Pramana – J. Phys. (2020) 94:18
https://doi.org/10.1007/s12043-019-1875-3

Exact solution of perturbed nonlinear Schrödinger equation using $(G'/\mathrm{G}, 1/G)$-expansion method

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MS received 14 April 2019; revised 31 August 2019; accepted 11 September 2019

Abstract. By constructing auxiliary equations and combining the expansion method of $(G'/G, 1/G)$, we study a class of nonlinear Schrödinger equation with perturbation terms which describes the propagation of the waves in optical metamaterials. More types of exact solutions, particularly solitary wave solutions, are obtained for the first time.

Keywords. Optical metamaterial equation; the expansion method of $(G'/G, 1/G)$; auxiliary equations; exact travelling solutions.

PACS Nos 02.30.Jr; 42.65.Tg; 81.05.Xj

1. Introduction

Metamaterials (MMs) are artificial structures that display properties not found in natural materials. MMs host a number of unusual electromagnetic properties that enable MMs to be potential candidates for stable soliton and other nonlinear phenomena, and have attracted the attention of many researchers.

In 2011, Xiang et al [1] derived a nonlinear Schrödinger equation (NLSE) with perturbation terms while studying the optical MMs equation in the following form:

$$\begin{aligned} iq_t + aq_{xx} + b|q|^2q &= i\mu q_x + i\lambda\left(|q|^2q\right)_x \\ &+ i\nu\left(|q|^2\right)_x q + \theta_1\left(|q|^2q\right)_{xx} \\ &+ \theta_2|q|^2q_{xx} + \theta_3 q^2 q^*_{xx}, \end{aligned} \tag{1.1}$$

where a and b are the group velocity and the self-phase modulation term, respectively, λ represents the self-steepening term in order to avoid the formation of shocks, υ is the nonlinear dispersion, μ denotes the intermodal dispersion, $\theta_j (j = 1, 2, 3)$ are the perturbation terms which appear in the context of MMs, x and t represent spatial and temporal variables, respectively, with $q(x, t)$ being the complex-valued wave profile [2–5].

A few years after eq. (1.1) was proposed, a plethora of results about its exact travelling solutions are being constantly reported [6–15].

In this paper, we use the $(G'/G, 1/G)$-expansion method [16] to study the exact solution of eq. (1.1). The method is concise, direct and effective in constructing the explicit travelling wave solutions of nonlinear development equations, and has been widely used in nonlinear differential equations, such as the Boussinesq-type equations, the DSSH equation, the shallow water wave equation, the nonlinear transport equation of MMs and so on [17–20]. However, we apply the $(G'/G, 1/G)$-expansion method to an auxiliary equation (to obtain accurate solution of the auxiliary equation) rather than eq. (1.1). Then using the relationship between the auxiliary equation and eq. (1.1), we can get plenty of solitary wave solutions of eq. (1.1) which are expressed by parameters. This method not only simplifies the computation process, but also greatly reduces the constraints on the parameters of eq. (1.1).

2. Outline of the $(G'/G, 1/G)$-expansion method

In this section, we give a brief introduction of the $(G'/G, 1/G)$-expansion method, and the main steps for solving travelling wave solutions in a particular nonlinear differential equation are designed by this method.

First, we consider the following second-order linear ordinary differential equations (ODE):

$$G''(\xi) + \gamma G(\xi) = \tau, \tag{2.1}$$

Published online: 18 December 2019

setting

$$\phi = \frac{G'}{G}, \quad \psi = \frac{1}{G}. \tag{2.2}$$

According to eqs (2.1) and (2.2)

$$\phi' = -\phi^2 + \tau\psi - \gamma, \quad \psi' = -\phi\psi. \tag{2.3}$$

Equation (2.1) can be solved by constant variation method. Due to different value ranges of γ, the solution forms of the equation can be divided into the following three types:

Case 1. When $\gamma < 0$, the general solution of eq. (2.1) is

$$G(\xi) = A_1 \sinh\left(\sqrt{-\gamma}\xi\right) + A_2 \cosh\left(\sqrt{-\gamma}\xi\right) + \frac{\tau}{\gamma}, \tag{2.4}$$

where A_1 and A_2 are arbitrary constants, and hence

$$\psi^2 = \frac{-\gamma}{\gamma^2\sigma + \tau^2}\left(\phi^2 - 2\phi\psi + \gamma\right), \tag{2.5}$$

where $\sigma = A_1^2 - A_2^2$.

