure 3. Natural logarithm of L_{11} against γ when $k_1 = 0.2$, $\xi_2 = 10$, $\lambda = 1$, for different values of α.

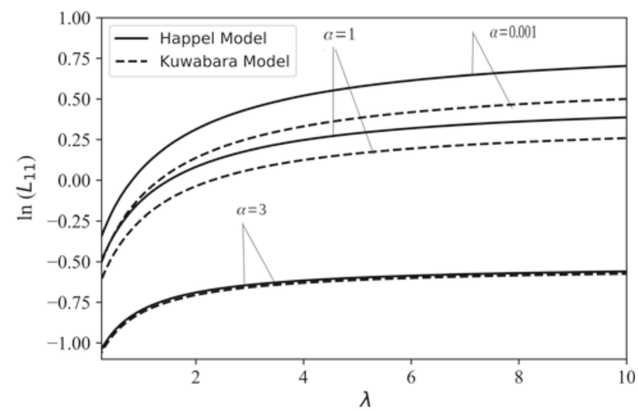

Figure 4. Natural logarithm of L_{11} against λ when $k_1 = 0.2$, $\xi_2 = 4$, $\gamma = 0.5$ for different values of α.

Influence of λ on the nature of dimensionless hydrodynamic permeability is demonstrated in figure 4. This figure indicates that the curve for ln L_{11} increases with increasing values of λ. Moreover, this increase is comparatively less under the magnetic influence which means that under magnetic effect the resistance to flow is higher.

The graph of ln L_{11} vs. α and ξ_2 for different values of λ are evaluated in figures 5 and 6. The graphs show that L_{11} decreases with increasing magnetic effect in both the fluid and porous region, which indicates that increasing magnetic effect reduces the ease of flow. Here $\lambda = 0$ indicates the flow past a semipermeable sphere and the location of the curve for the semipermeable sphere shows that L_{11} of the semipermeable sphere is lower compared to the weakly permeable sphere. Thus, the influence of the applied magnetic field on the nature of flow is seen to be significant and dominant. Moreover, it is noticed that the curve of ln L_{11} for all the

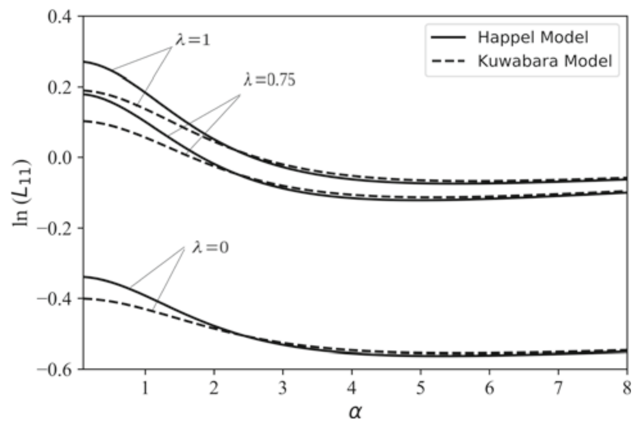

Figure 5. Natural logarithm of L_{11} against α when $\xi_2 = 0.2$, $k_1 = 0.2$, $\gamma = 0.5$ for different values of λ.

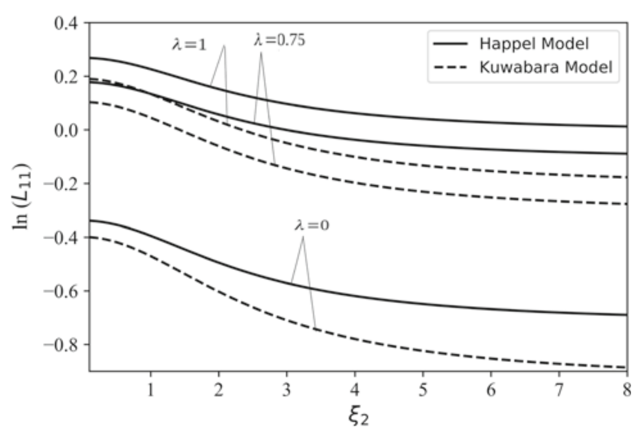

Figure 6. Natural logarithm of L_{11} against ξ_2 when $\alpha = 0.2$, $k_1 = 0.2$, $\gamma = 0.5$ for different values of λ.

above graphs are slightly higher for the Happel model compared to the Kuwabara model and this difference decreases with increasing magnetic effect.

6.2 *Kozeny constant*

Consider the resisting force acting on one particle multiplied by the number of particles per unit volume to be equal to the pressure drop of the particle per unit length [37]:

$$\frac{F_D(b/a)^3}{4\pi b^3/3} = -\frac{\Delta P}{L}, \tag{53}$$

where L is the characteristic length scale of the object (diameter).

