Pramana – J. Phys. (2020) 94:9
https://doi.org/10.1007/s12043-019-1877-1

On: New optical soliton solutions for nonlinear complex fractional Schrödinger equation via new auxiliary equation method and novel (G′/G)-expansion method

MARWAN AL-RAEEI[1],* and MOUSTAFA SAYEM EL-DAHER[2]

[1]Department of Physics, Faculty of Sciences, Damascus University, Damascus, Syria
[2]Faculty of Informatics and Communications, Arab International University, Daraa, Syria
*Corresponding author. E-mail: mhdm-ra@scs-net.org; mn41@live.com

MS received 25 July 2019; revised 9 September 2019; accepted 11 September 2019

Abstract. A comment on an article by Khater *et al* published in *Pramana – J. Phys.* **90**: 59 (2018) is presented here. We represent two quotes on the article, the two quotes about the space-dependent fractional Schrödinger equation type and about one of the constants used by the authors.

Keywords. Fractional Schrödinger equation; new auxiliary method; optical solitary waves.

PACS Nos 03.65.−w; 41.20.−; 02.30.jr

The most significant equation in the fractional quantum-mechanics found by Laskin is the fractional Schrödinger equation which takes the following formula in space form [1–8]:

$$\psi_t(\mathbf{r}, t) = -\frac{i}{\hbar}\hat{H}_\alpha \psi(\mathbf{r}, t). \qquad (1)$$

With the Hamiltonian operator:

$$\hat{H}_\alpha = V(\mathbf{r}) + D_\alpha(-\hbar^2\mathbf{\Delta})^{\alpha/2}, \qquad (2)$$

where $\psi_t(\mathbf{r}, t)$ represents the partial derivative of the wave function with respect to time, α is the space-fractional parameter and D_α represents a scale parameter.

The results obtained by Khater *et al* [9] are interesting, leading to a new representation of solitary waves but we need to comment on two points in this work.

Both the comments are about eq. (1.1) in [9] which takes the following form in their article:

$$ih\psi_t = D_\alpha(-\hbar^2\mathbf{\Delta})^{\alpha/2}\psi + V\psi. \qquad (3)$$

The first and the most important thing which we comment on, is the type of space-dependent fractional Schrödinger equation (eq. (1.1) in [9] and the third equation in this comment). This class of equations is a type of linear equations [3,5–7,10–15]. The fractional index in this equation corresponds to the order of derivative and so eq. (1.1) is a linear equation and Khater *et al* [9], in their work, state that the equation is nonlinear which is incorrect.

The second thing which we comment on in the same work is the Planck constant in the same equation. The constant appeared in this equation is the reduced Planck constant, not the Planck constant as in refs [1–4,7,13,16] as we stated in this article (eqs (1) and (2)) but Khater *et al* [9] stated that this constant is the Planck constant and they used the symbol h.

The studies in Banerjee *et al* [16], Tala-Tebue *et al* [17], Xie *et al* [18], Zayed *et al* [19], Feng *et al* [20], Mylonas *et al* [21], Wu and Jiang [22], represent solutions leading to the solitary waves from nonlinear Schrödinger equation, and the studies in Ma and Lee [23] and Ma *et al* [24] represent solutions to the non-linear equation with the transformed rational function method and the multiple exp-function method. It is very clear that the errors observed in the type of equation and Planck constant in ref. [9] are not present in the aforementioned references.

References

[1] N Laskin, *Chaos* **10**, 780 (2000), https://doi.org/10.1063/1.1050284
[2] N Laskin, *Chaos Solitons Fractals* **102**, 16 (2017), https://doi.org/10.1016/j.chaos.2017.04.010

Published online: 18 December 2019

[3] N Laskin, *Fractional quantum mechanics* (World Scientific Publishing, 2018)
[4] N Laskin, *Phys. Lett. A* **268**, 298 (2000)
[5] X Chang, *J. Math. Phys.* **54**, 061504 (2013), https://doi.org/10.1063/1.4809933
[6] Qun Liu, Fanhai Zeng and Changpin Li, *Int. J. Comp. Math.* **92**, 1439 (2015), https://doi.org/10.1080/00207160.2014.945440
[7] Marwan Al-Raeei and M S El-Daher, *Phys. Lett. A* **383**, 125831 (2019), https://doi.org/10.1016/j.physleta.2019.07.019
[8] Hasan A Fallahgoul, Sergio M Focardi and Frank J Fabozzi, *Fractional calculus and fractional processes with applications to financial economics* (Elsevier, 2017)
[9] Mostafa M A Khater, Aly R Seadawy and D Lu, *Pramana – J. Phys.* **90**: 59 (2018), https://doi.org/10.1007/s12043-018-1547-8
[10] Yong Zhou, *Basic theory of fractional differential equations* (World Scientific Publishing, 2014)
[11] Marwan Al-Raeei and M S El-Daher, *Chem. Phys. Lett.* **734**, 136729 (2019), https://doi.org/10.1016/j.cplett.2019.136729
[12] A M Mathai and Hans J Haubold, *Matrix methods and fractional calculus* (World Scientific Publishing, 2017)
[13] R Hilfer, *Applications of fractional calculus in physics* (World Scientific Publishing, 2000)
[14] Yong Zhou, JinRong Wang and Lu Zhang, *Basic theory of fractional differential equations* (World Scientific Publishing, 2017)
[15] Marwan Al-Raeei and M S El-Daher, *Pramana – J. Phys.* **90**: 60 (2018), https://doi.org/10.1007/s12043-018-1550-0
[16] Joydip Banerjee, Uttam Ghosh, Susmita Sarkar and Shantanu Das, *Pramana – J. Phys.* **88**: 70 (2017), https://doi.org/10.1007/s12043-017-1368-1
[17] E Tala-Tebue, Z I Djoufack, E Fendzi-Donfack, A Kenfack-Jiotsa and T C Kofané, *Optik* **127**, 11124 (2016), https://doi.org/10.1016/j.ijleo.2016.08.116
[18] Yingying Xie, Z Yang and Lingfei Li, *Phys. Lett. A* **382**, 2506 (2018), https://doi.org/10.1016/j.physleta.2018.06.023
[19] Elsayed M E Zayed, Ayad M Shahoot and Khaled A E Alurrfi, *Opt. Quant. Electron.* **50**, 96 (2018), https://doi.org/10.1007/s11082-018-1337-z
[20] Dahe Feng, Jianjun Jiao and Guirong Jiang, *Phys. Lett. A* **382**, 2081 (2018), https://doi.org/10.1016/j.physleta.2018.05.028
[21] I K Mylonas, C B Ward, P G Kevrekidis, V M Rothos and D J Frantzeskakis, *Phys. Lett. A* **381**, 3965 (2017), https://doi.org/10.1016/j.physleta.2017.10.043
[22] Hong Yu-Wu and Li Hong Jiang, *Pramana – J. Phys.* **89**, 40 (2017), https://doi.org/10.1007/s12043-017-1438-04
[23] Wen Xiu Ma and Jyh-Hao Lee, *Chaos Solitons Fractals* **42**, 1356 (2009), https://doi.org/10.1016/j.chaos.2009.03.043
[24] Wen Xiu Ma, Tingwen Huang and Yi Zhang, *Phys. Scr.* **82**, 065003 (2010), https://doi.org/10.1088/0031-8949/82/06/065003