Pramana – J. Phys. (2020) 94:13
https://doi.org/10.1007/s12043-019-1870-8

Ion-acoustic waves in magnetised plasma with nonthermal electrons and positrons

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MS received 13 August 2018; revised 9 August 2019; accepted 29 August 2019

Abstract. Zakharov–Kuznetsov (ZK) equation for ion-acoustic waves (IAWs) is derived using the reductive perturbation method (RPM) in magnetised plasma consisting of ions, positrons and nonthermal electrons in small but finite amplitude limit. Propagation characteristics of ion-acoustic solitary waves (IASWs) in three-dimensional space are analysed to determine their region of existence. Investigations reveal that ion-acoustic solitary pulses (IASPs) may exist in such plasmas and presence of nonthermal electrons significantly affects the amplitude and width of solitary pulses. Dependence of velocity, amplitude and width of solitary pulses on plasma parameters are presented graphically. The amplitude of soliton increases with increase in ion temperature ratio (σ) and positron concentration (α). However, it decreases with increase in nonthermal electron parameter (β) keeping other plasma parameters constant. Width of the soliton increases with increase in β, σ and α. Phase velocity of ion-acoustic wave (λ) increases with increase in nonthermal β and σ. In our analysis, we found that magnetisation of plasma affect the width of the soliton but not the amplitude.

Keywords. Zakharov–Kuznetsov equation; reductive perturbation method; solitary wave; nonthermal electron; positrons.

PACS Nos **52.35.Mw; 52.35.Sb**

1. Introduction

A large number of investigations [1–3] report the presence of energetic electrons in astrophysical plasma environment, which are highly nonthermal. Due to their streaming motion with respect to the background plasmas, nonthermal electron distribution can change the basic properties of coherent nonlinear waves and nonlinear structures. Observations of density depressions in magnetosphere [4,5] were explained theoretically by Cairns *et al* [6] using nonthermal electron distribution and it was pointed out that these depressions are due to rarefactive ion-acoustic solitary pulses (IASPs), which retain their shape and size and move with constant velocity with respect to the background plasma.

During the last two decades, many investigations [7–11], including the effect of nonthermal electrons, have been carried out in linear and nonlinear phenomena in plasmas. Study of ion-acoustic solitons (IASs) in multicomponent unmagnetised plasma consisting of isothermal and nonisothermal electrons with cold ion species was carried out by Das and Tagare [7]. Sabry *et al* [8] extended the study of IASs in plasma in the presence of negative ions and nonthermal electrons and investigated their effects on the characteristics of soliton. These studies opened a new field of research. Study of linear and obliquely propagating ion-acoustic solitary waves (IASWs) in magnetised plasma consisting of negative ions and nonisothermal electron by Mishra and Jain [10] shows that the amplitude and width of the solitons for slow and fast modes depend on the parameter of nonisothermal electrons.

Zakharov and Kuznetsov [12] derived the nonlinear evolution equation known as ZK equation for magnetised plasma consisting of cold ions and hot isothermal electrons for the ion-acoustic waves (IAWs). El-Labany *et al* [9] derived modified ZK equation for IASWs in magnetised negative ion plasma with nonisothermal electrons and found that compressive and

Published online: 18 December 2019

rarefactive IASWs strongly depend on the mass and density ratio of the positive to negative ions as well as on the nonthermal electron parameters. The ZK equations have been derived for different physical systems by many researchers [13–15]. El-Taibany and Sabry [16] derived ZK equation using reductive perturbation method (RPM) for studying small-amplitude nonlinear dust-acoustic waves with nonthermal ions. They reported that compressive and rarefactive solitons can exist in such plasmas.

Different studies related to nonthermal electrons or nonthermal ions in plasma or dusty plasma suggest that the presence of nonthermal electrons or ions moving faster with respect to background plasma can drastically change the characteristics of nonlinear structures. Hence, it is worthwhile to study the effect of nonthermal electrons on IAWs in a plasma containing ions, positrons and nonthermal electrons to explore the changes produced in velocity, amplitude and width of solitary waves governed by nonlinear ZK equation.

The aim of the present paper is to study the properties of small but finite-amplitude three-dimensional (3D) IASWs governed by nonlinear ZK equation in magnetised plasma consisting of warm ions, positrons and nonthermal electrons. The paper is organised as follows: In §2, we describe the basic equations governing our plasma system and using RPM, the nonlinear ZK equation is derived with appropriate boundary conditions. In §3 we determine the exact solutions of the ZK equation. Section 4 is devoted to the discussion of results and conclusions of our investigations.

