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Influence of magnetic field and Coulomb field on the Rashba effect in a triangular quantum well

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Abstract. The influence of magnetic field and Coulomb field on the Rashba spin–orbit interaction in a triangular quantum well was studied using Pekar variational method. We theoretically derived the expression of the bound magnetopolaron ground-state energy. The energy of the bound magnetopolaron splits under the influence of the Rashba effect. From this phenomenon, it is concluded that the effects of orbital and spin interactions on the polaron energy in different directions must be considered. Because of the contribution of the magnetic field cyclotron resonance frequency to the Rashba spin–orbit splitting, the energy spacing becomes larger as the magnetic field cyclotron resonance frequency increases. Compared to the bare electron, the bound polaron is more stable, and the energy of bound polaron split is more stable.

Keywords. Rashba effect; bound magnetopolaron; triangular quantum well.

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1. Introduction

Spintronics studies how to utilise the spin freedom of electrons in devices. It began with the giant magnetoresistance effect independently discovered by Fert and Gruenberg in 1988. In recent years, spintronics has become one of the most popular research fields in the field of physics [1,2]. It is a basic physical problem, and also has many applications. In spintronic devices, the charge and spin of electrons are simultaneously used to transmit and store information. This will greatly enhance the speed and efficiency of electronic devices. It is also possible to fabricate electronic devices with new physical properties by using electron spin. We believe that the spintronic devices can change people's lives.

At present, an important branch in spintronics studies the Rashba and Dresselhaus spin–orbit coupling effects in semiconductor heterojunctions or in semiconductor quantum wells, quantum wires, quantum dots, and which can be used in electronic devices such as spin filters, spin transistors and spin waveguides. The foundation of spintronics is that electrons of different spin states have different concentrations in the material, that is, the spin state splits in energy. Obviously, an external magnetic field can produce this splitting (the Zeeman splitting). In addition, if the inversion symmetry of the crystal structure is destroyed, the electron energy will still split (Rashba spin–orbit splitting) even without the magnetic field. In 1990, Datta and Das [3] first proposed the principle of transistors based on controlling the spin of electron. Since the publication of their article, many scholars around the world have carried out experimental and theoretical research on the Rashba effect in low-dimensional quantum systems [4–6], especially in quantum well systems. For example, Li and Xia [7] adopted the effective-mass envelope function theory and theoretically investigated the Rashba spin–orbit splitting in GaAs/GaAlAs quantum wells. Using Kane's 8-band $k \cdot p$ theory and envelope function approximation, Stanley *et al* [8] obtained a tightly bound Hamiltonian of the III–V semiconductor quantum well structure and accurately simulated the band structure and spin–orbit coupling. By applying a potential difference across the well, they calculated the Rashba spin split in the lowest conduction sub-band. Jin *et al* [9] studied the different growth directions and electron densities of Rashba splitting in asymmetric quantum wells. The strong Rashba effect in highly asymmetric quantum wells provides a potential candidate for spintronic devices. Rashba spin–orbit splitting is not a simple split. Sometimes the split

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will be mixed with Zeeman splitting. Although the two splits have different directions, there are still some controversies about how to distinguish their contributions. In the field of electron spin, the Zeeman effect is often ignored under a weak magnetic field, while in a high magnetic field, the Zeeman effect must be considered in energy splitting. Under an external magnetic field, Lipparini *et al* [10] explored the Rashba effect, the Dresselhaus effect and the Zeeman effect in quantum wells. They evaluated the contribution of spin orbits to the spin splitting of the Landau energy level. It is not difficult to find that much research work has been done on the Rashba effect of electrons in quantum wells, but there is very few in the field of polarons. The influence of magnetic field and Coulomb field on the Rashba effect in a triangular quantum well will be studied in the present paper.