Case 2. When $\gamma > 0$, the general solution of eq. (2.1) is

$$G(\xi) = A_1 \sin\left(\sqrt{\gamma}\xi\right) + A_2 \cos\left(\sqrt{\gamma}\xi\right) + \frac{\tau}{\gamma}, \tag{2.6}$$

where A_1 and A_2 are arbitrary constants and hence

$$\psi^2 = \frac{\gamma}{\gamma^2\rho - \tau^2}\left(\phi^2 - 2\phi\psi + \gamma\right), \tag{2.7}$$

where $\rho = A_1^2 + A_2^2$.

Case 3. When $\gamma = 0$, the general solution of eq. (2.1) is

$$G(\xi) = \frac{\tau}{2}\xi^2 + A_1\xi + A_2, \tag{2.8}$$

where A_1 and A_2 are arbitrary constants, and hence

$$\psi^2 = \frac{1}{A_1^2 - 2\tau A_2}\left(\phi^2 - 2\tau\psi\right). \tag{2.9}$$

The main steps of the $(G'/G, 1/G)$-expansion method are introduced now.

Suppose that the nonlinear evolution equation with three independent variables x, y, t is given as follows:

$$R\left(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, u_{xt}, \ldots\right) = 0, \tag{2.10}$$

where R is a polynomial and $u(x, y, t)$ is an unknown function.

Step 1. Do the travelling wave transform

$$u(x, y, t) = u(\xi), \quad \xi = x - cy - wt. \tag{2.11}$$

We can transform PDE into ODE by substituting eq. (2.11) into eq. (2.10), and the ODE is of the form

$$R\left(u, -wu', u', -cu', w^2u'', u'', c^2u'', wcu'', \ldots\right) = 0. \tag{2.12}$$

Step 2. Assume that the solution of eq. (2.12) can be expressed as

$$u(\xi) = \sum_{i=1}^{N} a_i\phi^i + \sum_{j=1}^{N} b_j\phi^{j-1}\psi, \tag{2.13}$$

where $G = G(\xi)$ satisfies formula (2.1) and N can be calculated by the homogeneous balance method.

Step 3. We plug eq. (2.13) back into eq. (2.12), then substitute into eq. (2.12), using eq. (2.3) and (2.5) (or (2.7)), to obtain a polynomial including powers of ϕ and ψ. Equating the coefficients of $\phi^i\psi^j$ to zero gives a system of algebraic equations. The undetermined coefficients γ and τ, a_i, b_i of algebraic equations were solved with the aid of Maple. Finally, the travelling wave solution of eq. (2.12) expressed by the hyperbolic function is obtained by taking them into formula (2.13).

Step 4. When $\gamma = 0$ or $\gamma > 0$, repeat Step 3 to calculate the values of γ, τ, a_i, b_i to get the travelling wave solution of eq. (2.12) which is expressed by trigonometric and rational functions.