2. Basic equations and derivation of the nonlinear ZK equation

We consider a 3D plasma consisting of warm ions, positron, and nonthermal electrons, which is confined in a magnetic field $B = B_0\hat{z}$ where $\hat{z}$ is the unit vector along the z-axis. The governing equations for IAWs in the above described plasma are as follows:

$$\frac{\partial(N)}{\partial t} + \vec{\nabla}\left(N \cdot \vec{V}\right) = 0 \tag{1}$$

$$\frac{\partial(V)}{\partial t} + \left(\vec{V} \cdot \nabla\right)\vec{V} = -\vec{\nabla}\phi + B\left(V \times \hat{z}\right) - 2\sigma\vec{\nabla}N \tag{2}$$

$$\nabla^2\phi = \left(1 - \beta\phi + \beta\phi^2\right)e^{\phi} - \alpha e^{-\gamma\phi} - (1-\alpha)N. \tag{3}$$

Here eq. (1) is the continuity equation for the ion fluid and eq. (2) is the momentum equation for the ion fluid including the effect of magnetic field and finite ion temperature, whereas eq. (3) is the Poisson's equation including the contribution of positron fluid in plasma and nonthermal electrons. In the above equations N, $\vec{V}$ and ϕ are normalised density, fluid velocity of the plasma ions and the electric potential, respectively. These quantities have been rendered dimensionless in terms of equilibrium plasma density (n_0), ion sound speed $(T_e/m_i)^{1/2}$ and characteristic potential (T_e/e), respectively. The space coordinate (x) has been normalised in terms of Debye length $\lambda_D = (\varepsilon_0 T_e/n_0 e^2)^{1/2}$ and time coordinates by the inverse of ion plasma frequency. Here $\beta = 4\rho/(1+\rho)$ where ρ is the electron nonthermal parameter which determines the ratio of fast energetic and thermal electrons. The temperature ratio of positron with electron fluid and the ratio of positron concentration with equilibrium density of ion fluid is $\gamma = T_p/T_e$ and $\alpha = n_p^{(0)}/n^{(0)}$ respectively. $\sigma = 3T = 3T_i/T_e$ where T_i and T_e are the temperature of the ion and the electron fluid respectively.

To derive the ZK equation from the basic set of equations, viz. eqs (1)–(3), we use RPM, introducing the following stretched coordinates (ξ) and (τ) as

$$X = \varepsilon x, \quad Y = \varepsilon y, \quad Z = \varepsilon(z - \lambda t) \text{ and } \tau = \varepsilon^3 t, \tag{4}$$

where ε is a small parameter and λ is the phase velocity of the wave in the ion-acoustic wave frame which is to be determined later.

We expand the dependent variables in the power series of ε about their equilibrium values as follows:

$$\begin{bmatrix} N \\ \phi \\ V_z \\ V_{x,y} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} N^{(1)} \\ \phi^{(1)} \\ V_z^{(1)} \\ 0 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} N^{(2)} \\ \phi^{(2)} \\ V_z^{(2)} \\ V_{x,y}^{(1)} \end{bmatrix} + \cdots \tag{5}$$

Substituting expansion (5) into eqs (1)–(3), we obtain the lowest order of ε as

$$N^{(1)} = \frac{1}{\left(\lambda^2 - 2\sigma\right)}\phi^{(1)}, \tag{6}$$

$$V_x^{(1)} = -B\frac{\lambda^2}{\left(\lambda^2 - 2\sigma\right)}\frac{\partial\phi^{(1)}}{\partial Y}, \tag{7}$$

$$V_y^{(1)} = B\frac{\lambda^2}{\left(\lambda^2 - 2\sigma\right)}\frac{\partial\phi^{(1)}}{\partial X}, \tag{8}$$

$$V_z^{(1)} = \frac{\lambda}{\left(\lambda^2 - 2\sigma\right)}\phi^{(1)}. \tag{9}$$

Using eq. (6) in Poisson equation (3) and keeping lowest order $(O(\varepsilon))$ term, we get the following relation:

$$\lambda^2 = \frac{[(1-\alpha) + 2\sigma\{(1-\beta) + \alpha\gamma\}]}{[(1-\beta) + \alpha\gamma]}. \tag{10}$$