Substituting eq. (53) in eq. (32), we obtain

$$U = -\frac{2b^2}{\gamma^3}\left(\frac{1}{\alpha^2(B - 2A - 2CS_2 - 2DU_2)}\right)\frac{\Delta P}{\mu L}. \tag{54}$$

Table 1. Numerical value of k_z for solid sphere ($\alpha \to 0$, $\xi_2 = 0$, $\lambda = 0$, $\xi_1 \to \infty$), semipermeable sphere ($\alpha = 4$, $\xi_2 = 8$, $\lambda = 0$, $k_1 = 0.1$) and weakly permeable sphere ($\alpha = 4$, $\xi_2 = 8$, $\lambda = 1$, $k_1 = 0.1$).

ε	k_z		
	Solid	Semipermeable	Weakly permeable
	Happel model		
0.1	4.43455	1.00037$E(-2)$	0.97694 $E(-2)$
0.2	4.41842	0.10413	0.08675
0.3	4.44404	0.44641	0.30229
0.4	4.53954	1.25464	0.72865
0.5	4.73842	2.67338	1.47801
	Kuwabara model		
0.1	4.66430	1.04595$E(-2)$	0.98123$E(-2)$
0.2	4.86372	0.10670	0.08751
0.3	5.11178	0.44848	0.31379
0.4	5.43043	1.25625	0.77402
0.5	5.85782	2.72570	1.58403

Now, the relationship describing Darcy's equation for incompressible flow through a porous medium with the superficial velocity (U) is given as [17,48]

$$U = \frac{\varepsilon R_H^2}{k_z}\frac{\Delta P}{\mu L}, \tag{55}$$

where k_z, known as Kozeny constant, is a dimensionless number and R_H^2 is the hydraulic radius defined for the porous media.

For the spherical particle system, we have

$$R_H = \frac{4\pi(a^3 - b^3)/3}{4\pi b^2} = \frac{b}{3}\left(\frac{1-\gamma^3}{\gamma^3}\right) = \frac{b}{3}\left(\frac{\varepsilon}{1-\varepsilon}\right). \tag{56}$$

Substituting the values from eqs (54)–(56), we have an expression for k_z for flow past a spherical particle as [37]

$$k_z = -\frac{\varepsilon^3\alpha^2}{18(1-\varepsilon)}[B - 2A - 2CS_2 - 2DU_2]. \tag{57}$$

Numerical value of k_z against ε for varying geometry are seen in table 1. Previously, Kim and Yuan [49] estimated the range of volume fraction $(1 - \varepsilon)$ for the purpose of using Carman–Kozenys model to be valid for $1 - \varepsilon \geq 0.3$, i.e. $\varepsilon \leq 0.7$ [17]. Moreover, Prakash and Raja Sekhar [42] analysed that for closely packed permeable spherical particles (i.e., $1 - \varepsilon > 0.4$ or $\varepsilon < 0.6$), the effect of Darcy's number (presently k_1) is significant. As in the case, the interstitial spacing is very less and thus fluid can mostly pass through the particles in place of these spaces. Based on the above discussion, the range of ε is chosen to be $0.1 \leq \varepsilon \leq 0.5$, validating the considered problem, i.e. low permeable porous media [43]. The values of k_z for flow past a solid spherical particle in the cell in the absence of any magnetic field are similar to the values obtained by Vasin and Filippov [46]. New results for MHD flow past a semipermeable sphere and a weakly permeable sphere are presented. From the table, it is clearly concluded that k_z for weakly permeable sphere is lower than that for the semipermeable sphere ($\lambda = 0$). Therefore, the drag force acting on the weakly permeable sphere is less than on the semipermeable sphere.

7. Conclusion

The present work is aimed to evaluate the solution of MHD flow through a weakly permeable sphere in unit cell model using an analytical approach. The modified Stokes and Darcy's equations govern the flow field and explicit expressions for the drag force, L_{11} and k_z which act on the weakly permeable spherical particle are evaluated. Representation of L_{11} together with permeability parameter, γ, λ, α and ξ_2 for the fluid and the porous region are shown by graphs. From the obtained results, we conclude that this problem is applicable for low range of permeability and significant variation on the values of L_{11} under the effect of magnetic fields is illustrated. Also, it is seen that as α acting on the fluid region increases, there is a noticeable decrease in the value of L_{11}. Therefore, applying magnetic field on the creeping flow of weakly permeable sphere has significant effect on its flow dynamics. Moreover, numerical values of k_z for the weakly permeable and semipermeable spheres are presented and it is found that this value is lower for the weakly permeable sphere compared to semipermeable case. In limiting case, various well-known earlier published results are obtained.