2. Theoretical model

A triangular quantum well composed of two polar materials grows in the z direction. An electron bound by hydrogenated impurities interacts with the bulk longitudinal optical phonon field in the quantum well. A magnetic field is applied in the z direction with vector of $\mathbf{A} = (-\mathbf{B}y/2, \mathbf{B}x/2)$. Hamiltonian of the system with hydrogenated impurities at the centre of the well is expressed as

$$\begin{aligned} H = {} & \frac{1}{2m}\left[\left(P_x - \frac{\bar{\beta}^2}{4}y\right)^2 + \left(P_y + \frac{\bar{\beta}^2}{4}x\right)^2\right] \\ & + \frac{P_z^2}{2m} + V(z) + \sum_{\mathbf{k}} \hbar\omega_{\mathrm{LO}} a_{\mathbf{k}}^{+} a_{\mathbf{k}} \\ & + \sum_{\mathbf{k}} \left(V_k a_{\mathbf{k}} \mathrm{e}^{\mathbf{i}\,\mathrm{k}\cdot\mathbf{r}} + \mathrm{h.c.}\right) \quad - \frac{\beta}{r} \\ & + \frac{\alpha_{\mathrm{R}}}{\hbar}\left[\left(P_y + \frac{\bar{\beta}^2}{4}x\right)\sigma_x - \left(P_x - \frac{\bar{\beta}^2}{4}y\right)\sigma_y\right], \end{aligned} \tag{1}$$

where $\beta' = 2eB/c$. The physical meaning of m, $\mathbf{p}$, $\mathbf{r}$, ω_{LO} and $\mathbf{k}$ are the same as that in ref. [11]. The Coulomb bound potential between the electron and hydrogenated impurities is represented by β. The black body of electron is delimited by $\boldsymbol{\sigma}$. α_{R} represents the Rashba spin–orbit parameter.

The conduction band bending potential is replaced by a triangular potential approximation, and it can be written as

$$V(z) = \begin{cases} eF_s z, & z \geq 0, \\ \infty, & z < 0, \end{cases} \tag{2}$$

where F_s is the built-in electric field which is expressed as

$$F_s = \frac{4\pi e n_s}{\varepsilon_{01}}. \tag{3}$$

The electron areal density is indicated by n_s and ε_{01} represents the static dielectric constant. The interaction Fourier coefficient is described as

$$V_k = i\left(\frac{\hbar\omega_{\mathrm{LO}}}{k}\right)\left(\frac{\hbar}{2m\omega_{\mathrm{LO}}}\right)^{1/4}\left(\frac{4\pi\alpha}{V}\right)^{1/2}, \tag{4}$$

where V and α are the crystal volume and the electron–LO phonon coupling strength, respectively.

We expand the Coulomb bound potential into the Fourier series form

$$-\frac{\beta}{r} = -\frac{4\pi\beta}{V}\sum_k \frac{1}{k^2}\exp(i\mathbf{k}\cdot\mathbf{r}). \tag{5}$$

Perform unitary transformation on eq. (1), and take the unitary transformation operator as

$$U = \exp\left[\sum_{\mathbf{k}}\left(a_{\mathbf{k}}^{+} f_k - a_{\mathbf{k}} f_k^{*}\right)\right], \tag{6}$$

where f_k and f_k^{*} are variational functions, which may be obtained by varying energy and taking minimum values. Then the transformed Hamiltonian can be expressed as

$$\begin{aligned} H' = {} & U^{-1} H U \\ = {} & \frac{1}{2m}\left[\left(P_x - \frac{\bar{\beta}^2}{4}y\right)^2 + \left(P_y + \frac{\bar{\beta}^2}{4}x\right)^2\right] \\ & + \frac{P_z^2}{2m} + eF_s z + \sum_k \hbar\omega_{\mathrm{LO}}(a_{\mathbf{k}}^{+} + f_k^{*})(a_{\mathbf{k}} + f_k) \\ & + \sum_k \left[V_k(a_k + f_k)\exp(i\mathbf{k}\cdot\mathbf{r}) + \mathrm{h.c.}\right] \\ & - \frac{4\pi\beta}{V}\sum_k \frac{1}{k^2}\exp(-i\mathbf{k}\cdot\mathbf{r}) \\ & + \frac{\alpha_{\mathrm{R}}}{\hbar}\left[\left(P_y + \frac{\bar{\beta}^2}{4}x\right)\sigma_x - \left(P_x - \frac{\bar{\beta}^2}{4}y\right)\sigma_y\right]. \end{aligned} \tag{7}$$