Considering the next higher-order term, i.e., $O(\varepsilon^2)$, we get the second-order solutions:

$$N^{(2)} = \frac{1}{(\lambda^2 - 2\sigma)}\phi^{(2)} + \left[\frac{\lambda^2 + 4\sigma(\lambda^2 - 2\sigma) + 2(\lambda^2 - 2\sigma)}{2(\lambda^2 - 2\sigma)^3}\right]\phi^{(1)^2}, \tag{11}$$

$$V_x^{(2)} = B\left[-\frac{\partial \phi^{(2)}}{\partial Y} - 2\sigma\frac{\partial N^{(2)}}{\partial Y} + B\frac{\lambda^3}{(\lambda^2 - 2\sigma)}\frac{\partial^2 \phi^{(1)}}{\partial Z \partial X}\right], \tag{12}$$

$$V_y^{(2)} = B\left[\frac{\partial \phi^{(2)}}{\partial Y} + 2\sigma\frac{\partial N^{(2)}}{\partial Y} + B\frac{\lambda^3}{(\lambda^2 - 2\sigma)}\frac{\partial^2 \phi^{(1)}}{\partial Y \partial Z}\right], \tag{13}$$

$$V_z^{(2)} = \frac{\lambda}{(\lambda^2 - 2\sigma)} \times \left[\phi^{(2)} + \frac{\lambda^2 + 4\sigma(\lambda^2 - 2\sigma)}{2(\lambda^2 - 2\sigma)^2}\phi^{(1)^2}\right]. \tag{14}$$

From third-order continuity equations (1)–(3), with the help of eqs (6)–(9) and eqs (11)–(14), we obtain the following ZK equation:

$$\frac{\partial \phi}{\partial \tau} + P\phi\frac{\partial \phi}{\partial Z} + Q\frac{\partial^3 \phi}{\partial Z^3} + R\left[\frac{\partial^3 \phi}{\partial X^2 \partial Z} + \frac{\partial^3 \phi}{\partial Y^2 \partial Z}\right] = 0, \tag{15}$$

where

$$P = \frac{Q}{2[(1-\beta) + \alpha\gamma]} \times \left[(1-\alpha)\frac{\{\lambda^2 + (4\sigma + 2)(\lambda^2 - 2\sigma)\}}{(\lambda^2 - 2\sigma)^3} + [\alpha\gamma^2 - \{1 + 2(1-\beta)\}]\right], \tag{16}$$

$$Q = \frac{(\lambda^2 - 2\sigma)^2}{2\lambda(1-\alpha)} \tag{17}$$

and

$$R = Q\left[1 - B\frac{\lambda^4(1-\alpha)}{(\lambda^2 - 2\sigma)^2}\right]. \tag{18}$$

In eq. (15) ϕ is used in place of $\phi^{(1)}$, to avoid superscript.

3. Solutions of the nonlinear ZK equation

Denoting $\phi(X, Y, Z, \tau) = f(\xi)$ in eq. (15), it reduces to the nonlinear ordinary differential equation

$$-vf' + P_l f f' + Q_l f''' = 0, \tag{19}$$

where

$$\xi = A_X X + A_Y Y + A_Z Z - v\tau.$$

Here A_X, A_Y and A_Z are the directional cosines (satisfying $A_X^2 + A_Y^2 + A_Z^2 = 1$) of the wave vector along the X-, Y- and Z-axes, v is a constant soliton speed, $P_l = PA_Z$ and $Q_l = QA_Z^3 + RA_Z(A_X^2 + A_Y^2)$.

Following El-Labany *et al* [9], we find

$$\phi = \phi_0 \operatorname{sech}^2\left(\frac{\xi}{W}\right). \tag{20}$$

Solution (20) represents a solitary wave solution of the ZK equation (15), which has amplitude ϕ_0 and width (W) where

$$\phi_0 = \frac{3v}{P_l}, \tag{21}$$

$$W = \left(\frac{4Q_l}{v}\right)^{1/2}. \tag{22}$$

4. Discussion of results and conclusions

Our numerical calculations with different sets of plasma parameters using eqs (10), (21) and (22) for the change in phase velocity (λ), ϕ_0 and width (W) of solitons due to the presence of nonthermal electrons and positrons in a plasma is shown in figure 1. From this graph, it may be noted that λ of the ion-acoustic wave increases with increase in nonthermal parameter (β) for a given positron temperature ratio $(\gamma) = 0.005$ and relative positron concentration $(\alpha) = 0.001$. It may also be noted that λ also increases as ion temperature ratio (σ) increases.