3. The exact solution of the nonlinear Schrödinger equation

Doing the travelling wave transform for eq. (1.1), let

$$q(x, t) = u(\xi)e^{i(-kx+wt)}, \quad \xi = x - ct. \tag{3.1}$$

Inserting eq. (3.1) into eq. (1.1), and decomposing into real and imaginary parts yield the following pair of relations:

$$\left(\mu k + w + ak^2\right)u + \left(\lambda k - b - (\theta_1 + \theta_2 + \theta_3)k^2\right)u^3 - au'' + 6\theta_1 u\left(u'\right)^2 + (3\theta_1 + \theta_2 + \theta_3)u^2u'' = 0 \tag{3.2}$$

and

$$(\mu + c + 2ak)u' + (3\lambda + 2v - 2(3\theta_1 + \theta_2 - \theta_3)k) \times u^2u' = 0. \tag{3.3}$$

Let the coefficients of eq. (3.3) be equal to zero. Then we have

$$c = -\frac{(3\lambda + 2v)a}{3\theta_1 + \theta_2 - \theta_3} - \mu, \quad k = \frac{3\lambda + 2v}{2(3\theta_1 + \theta_2 - \theta_3)}. \tag{3.4}$$

Suppose eq. (3.2) satisfies the following auxiliary equation:

$$u'' = H_1u + H_3u^3. \tag{3.5}$$

By substituting eq. (3.5) into eq. (3.2), then dividing both sides by u, we obtain

$$\begin{aligned}&\left(\left(\mu k + w + ak^2\right) - aH_1\right)\\&+\left(\left(\lambda k - b - (\theta_1+\theta_2+\theta_3)k^2\right) - aH_3\right.\\&\left.+(3\theta_1+\theta_2+\theta_3)H_1\right)u^2\\&+(3\theta_1+\theta_2+\theta_3)H_3u^4+6\theta_1\left(u'\right)^2=0.\end{aligned} \tag{3.6}$$

On the other hand, multiplying both sides of eq. (3.5) by u', and integrating it with respect to ξ, we get

$$\left(u'\right)^2 = H_1u^2 + \frac{1}{2}H_3u^4 + M, \tag{3.7}$$

where

$$M = \left(\left(u'\right)^2 - H_1u^2 - \frac{1}{2}H_3u^4\right)\bigg|_{\xi=\xi_0}. \tag{3.8}$$

ξ_0 is a value in the domain of u.

Comparing eq. (3.6) with eq. (3.7), we acquire

$$\frac{1}{2}H_3 = -\frac{(3\theta_1+\theta_2+\theta_3)H_3}{6\theta_1},\quad H_1 = -\frac{\left(\lambda k - b - (\theta_1+\theta_2+\theta_3)k^2\right) - aH_3 + (3\theta_1+\theta_2+\theta_3)H_1}{6\theta_1},$$
$$M = -\frac{\left(\mu k + w + ak^2\right) - aH_1}{6\theta_1}. \tag{3.9}$$

From eq. (3.9), we lead to the constraint on coefficients of eq. (1.1)

$$-6\theta_1 = \theta_2 + \theta_3$$

and

$$H_3 = \frac{3\theta_1H_1 + \left(\lambda k - b + 5\theta_1k^2\right)}{a},$$

$$w = aH_1 - 6\theta_1M - ak^2 - \mu k, \tag{3.10}$$

where $\theta_1 \neq 0$ and $a \neq 0$.

Inserting eq. (2.13) into eq. (3.5), and balancing the highest-order derivative term u'' and the nonlinear term u^3, we get $N = 1$. So we assume the solution of eq. (3.5) as follows:

$$u(\xi) = a_1\phi(\xi) + b_1\psi(\xi), \tag{3.11}$$

where a_1 and b_1 are constants and $a_1^2 + b_1^2 \neq 0$.

Case 1. When $\gamma<0$, substituting eq. (3.11) into eq. (3.5), and using eqs (2.3) and (2.5), the left-hand side of eq. (3.5) becomes a polynomial in ϕ and ψ. Setting its coefficients to zero yields a system of algebraic equations as follows:

$$\phi^3 : 2a_1 - H_3\left(a_1^3 - \frac{3a_1b_1^2\gamma}{\gamma^2\sigma+\tau^2}\right) = 0$$

$$\phi^2\psi : 2b_1 - H_3\left(3a_1^2b_1 - \frac{b_1^3\gamma}{\gamma^2\sigma+\tau^2}\right) = 0$$