The ground-state trial wave function of the system is selected as

$$\begin{aligned} |\psi\rangle = {} & \left(\frac{1}{2\pi}\right)^{1/2}\delta\mathrm{e}^{-\delta\rho/2}\left(\frac{\beta'}{2}\right)^{1/2} z\mathrm{e}^{-\beta' z/2} \\ & \times (a\chi_{1/2} + b\chi_{-1/2})|0\rangle_{ph}. \end{aligned} \tag{8}$$

Applying $|\psi\rangle$ to eq. (7), the expected value of energy can be obtained and written as

$$F\left(f_k, f_k^{*}, \delta, \beta'\right) = \langle\psi|H'|\psi\rangle. \tag{9}$$

Using the variational method, f_k and f_k^* are obtained. Substituting them into the energy expectation value, and the expression of bound magnetopolaron ground-state energy can be obtained, which is

$$E = \frac{\hbar^2 k^2}{2m} + \frac{3m\omega_c^2}{4\delta^2} + \frac{\beta'^2\hbar^2}{8m} + \frac{12e^2\pi n_s}{\beta'\varepsilon_{01}} - \frac{3\pi}{16}\alpha\hbar\omega_{\mathrm{LO}}\delta\left(\frac{\hbar}{2m\omega_{\mathrm{LO}}}\right)^{1/2} - \frac{\pi}{2}\beta\delta \pm \alpha_{\mathrm{R}}\left(k + \frac{m\omega_c}{\hbar\delta}\right), \quad (10)$$

where $\omega_c = eB/mc$, represents the magnetic field cyclotron resonance frequency.

3. Numerical results and discussion

To more clearly explain the influence of magnetic field and Coulomb field on the Rashba effect of the polaron, we numerically calculated the ground-state energy of the bound magnetopolaron in a triangular quantum well. For simplicity, we take the usual polaron units in the calculation progress ($\hbar = \omega_{\mathrm{LO}} = 2m = 1$). The relations between the ground-state energy of the bound magnetopolaron and the wave vector, the electron area density and the magnetic field cyclotron resonance frequency are further discussed. Figures 1–5 show the results of numerical calculations. The physical meaning of the curves in all the figures is the same as that in ref. [9], that is, the ground-state energy (E_0) of the zero spin–orbit interaction is represented by the solid line, the dash–dotted line and the dotted line represent the spin-up splitting energy (E_+) and spin-down splitting energy ($E\_$), respectively.

Figure 1 shows the functional relationship between ground-state energy (E) of the bound magnetopoloran and the wave vector (k) when the Coulomb bound potential (β) takes different values. When the magnetic field cyclotron resonance frequency (ω_c) takes different values, figure 2 demonstrates the functional relationship between E and k. The two figures reflect that the ground-state energy of the bound magnetopolaron parabolically increases as the wave vector increases. We know from eq. (10), that the wave vector contributes a positive value to the energy of the bound magnetopolaron. That is why E increases with the increase of k.

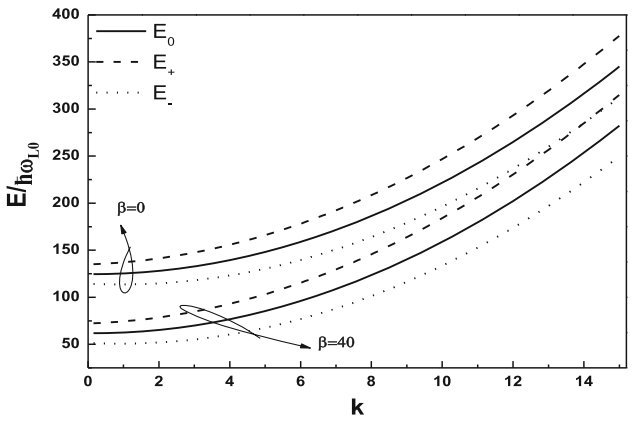

Figure 1. The relation between E and k when β takes different values.