Figure 2 depicts the variations of amplitude of the solitons with β, for different values of $\sigma = 0.01$ (red), 0.03 (green), 0.05 (yellow), 0.08 (black) and 0.1 (blue) keeping $\gamma = 0.05$, $\alpha = 0.001$, $A_X = 0.03$, $A_Y = 0.01$ and $v = 0.09$. From the graph, we may note that as β increases, the amplitude of the soliton decreases for any given value of σ. But for a given value of β, the amplitude of the soliton increases as σ increases. For further discussion of figures 3–6, we have presented the variation in amplitude and width of the soliton against the parameter shown on the graph. Please note that the other parameters are kept constant in this study.

Variation in the amplitude of the soliton with β for different values of α are shown in figure 3. From the

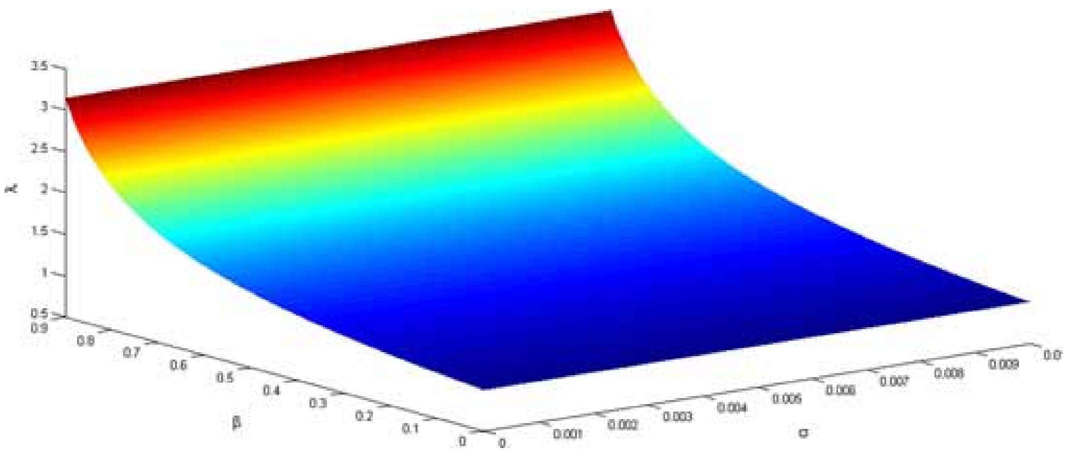

Figure 1. Ion-acoustic wave phase velocity (λ) is plotted against nonthermal parameters (β) and ion temperature ratio (σ) for $\gamma = 0.05$ and $\alpha = 0.001$.

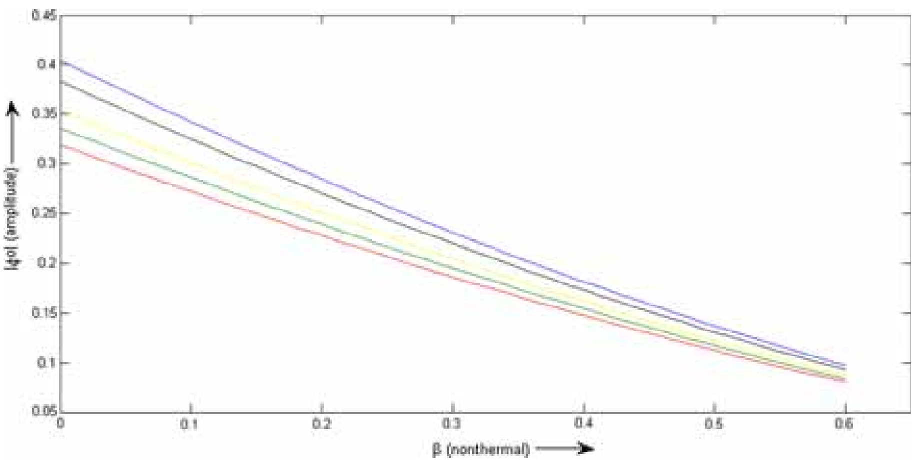

Figure 2. The amplitude (ϕ_0) of the soliton is plotted against nonthemal parameter (β), for σ (= 0.01 (red), 0.03 (green), 0.05 (yellow), 0.08 (black) and 0.1 (blue)). The other parameters are: $\gamma = 0.05$, $\alpha = 0.001$, $v = 0.09$, $A_X = 0.03$ and $A_Y = 0.01$.