$$\phi^2 : b_1\tau\gamma + \frac{2H_3b_1^3\tau\gamma^2}{\lambda^2\sigma+\tau^2} = 0$$

$$\phi\psi : -a_1\tau - \frac{2H_3a_1^2b_1\tau\gamma}{\gamma^2\sigma+\tau^2} = 0$$

$$\phi : 2a_1\gamma - H_1a_1 + \frac{3H_3a_1b_1^2\gamma^2}{\gamma^2\sigma+\tau^2} = 0$$

$$\psi : b_1\gamma\left(\gamma^2\sigma-\tau^2\right) - H_1b_1\left(\gamma^2\sigma+\tau^2\right) - \frac{H_3b_1^3\gamma^2\left(3\tau^2-\gamma^2\sigma\right)}{\gamma^2\sigma+\tau^2} = 0$$

$$\psi^0 : b_1\tau\gamma^2 + \frac{2H_3b_1^3\tau\gamma^2}{\left(\gamma^2\sigma+\tau^2\right)^2} = 0.$$

Solving the above set of equations by Maple, we acquire the following cases:

Case (i): If $H_1 < 0$ and $H_3 > 0$, then

$$a_1 = \pm\sqrt{\frac{2}{H_3}},\quad b_1 = 0,$$

$$\gamma = \frac{1}{2}H_1,\quad \tau = 0,\quad \sigma = \text{ an arbitrary constant.}$$

It follows that

$$u = \pm\sqrt{\frac{2}{H_3}}\phi,$$

and through eqs (3.8) and (3.10) we obtain

$$M = \frac{H_1^2}{2H_3},$$

$$w = aH_1 - \frac{3a\theta_1H_1^2}{3\theta_1H_1 + 5\theta_1k^2 + \lambda k - b} - ak^2 - \mu k.$$

So the solution of eq. (1.1) is as follows:

$$q(x,t) = u(\xi)\mathrm{e}^{i(-kx+wt)},$$

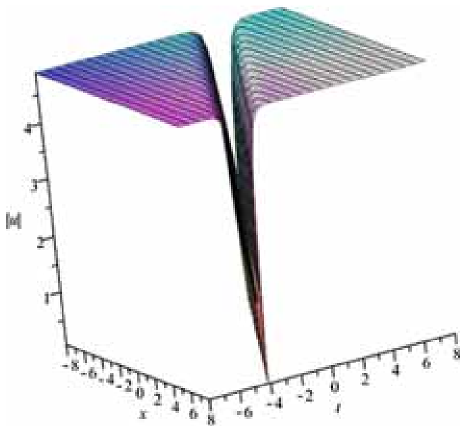

Figure 1. Profile of $|u|$ for $a = 1, b = 1, \mu = 1, \theta_1 = -1/3, \theta_2 = 1, \theta_3 = 1, \lambda = -1$ and $H_1 = -2$.

where

$$k = \frac{3\lambda + 2v}{2(3\theta_1 + \theta_2 - \theta_3)},$$

$$w = aH_1 - \frac{3a\theta_1 H_1^2}{3\theta_1 H_1 + 5\theta_1 k^2 + \lambda k - b} - ak^2 - \mu k,$$

$$u = \pm\sqrt{\frac{2}{H_3}}\,\frac{\sqrt{-\gamma}\left(A_1 \cosh\left(\sqrt{-\gamma}\xi\right) + A_2 \sinh\left(\sqrt{-\gamma}\xi\right)\right)}{A_1 \sinh\left(\sqrt{-\gamma}\xi\right) + A_2 \cosh\left(\sqrt{-\gamma}\xi\right)},$$

$$\xi = x + \left(\frac{(3\lambda + 2v)a}{3\theta_1 + \theta_2 - \theta_3} + \mu\right)t.$$

The waveform of $|u|$ is shown in figure 1.