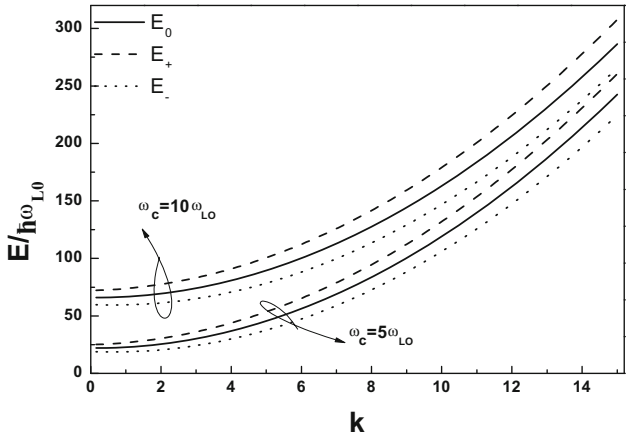

Figure 2. The relation between E and k when ω_c takes different values.

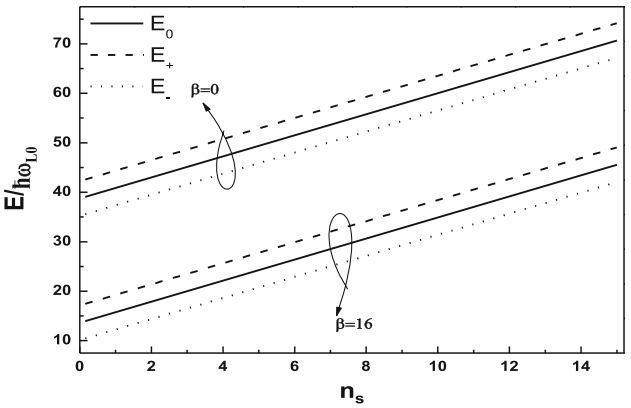

Figure 3. The relation between E and n_s when β takes different values.

When β takes different values, figure 3 illustrates the relationship between E of the bound magnetopolaron and the electron areal density (n_s). Figure 4 demonstrates the relation between E and n_s for different values of ω_c. From the two figures, it can be found that E is an increasing function of n_s. Because the band bending becomes larger as n_s increases, it keeps the electron away from the interface. So the electron–LO phonon interaction is enhanced. As a result, E increases with n_s.

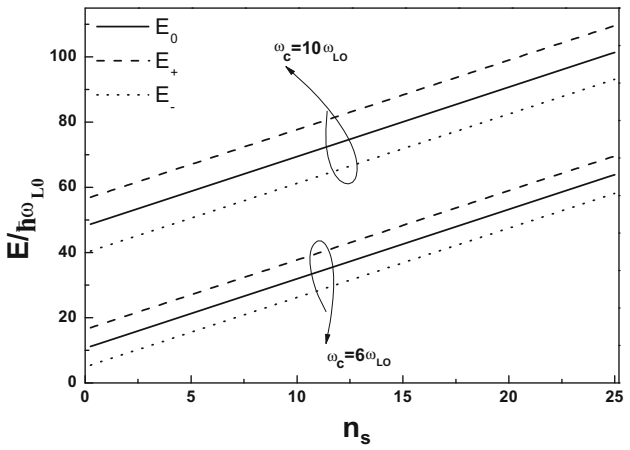

Figure 4. The relation between E and n_s when ω_c takes different values.

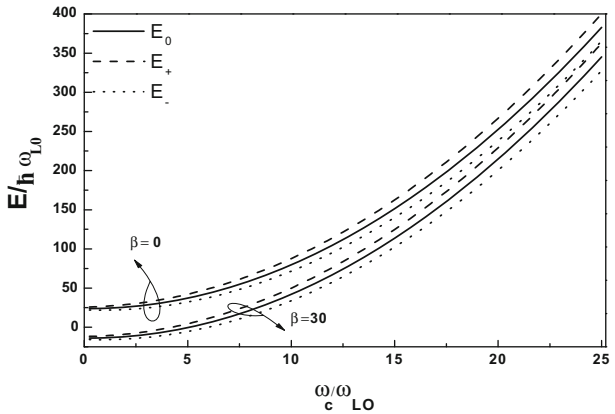

Figure 5. The relation between E and ω_c when β takes different values.

