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Chirped solitons in optical monomode fibres modelled with Chen–Lee–Liu equation

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Abstract. The paper studies the extraction of chirped soliton to Chen–Lee–Liu equation (CLLE) with the group velocity dispersion (GVD) and self-steeping coefficients that describe pulse transmission through optical monomode fibres. The chirped bright, dark and singular optical solitons are obtained and the results show that nonlinear chirp parameters strongly vary on self-steeping, GVD and spreading effects. The constraint conditions for the existence of solitons are also derived during the derivation. The results are helpful and important for understanding the propagation of optical pulses.

Keywords. Chirped solitons; Chen–Lee–Liu equation; dual power law of nonlinearity.

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1. Introduction

Optical solitons attracted the attention of researchers due to the capability of propagation of waves without scattering over long distance, i.e., they do not change their shape over long distance [1–6]. They are important in optical fibre communication due to this property. The nonlinear wave phenomena can be observed in different scientific fields such as, fluid dynamics, plasma physics, solitary waves, optical fibres etc. [7–15].

The pulse propagation of chirped soliton in optical fibres has wide applications in pulse amplification and is particularly useful in optical pulse compressors (see also [16–32]). They are also important in soliton-based communication links and in designing fibre-optic amplifiers. The propagation of these pulses are modelled by Chen–Lee–Liu equation (CLLE) [33,34] and is given by

$$iq_t + aq_{xx} + ib|q|^{2n}q_x = 0, \tag{1}$$

where $q(x,t)$ is a wave profile with the spatial and temporal variables x and t, respectively. The first term on the left side represents linear development, while second term represents group velocity dispersion (GVD) and the third one represents the self-phase modulation due to dual power law of nonlinearity [15–20]. It appears when GVD is low, which happens in reality. Hence, the coefficients a and b are connected with GVD and self-steeping phenomena, respectively.

The parameter n in the above equation describes the dual power law of nonlinearity. It should be noted that, it is imperative to maintain,

$$0 < n < 2. \tag{2}$$

For stability of the soliton $n \neq 2$ to eliminate self-converging singularity. The recent achievement in the concept of solitons has caused the simplification of various nonlinear evolution equations that have extensive applications in mathematics and physics [15–25].

2. Extraction of solitons

The following complex function is considered for the extraction of chirped solitons:

$$q(x,t) = \rho(\xi)e^{i[\chi\xi - \Omega t]}, \tag{3}$$

where the real valued functions are given by ρ and χ of the coordinates $\xi = kx - vt$, and it is named as travelling wave. The related chirp is given by the equation

$$\delta\omega(t,x) = \frac{\partial}{\partial x}[\chi\xi - \Omega t] = -k\chi'(\xi).$$

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The substitution of eq. (3) into (1) gives equations in real and imaginary parts. The simplification leads to the following equations in the form of ρ and χ:

$$-\chi'\nu + ak^2\rho'' + \chi' k\rho^{2n+1} - ak^2\rho\chi'^2 = 0 \tag{4}$$

and

$$-\nu\rho' - ak^2\chi'\rho' - \chi'' ak^2\rho - ak^2\chi'\rho' - \rho'\rho^{2n}kb = 0. \tag{5}$$

Furthermore, we now consider an ansatz to solve eqs (4) and (5). This ansatz depends on wave amplitude and is given by

$$\chi' = \sigma\rho^{2n} + \eta. \tag{6}$$

Consequently, the resultant chirp transforms into $\delta\omega(t,x) = -(\sigma\rho^{2n} + \eta)$, where σ and η are chirp parameters which are nonlinear and constant in nature. This shows that the chirp related to propagating pluses depends on intensity (i.e., $\delta\omega(t,x) = -\sigma\rho^{2n} - \eta$, where $I = |q|^2 = \rho^2$) and includes linear and nonlinear terms.

If one substitutes eq. (6) into eq. (5), the relation for σ and η can be obtained in the following forms. The relation for the nonlinear chirp parameter (σ) is given by

$$\sigma = \frac{b}{2ak(1+nk)}. \tag{7}$$

This relation of chirp constraint σ strongly vary on the self steeping, GVD, and nonlinear spreading effects. The relation for constant chirp parameter (η) is given by

$$\eta = -\frac{\nu}{2ak^2}. \tag{8}$$

This relation of constant chirp parameter depends on GVD. Thus, the difference of these coefficients allows to calculate the effective power of the breadth of chirping using eqs (6)–(8) into eq. (4), and one attains,

$$C_1\rho + C_2\rho'' + C_3\rho^{2n+1} + C_4\rho^{4n+1} = 0. \tag{9}$$

Equation (9) is elliptic in nature, and describes the growth of the wave breadth in a monomode fibre. This phenomenon is described by the modified CLLE, given in eq. (1) analytically for $C_j \neq 0$ ($j = 1, \ldots, 4$) to get chirped solitons for bright, dark and singular soliton solutions for the model given in eq. (1), where the values of C_j's are

$$C_1 = \frac{\nu^2 - 2a^2k^4\eta^2}{2ak^2},$$

$$C_2 = ak^2, \quad C_3 = \frac{\nu(k + k^2n - b) - 4a^2nk\sigma}{2ak^2(1+nk)}$$

and

$$C_4 = \frac{b - 2a^2k^2\sigma^2(1+nk)}{2a}.$$

Moreover, we construct relations of chirped soliton for CLLE with GVD and self-steepening coefficients in the following sections.