Case (ii): If $H_1 > 0$ and $H_3 < 0$ (or > 0), then

$$a_1 = 0, \quad b_1 = \pm\sqrt{\frac{2H_1\sigma}{H_3}},$$
$$\gamma = -H_1, \quad \tau = 0, \quad \sigma < 0\,(\text{or} > 0).$$

It follows that

$$u = \pm\sqrt{\frac{2H_1\sigma}{H_3}}\psi,$$

and through eqs (3.8) and (3.10) we respectively obtain

$$M = 0,$$
$$w = aH_1 - ak^2 - k\mu.$$

So the solution of eq. (1.1) is as follows:

$$q(x, t) = u(\xi)\mathrm{e}^{i(-kx+wt)},$$

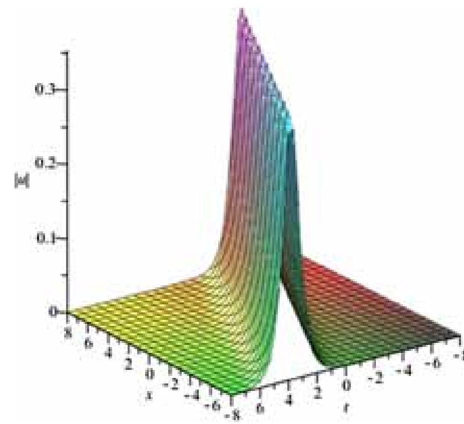

Figure 2. Profile of $|u|$ for $a = 1, b = 1, \mu = 1, \theta_1 = -1/3, \theta_2 = 1, \theta_3 = 1, \lambda = -1$ and $H_1 = -1$.

where

$$k = \frac{3\lambda + 2v}{2(3\theta_1 + \theta_2 - \theta_3)}, \quad w = aH_1 - ak^2 - \mu k,$$

$$u = \pm\sqrt{\frac{2H_1\sigma}{H_3}}\,\frac{1}{A_1 \sinh\left(\sqrt{-\gamma}\xi\right) + A_2 \cosh\left(\sqrt{-\gamma}\xi\right)},$$

$$\xi = x + \left(\frac{(3\lambda + 2v)a}{3\theta_1 + \theta_2 - \theta_3} + \mu\right)t.$$

The waveform of $|u|$ is shown in figure 2.

Case (iii): If $H_1 < 0$ and $H_3 > 0$, then

$$a_1 = \pm\sqrt{\frac{2}{H_3}}, \quad b_1 = \pm\sqrt{-\frac{4H_1^2\sigma + \tau^2}{4H_1H_3}},$$

$$\gamma = 2H_1, \quad \tau = \tau, \quad \sigma \geq -\frac{\tau^2}{4H_1^2}.$$

It follows that

$$u = \pm\sqrt{\frac{2}{H_3}}\phi \pm \sqrt{-\frac{4H_1^2\sigma + \tau^2}{4H_1H_3}}\psi,$$

and through eqs (3.8) and (3.10) we obtain

$$M = \frac{H_1^2}{2H_3},$$

$$w = aH_1 - \frac{3a\theta_1 H_1^2}{3\theta_1 H_1 + 5\theta_1 k^2 + \lambda k - b} - ak^2 - \mu k.$$

So the solution of eq. (1.1) is as follows:

$$q(x, t) = u(\xi)\mathrm{e}^{i(-kx+wt)},$$

where

$$k = \frac{3\lambda + 2v}{2(3\theta_1 + \theta_2 - \theta_3)},$$

$$w = aH_1 - \frac{3a\theta_1 H_1^2}{3\theta_1 H_1 + 5\theta_1 k^2 + \lambda k - b} - ak^2 - \mu k,$$