Appendix

$$S_1 = K_{1/2}(\alpha), \quad S_2 = K_{3/2}(\alpha),$$

$$S_3 = K_{1/2}(\alpha/\eta), \quad S_4 = K_{3/2}(\alpha/\eta),$$

$$U_1 = I_{1/2}(\alpha), \quad U_2 = I_{3/2}(\alpha),$$

$$U_3 = I_{1/2}(\alpha/\eta), \quad U_4 = I_{3/2}(\alpha/\eta),$$

$$W_1 = S_1U_4 + S_4U_1, \quad W_2 = S_2U_4 - S_4U_2,$$

$$W_3 = W_1\alpha + W_2, \quad W_4 = W_1\alpha + 3W_2,$$

$$W_5 = S_1U_2 + S_2U_1, \quad W_6 = S_3U_1 - S_1U_3,$$

$$W_7 = S_3U_2 + S_2U_3, \quad W_8 = S_3U_4 + S_4U_3,$$

$$W_9 = W_6\alpha - 3W_7, \quad W_{10} = W_5\eta + W_8,$$

$$W_{11} = W_6\alpha - W_7, \quad W_{12} = W_9\beta^2 - W_{11}\alpha^2,$$

$$W_{13} = W_3\alpha^2 - W_4\beta^2, \quad W_{14} = 2W_1\beta^2 + W_3\,\alpha,$$

$$W_{15} = 2W_6\,\beta^2 + W_{11}\alpha, \quad W_{16} = -2W_{12}\,\eta + W_{13}\alpha,$$

$$W_{17} = 2W_{16}\,\eta^3 + 6W_{14}\,\eta^2, \quad W_{18} = 2\alpha\eta W_{15} + \alpha^2 W_{14},$$

$$W_{19} = W_2\alpha^2 + W_4, \quad W_{20} = W_9 - W_7\alpha^2,$$

$$W_{21} = W_2\alpha + W_4, \quad W_{22} = W_6 - W_7\alpha,$$

$$W_{23} = 2W_{21}\beta^2 + W_{19}\,\alpha, \quad W_{24} = 2W_{22}\,\beta^2 + W_{20}\,\alpha,$$

$$Z = 6\eta^2 + \alpha^2,$$

$$\Delta_1 = W_{19}\beta^2 - 2W_2\,\alpha^2,$$

$$\Delta_2 = W_{19}(2(\beta^2 - \alpha^2)\eta^3 - \alpha^2) - 2\alpha(W_2\alpha + W_1)\beta^2,$$

$$\Delta_3 = W_{17} + W_{18},$$

$$\Delta_4 = W_4 Z + 2W_9\alpha\eta,$$

$$\Delta_5 = 2\alpha(W_{20}\beta^2 + 2W_7\alpha^2)\eta + Z\Delta_1,$$

$$\Delta_6 = -4W_{20}(\beta^2 - \alpha^2)\eta^4 + 6W_{23}\eta^2 + 2W_{24}\alpha\eta - \Delta_2\alpha,$$

$$\Delta_7 = -12(W_5\,\beta^2\eta^4 + W_8(\beta^2 - \alpha^2)\eta^3),$$

$$\Delta_8 = W_1 Z - 2W_9\eta^4 - \eta(W_4\alpha\eta^2 + 6W_{10}\,\eta^{3/2} - 2W_6\alpha).$$

References

[1] H P G Darcy, *Proc. Roy. Soc. London Ser.* **83**, 357 (1910)
[2] H C Brinkman, *Appl. Sci. Res. A* **1**, 27 (1947)
[3] G S Beavers and D D Joseph, *J. Fluid Mech.* **30**, 197 (1967)
[4] P G Saffman, *Studies Appl. Math.* **50**, 93 (1971)
[5] D D Joseph and L N Tao, *J. Appl. Math. Mech.* **44(8–9)**, 361 (1964)
[6] M P Singh and J L Gupta, *J. Appl. Math. Mech.* **51(1)**, 27 (1971)
[7] V M Shapovalov, *J. Appl. Mech. Tech. Phys.* **50(4)**, 584 (2009)
[8] J Happel and H Brenner, *Low Reynolds number hydrodynamics* (Prentice-Hall, Englewood Cliffs, NJ, 1965)
[9] J Happel, *AIChE J.* **4**, 197 (1958)
[10] S Kuwabara, *J. Phys. Soc. Jpn* **14**, 527 (1959)
[11] G Dassios, M Hadjinicolaou, F A Coutelieris and A C Payatakes, *Int. J. Eng. Sci.* **33(10)**, 1465 (1995)
[12] P Vainshtein, M Shapiro and C Gutfinger, *Int. J. Multiph. Flow* **28**, 1945 (2002)
[13] H Ramkissoon and K Rahaman, *Int. J. Eng. Sci.* **41**, 283 (2003)
[14] D Srinivasacharya, *C R Mecanique* **333(8)**, 612 (2005)
[15] S Senchenko and H J Keh, *Phys. Fluids* **18**, 088 (2006)