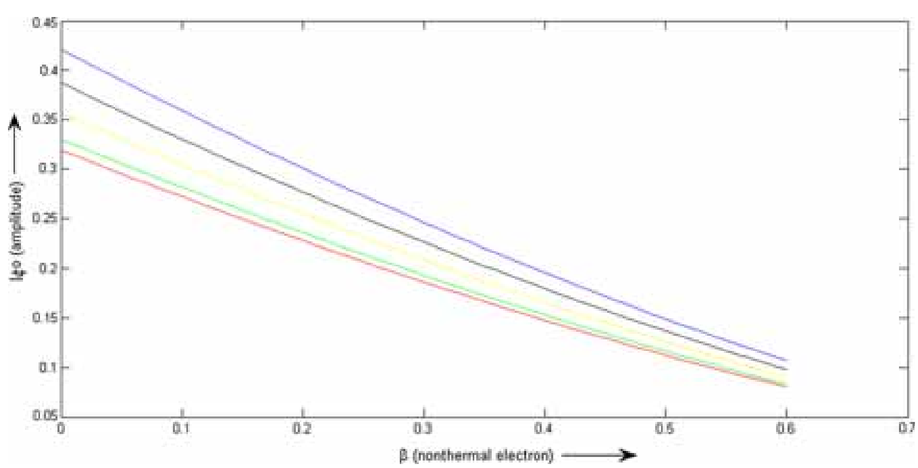

Figure 3. The amplitude (ϕ_0) of the soliton is plotted against nonthemal parameter (β), for different values of α(= 0.001 (red), 0.01 (green), 0.03 (yellow), 0.05 (black) and 0.07 (blue)). The other parameters are: $\gamma = 0.05$, $\alpha = 0.001$, $v = 0.09$, $A_X = 0.03$ and $A_Y = 0.01$.

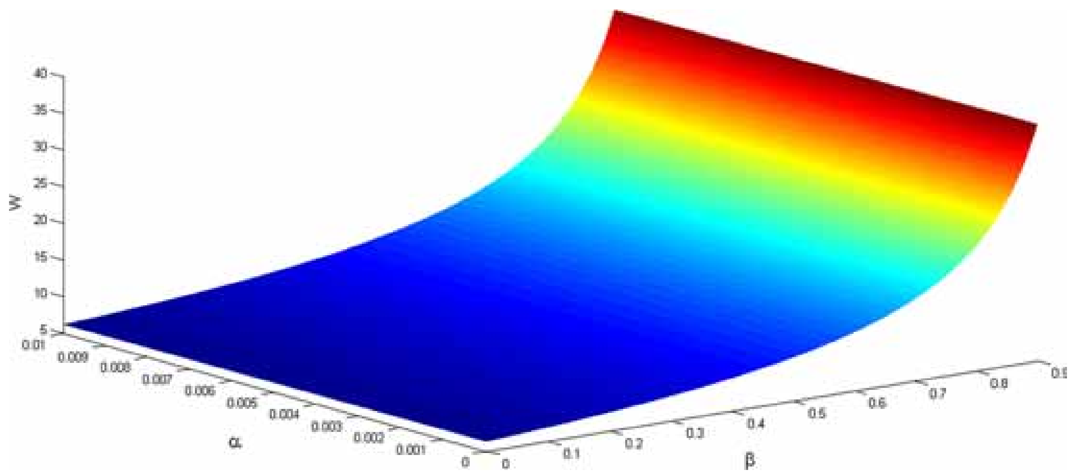

Figure 4. The width (W) of the soliton is plotted against positron concentration (α) and nonthemal parameter (β), for $\gamma = 0.05$, $\alpha = 0.001$, $A_X = 0.03$, $A_Y = 0.01$, $v = 0.09$ and $B = 5$.