Figure 5 shows the relation of E with ω_c when β takes different values. It can be seen that E parabolically increases with the increase of ω_c, because the maximum of the electron wave function lies in the quantum well, but the wave function has a large overlap. When the magnetic field is stronger, the wave function becomes more and more local, and the overlap integral of the electron wave function increases. Therefore, we get the above conclusion.

From figures 1–5, we find a common phenomenon that E is split into spin-up and spin-down branches. The spin properties of the electron in the semiconductor are determined not only by its own magnetic moment, but also by its orbital motion. If the inversion symmetry of the crystal is destroyed, the energy will split even without the influence of the applied magnetic field. In narrow band-gap semiconductors, Rashba spin–orbit splitting is mainly caused by structural inversion asymmetry. Rashba splitting caused by spin–orbit coupling is more obvious in semiconductors. In order to more clearly reflect the influence of the magnetic field on the Rashba spin–orbit splitting, we only consider the Rashba spin–orbit splitting and ignore the influence of the Zeeman splitting. Each branch in all figures is not defined as the energy of the spin-up or spin-down, but is defined as the spin-up or spin-down splitting energy, respectively. Figures 1, 3 and 5 show that β has no influence on the splitting. From eq. (10), we know that the contribution of β to E is negative, and the existence of β reduces the total energy of the system. Compared to the bare electron, the bound polaron is more stable, and the energy of the bound polaron split is more stable.

When values of k, n_s and ω_c are fixed, it can be seen in figures 1, 3 and 5 that β is larger, and E is smaller. There is a Coulomb bound potential between the electron and the hydrogenated impurities due to the existence of the hydrogenated impurities in quantum well. Electron is subject to a new limitation due to the existence of the Coulomb bound potential, which causes larger electron wave functions to overlap each other leading to the enhancement of electron–LO phonon interaction. However, it can be seen in eq. (10) that the electron–phonon interaction contributes a negative value to the magnetopolaron energy, and so E is a decreasing function of β.

From figures 2, 4 and 5 one can see that the energy spacing increases with the increase of ω_c. From the last term in eq. (10), we know that the contribution of ω_c to the splitting energy is positive. So the energy spacing will increase with the increase of ω_c. We can also see that the energy can still split under the zero magnetic field. This is because the electron spin–orbit coupling interaction causes zero-field spin splitting of electron.

4. Conclusion

Due to the influence of the Rashba effect, E of the bound magnetopolaron is split into spin-up and spin-down branches. This phenomenon fully demonstrates that the influence of orbit and spin interactions in different directions on the energy of the polaron must be premeditated. Because the contribution of ω_c to the Rashba spin-orbit splitting is positive, the energy spacing becomes larger as ω_c increases. The existence of β reduces the total energy of the system. Therefore, the bound polaron compared with the bare electron is more stable, and the energy of the bound polaron split is more stable.

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References

[1] E I Rashba and Al L Efros, *Phys. Rev. Lett.* **91**, 126405 (2003)

[2] T-N Xu, H-Z Wu and C-H Sui, *Chin. Phys. Soc.* **57**, 7865 (2008)

[3] S Datta and B Das, *Appl. Phys. Lett.* **56**, 665 (1990)

[4] J Lee and H N Specror, *J. Appl. Phys.* **99**, 113708 (2006)

[5] T P Pareek and P Bruno, *Pramana – J. Phys.* **58**, 293 (2002)

[6] S Ullah, G M Gusev, A K Bakarov and F G G Hemandez, *Pramana – J. Phys.* **91**: 34 (2018)

[7] S-S Li and J-B Xia, *Nano. Res. Lett.* **4**, 178 (2009)

[8] J P Stanley, N Pattinson, C J Lambert and J H Jefferson, *Physica E* **20**, 433 (2004)

[9] S Jin, H Wu and T Xu, *Appl. Phys. Lett.* **95**, 132105 (2009)

[10] E Lipparini, M Barranco, F Malet and M Pi, *Phys. Rev. B* **74**, 115303 (2006)

[11] S Shuping, *Chin. Quant. Elec.* **5**, 265 (2018)