[16] P K Yadav, A Tiwari, S Deo, A Filippov and S Vasin, *Acta Mech.* **215**, 193 (2010)
[17] J Prakash, G P Raja Sekhar and M Kohr, *Z. Angew. Math. Phys.* **62**, 1027 (2011)
[18] E I Saad, *J. Porous Media* **15(9)**, 849 (2012)
[19] M K Prasad and M Kaur, *J. Braz. Soc. Mech. Sci. Eng.* **40**, 114 (2018)
[20] M K Prasad and T Bucha, *Int. J. Appl. Comput. Math.* **5**, 109 (2019)
[21] R Krishnan and P Shukla, *Int. J. Fluid Mech. Res.* **46(3)**, 219 (2019)
[22] D Yu Khanukaeva, A N Filippov, P K Yadav and A Tiwari, *Eur. J. Mech. B Fluids* **76**, 73 (2019)
[23] S Globe, *Phys. Fluids* **2**, 404 (1959)
[24] R R Gold, *J. Fluid. Mech.* **13(4)**, 505 (1962)
[25] N Rudraiah, B K Ramaiah and B M Rajasekhar, *Int. J. Eng. Sci.* **13**, 1 (1975)
[26] H P Mazumdar, U N Ganguly and S K Venkatesan, *Indian J. Pure Appl. Math.* **27(5)**, 519 (1996)
[27] K Haldar and S N Ghosh, *Indian J. Pure Appl. Math.* **25**, 345 (1994)
[28] K R Cramer and S I Pai, *Magnetofluid dynamics for engineers and applied physicists* (McGraw-Hills, New York, 1973)
[29] P A Davidson, *An introduction to magnetohydrodynamics* (Cambridge University Press, 2001)
[30] G E Geindreau and J L Aurialt, *J. Fluid Mech.* **466**, 343 (2002)
[31] V K Verma and S Datta, *Adv. Theory Appl. Mech.* **3(2)**, 53 (2010)
[32] D V Jayalakshmamma, P A Dinesh and M Sankar, *Mapana J. Sci.* **10(2)**, 11 (2011)
[33] B G Srivastava and S Deo, *Appl. Math. Computat.* **219(17)**, 8959 (2013)
[34] V K Verma and S K Singh, *J. Porous Media* **18(9)**, 923 (2015)
[35] B G Srivastava, P K Yadav, S Deo, P K Singh and A Flippov, *Colloid J.* **76(6)**, 725 (2014)
[36] P K Yadav, S Deo, S P Singh and A Filippov, *Colloid J.* **79(1)**, 160 (2017)
[37] E I Saad, *J. Porous Media* **21(7)**, 637 (2018)
[38] M Kumar, G J Reddy and N Dalir, *Pramana – J. Phys.* **91**: 60 (2018)
[39] K P Kashyap, O Ojjela and S K Das, *Pramana – J. Phys.* **92**: 73 (2019)
[40] M K Prasad and T Bucha, *Int. J. Appl. Comput. Math.* **5**, 98 (2019)
[41] M K Prasad and T Bucha, *J. Braz. Soc. Mech. Sci. Eng.* **41**, 320 (2019)
[42] J Prakash and G P Raja Sekhar, *J. Eng. Math.* **80**, 63 (2013)
[43] D A Nield and A Bejan, *Convection in porous media* (Springer, 2006)
[44] A Tiwari, P K Yadav and P Singh, *Natl. Acad. Sci. Lett.* **4(1)**, 53 (2018)
[45] S de Groot and P Mazur, *Non-equilibrium thermodynamics* (North Holland, Amsterdam; Wiley, New York, 1962)
[46] S I Vasin and A N Filippov, *Colloid J.* **71(2)**, 141 (2009)
[47] S I Vasin, A N Filippov and V M Starov, *Adv. Colloid Interface Sci.* **139**, 83 (2008)
[48] P C Carman, *Flow of gases through porous media* (Academic Press Inc., New York, 1956)
[49] A S Kim and R Yuan, *J. Mem. Sci.* **249**, 89 (2005)