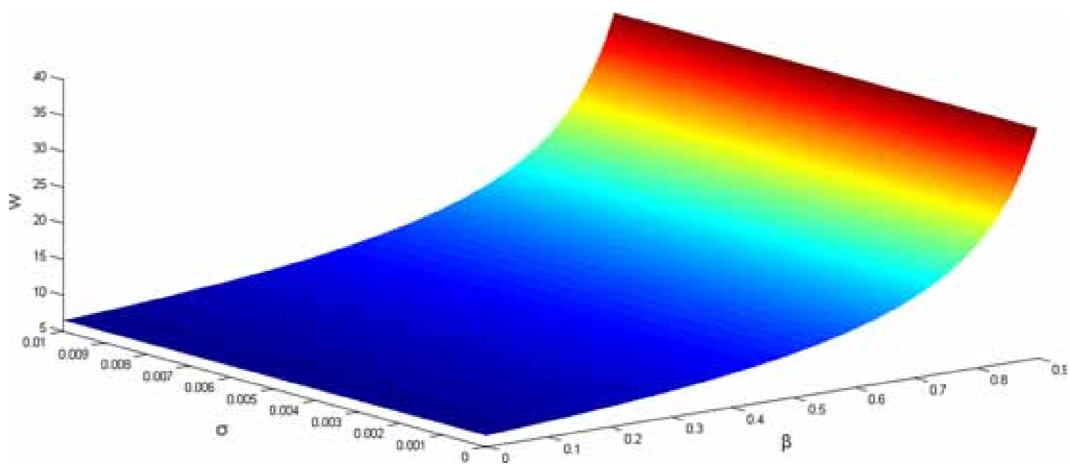

Figure 5. The width (W) of the soliton is plotted against ion temperature ratio (σ) and nonthemal parameter (β), for $\gamma = 0.05$, $\alpha = 0.001$, $v = 0.09$, $A_X = 0.03$, $A_Y = 0.01$ and $B = 5$.

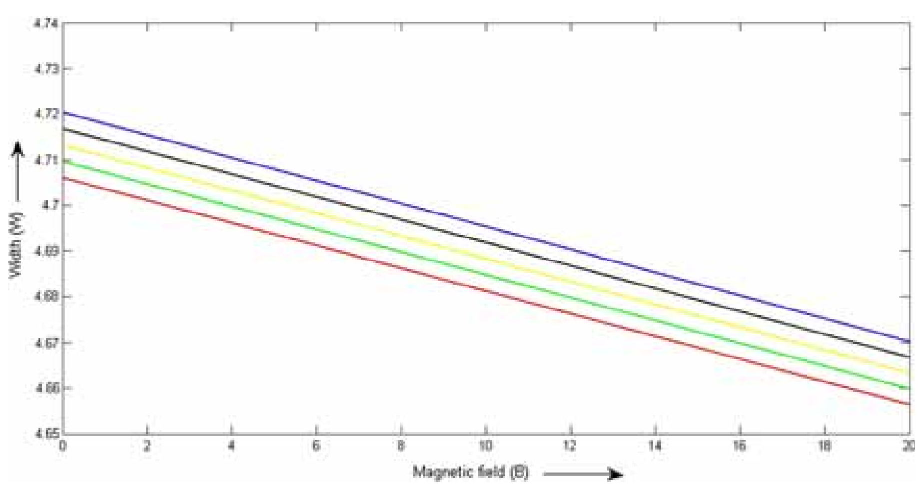

Figure 6. The width (W) of the soliton is plotted against magnetic field (B), for different values of β(= 0.005 (red), 0.006 (green), 0.007 (yellow), 0.008 (black) and 0.009 (blue)). The other parameters are: $\gamma = 0.05$, $\alpha = 0.001$, $\sigma = 0.005$, $A_X = 0.03$, $A_Y = 0.01$ and $v = 0.09$.

Table 1. Amplitude of the solitary wave with $\beta = 0.1$, $\gamma = 0.05$, $\alpha = 0.001$, $v = 0.09$, $A_X = 0.03$ and $A_Y = 0.01$ for different values of σ.

Ion temperature ratio (σ)	Amplitude (ϕ_0)
0.01	0.28
0.03	0.30
0.05	0.32
0.08	0.34
0.1	0.35

Table 2. Amplitude of the solitary wave with $\sigma = 0.01$, $\gamma = 0.05$, $\alpha = 0.001$, $v = 0.09$, $A_X = 0.03$ and $A_Y = 0.01$ for different values of β.