$$u = \pm\sqrt{\frac{2}{H_3}} \times \frac{\sqrt{-\gamma}\left(A_1 \cosh\left(\sqrt{-\gamma}\xi\right) + A_2 \sinh\left(\sqrt{-\gamma}\xi\right)\right)}{A_1 \sinh\left(\sqrt{-\gamma}\xi\right) + A_2 \cosh\left(\sqrt{-\gamma}\xi\right) + \frac{\tau}{\gamma}} \pm \sqrt{-\frac{4H_1^2\sigma + \tau^2}{4H_1 H_3}} \times \frac{1}{A_1 \sinh\left(\sqrt{-\gamma}\xi\right) + A_2 \cosh\left(\sqrt{-\gamma}\xi\right) + \frac{\tau}{\gamma}},$$

$$\xi = x + \left(\frac{(3\lambda + 2v)a}{3\theta_1 + \theta_2 - \theta_3} + \mu\right)t.$$

Case 2. When $\gamma > 0$, similar to Case 1, by solving algebraic equations, we obtain the following cases:

Case (iv): If $H_1 > 0$ and $H_3 > 0$, then

$$a_1 = \pm\sqrt{\frac{2}{H_3}}, \quad b_1 = 0,$$

$$\gamma = \frac{1}{2}H_1, \quad \tau = 0, \quad \rho = \text{ an arbitrary constant.}$$

It follows that

$$u = \pm\sqrt{\frac{2}{H_3}}\phi,$$

and through eqs (3.8) and (3.10) we obtain

$$M = \frac{H_1^2}{2H_3},$$

$$w = aH_1 - \frac{3a\theta_1 H_1^2}{3\theta_1 H_1 + 5\theta_1 k^2 + \lambda k - b} - ak^2 - \mu k.$$

So the solution of eq. (1.1) is as follows:

$$q(x,t) = u(\xi)e^{i(-kx+wt)},$$

where

$$k = \frac{3\lambda + 2v}{2(3\theta_1 + \theta_2 - \theta_3)},$$

$$w = aH_1 - \frac{3a\theta_1 H_1^2}{3\theta_1 H_1 + 5\theta_1 k^2 + \lambda k - b} - ak^2 - \mu k,$$

$$u = \pm\sqrt{\frac{2}{H_3}}\frac{\sqrt{\gamma}\left(A_1 \cos\left(\sqrt{\gamma}\xi\right) - A_2 \sin\left(\sqrt{\gamma}\xi\right)\right)}{A_1 \sin\left(\sqrt{\gamma}\xi\right) + A_2 \cos\left(\sqrt{\gamma}\xi\right)},$$

$$\xi = x + \left(\frac{(3\lambda + 2v)a}{3\theta_1 + \theta_2 - \theta_3} + \mu\right)t.$$

Case (v): If $H_1 < 0$, $H_3 > 0$ (or < 0), then

$$a_1 = 0, \quad b_1 = \pm\sqrt{-\frac{2H_1\rho}{H_3}},$$

$$\gamma = -H_1, \quad \tau = 0, \quad \rho > 0 \text{ (or } < 0).$$

It follows that

$$u = \pm\sqrt{-\frac{2H_1\rho}{H_3}}\psi,$$

and through eqs (3.8) and (3.10) we obtain

$$M = 0,$$
$$w = aH_1 - ak^2 - k\mu.$$

So the solution of eq. (1.1) is as follows:

$$q(x,t) = u(\xi)e^{i(-kx+wt)},$$

where

$$k = \frac{3\lambda + 2v}{2(3\theta_1 + \theta_2 - \theta_3)},$$

$$w = aH_1 - ak^2 - k\mu.$$

$$u = \pm\sqrt{-\frac{2H_1\rho}{H_3}}\frac{1}{A_1 \sin\left(\sqrt{\gamma}\xi\right) + A_2 \cos\left(\sqrt{\gamma}\xi\right)},$$

$$\xi = x + \left(\frac{(3\lambda + 2v)a}{3\theta_1 + \theta_2 - \theta_3} + \mu\right)t$$

Case (vi): If $H_1 > 0$ and $H_3 > 0$, then

$$a_1 = \pm\sqrt{\frac{2}{H_3}}, \quad b_1 = \pm\sqrt{\frac{4H_1^2\rho - \tau^2}{4H_1 H_3}},$$

$$\gamma = 2H_1, \quad \tau = \tau, \quad \rho \geq \frac{\tau^2}{4H_1^2}.$$

It follows that

$$u = \pm\sqrt{\frac{2}{H_3}}\phi \pm \sqrt{\frac{4H_1^2\rho - \tau^2}{4H_1 H_3}}\psi,$$

and through eqs (3.8) and (3.10) we obtain

$$M = \frac{H_1^2}{2H_3},$$

$$w = aH_1 - \frac{3a\theta_1 H_1^2}{3\theta_1 H_1 + 5\theta_1 k^2 + \lambda k - b} - ak^2 - \mu k.$$

So the solution of eq. (1.1) is as follows:

$$q(x,t) = u(\xi)e^{i(-kx+wt)},$$

where

$$k = \frac{3\lambda + 2v}{2(3\theta_1 + \theta_2 - \theta_3)},$$

$$w = aH_1 - \frac{3a\theta_1 H_1^2}{3\theta_1 H_1 + 5\theta_1 k^2 + \lambda k - b} - ak^2 - \mu k,$$

$$u = \pm\sqrt{\frac{2}{H_3}}\frac{\sqrt{\gamma}\left(A_1\cos\left(\sqrt{\gamma}\xi\right) - A_2\sin\left(\sqrt{\gamma}\xi\right)\right)}{A_1\sin\left(\sqrt{\gamma}\xi\right) + A_2\cos\left(\sqrt{\gamma}\xi\right) + \frac{\tau}{\gamma}}$$

$$\pm\sqrt{\frac{4H_1^2\rho - \tau^2}{4H_1 H_3}}\frac{1}{A_1\sin(\sqrt{\gamma}\xi) + A_2\cos(\sqrt{\gamma}\xi) + \frac{\tau}{\gamma}},$$

$$\xi = x + \left(\frac{(3\lambda + 2v)a}{3\theta_1 + \theta_2 - \theta_3} + \mu\right)t.$$

Case 3. When $\gamma = 0$, similar to Case 1, after solving the system of algebraic equations, we obtain the following cases:

Case (vii): If $H_1 = 0$ and $H_3 > 0$, then

$$a_1 = 0, \quad b_1 = \pm\sqrt{\frac{2}{H_3}}, \quad \tau = 0.$$

It follows that

$$u = \pm\sqrt{\frac{2}{H_3}}\psi,$$

and through eqs (3.8) and (3.10) we obtain

$$M = 0,$$

$$w = -ak^2 - k\mu.$$

So the solution of eq. (1.1) is as follows:

$$q(x,t) = u(\xi)e^{i(-kx+wt)},$$

where

$$k = \frac{3\lambda + 2v}{2(3\theta_1 + \theta_2 - \theta_3)}, \quad w = -ak^2 - k\mu,$$

$$u = \pm\sqrt{\frac{2}{H_3}}\frac{1}{A_1\xi + A_2},$$

$$\xi = x + \left(\frac{(3\lambda + 2v)a}{3\theta_1 + \theta_2 - \theta_3} + \mu\right)t.$$

4. Conclusion

In this study, we propose the $(G'/G, 1/G)$-expansion method for finding multiple exact solutions for NLSE with perturbation terms. For eq. (1.1), by constructing the auxiliary equation, we have obtained new types of exact solutions for NLSE (1.1). To our knowledge, these results have not been reported in the existing literature. We believe that our results will be helpful for the further investigation of the wave propagation in optical MMs. In fact, optical solitons are being used as carriers of ultra-high-speed no distortion signals because of its low bit error rate (BER) and strong anti-interference ability, causing revolutionary changes in optical communication technology.

Acknowledgement

This work is supported by Natural Science Foundation of Guangxi 2017GXNSFBA198056 (Grant No. 80000(CNY)).

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