Nonthemal parameter (β)	Amplitude (ϕ_0)
0.1	0.28
0.2	0.25
0.3	0.21
0.4	0.176
0.5	0.148

figure, we note that for a given value of α, amplitude of the soliton decreases as β increases. But for a given value of β, the amplitude of the soliton increases as α increases.

Figure 4 shows the variation of the width of the soliton with β and α. From the 3D view of the graph, we may note that width of the soliton increases as β and α increase. Similar increase in width of the soliton is also noted as a function of β and σ as shown in figure 5. From figure 6, we note that for a given value of β, the width of soliton decreases as magnetic field (B) increases. But for a given value of B, the width of the soliton increases as β increases.

It may be noted from table 1 that when $\beta = 0.1$, for $\gamma = 0.05$, $\alpha = 0.001$, $v = 0.09$, $A_X = 0.03$, $A_Y = 0.01$ and $B = 5$, by increasing σ, the amplitude of the solitary wave increases. Table 2 shows that when $\sigma = 0.01$, for $\gamma = 0.05$, $\alpha = 0.001$, $v = 0.09$, $A_X = 0.03$, $A_Y = 0.01$ and $B = 5$, for different values of β, the amplitude of the solitary wave decreases. From table 3, we can find that when $\beta = 0.1$, $\gamma = 0.05$, $\alpha = 0.001$, $A_X = 0.03$, $A_Y = 0.01$, $v = 0.09$ and $B = 5$, for different values of α, the amplitude of the solitary wave increases. Table 4 shows that when $\beta = 0.009$, $\gamma = 0.05$, $\sigma = 0.01$, $\alpha = 0.001$, $A_X = 0.03$, $A_Y = 0.01$ and $v = 0.09$ for different values of B, the width of the solitary wave decreases. For table 5, we find that when $B = 5$, $\gamma = 0.05$, $\sigma = 0.01$, $\alpha = 0.001$, $A_X = 0.03$, $A_Y = 0.01$ and $v = 0.09$ for different values of β, the width of the solitary wave increases.

Table 3. Amplitude of the solitary wave with $\beta = 0.1$, $\gamma = 0.05$, $\alpha = 0.001$, $A_X = 0.03$, $A_Y = 0.01$ and $v = 0.09$ for different values of α.

Positron concentration (α)	Amplitude (ϕ_0)
0.01	0.27
0.02	0.30
0.03	0.32
0.045	0.34
0.07	0.37

Table 4. Width of the solitary wave with $\beta = 0.009$, $\gamma = 0.05$, $\sigma = 0.01$, $\alpha = 0.001$, $A_X = 0.03$, $A_Y = 0.01$ and $v = 0.09$ for different values of B.

Magnetic field (B)	Width (W)
0	4.7203
5	4.7079
10	4.6954
15	4.6828
20	4.6702

Table 5. Width of the solitary wave with $B = 5$, $\gamma = 0.05$, $\sigma = 0.01$, $\alpha = 0.001$, $A_X = 0.03$, $A_Y = 0.01$ and $v = 0.09$ for different values of β.

Nonthemal parameter (β)	Width (W)
0.005	4.695
0.006	4.705
0.007	4.707
0.008	4.709
0.009	4.712

In the present study, our focus was to investigate the effect of α, β and σ on the characteristics of solitons in magnetised plasma. Our main conclusions of the study are as follows:

(1) We have presented a comprehensive study of 3D IASWs in magnetised plasma using RPM to derive nonlinear ZK equation and investigated the effect of α, β and σ on the amplitude, width and velocity of solitary pulses, whereas El-Labany *et al* [9] have considered the propagation of solitary waves in magnetised negative ion plasma consisting of ions, positrons and isothermal electrons.
(2) The analysis of our ZK equation shows that the magnetisation of plasma affect the width of solitary pulses but do not affect the amplitude. Considering isothermal electrons in plasma, such behaviour is also investigated by Chawla and Mishra [17].
(3) For a given set of plasma parameters, on increasing β, the amplitude of the solitary pulses

decreases, but it increases with increase in α and σ.

(4) Width of the soliton increases as α, σ or B decreases but it increases with increase in β, for a given set of parameters.

(5) Phase velocity of IAWs also increases as β and σ increase.

Acknowledgement

Dr R S Tiwari is grateful to the management of Poornima College of Engineering, Sitapura, Jaipur for their encouragement for doing this research and also for providing the necessary facilities.

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