3. Chirped solitons

This section deals with the extraction of different types of chirp solitons, which are given in the form of non-linear pluses that depend on the intensity of the pulse force. Two types of bright solitons are extracted and the conditions of chirping are also given.

Type-I: Let the first form of the bright soliton be given as

$$\rho(\xi) = \frac{D}{[1 + E\cosh(\mu\xi)]^{1/2n}}, \tag{10}$$

where μ, D and E are given as

$$\mu = [-4nC_1]^{1/2}, \tag{11}$$

$$D = \left[-\frac{2C_1(1+C_2)}{C_3}\right]^{1/2n} \tag{12}$$

and

$$E = \left[\frac{4C_1C_4(-1 - C_2^2 - 2C_2) + C_3^2(1+2C_2)}{C_2C_3^2(2n+1)}\right]^{1/2}. \tag{13}$$

The existence condition for the bright soliton for even values of n is given by $C_2C_3^2(2n+1) < 0$. However, if we take n as an odd integer then the soliton will be pointing downward. Hence, the bright soliton is given by the following equation:

$$q(x,t) = \frac{D}{[1 + E\cosh(\mu\xi)]^{1/2n}} e^{i[\chi(\xi) - \Omega t]} \tag{14}$$

and the frequented chirping of this solution is given by the relation

$$\delta\omega(t,x) = -\left(\frac{\sigma D^{2n}}{1 + E\cosh(\mu\xi)} + \eta\right), \tag{15}$$

where μ, D and E are already given by eqs (11)–(13).

The dynamical behaviour of this solution is shown in figure 1a for $D = 2, \mu = 2, E = 0.707, \Omega = 2, n = 1, \xi = 2$ and $\beta = -1.5$.

Type-II: Let the second form of the bright soliton be given as

$$\rho(\xi) = \frac{k}{[1 + \phi\cosh^2(\mu\xi)]^{1/2n}}, \tag{16}$$

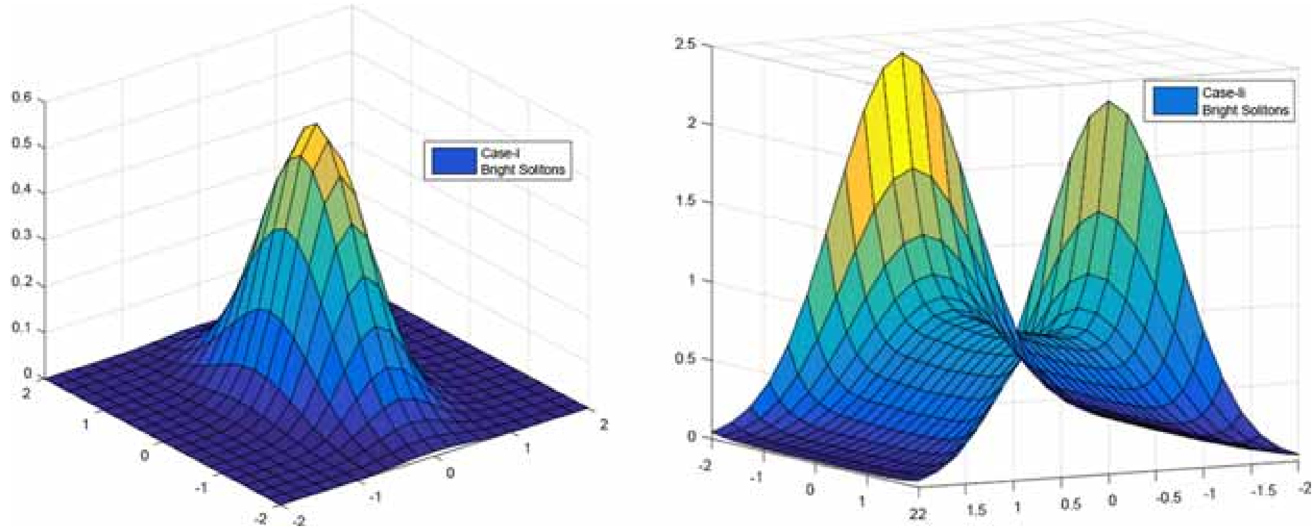

Figure 1. The dynamical behaviour of solutions (14) and (20) for different values of parameters.

where μ, k and ϕ are defined as

$$\mu = \left[-\frac{n^2 C_1}{C_2} \right]^{1/2}, \tag{17}$$

$$k = \left[-\frac{C_1(1+3n)}{C_3} \right]^{1/2n} \tag{18}$$

and

$$\phi = \left[\frac{C_3^2(1+3n) - C_1 C_4(1+6n+9n^2)}{n C_3^2} \right]. \tag{19}$$

The condition for the bright soliton is $nC_3^2 > 0$ for even values of n. However, if we take n as an odd integer then the above inequality will remain the same for the mentioned results of eq. (1).

$$q(x,t) = \frac{k}{[1+\phi \cosh^2(\mu\xi)]^{1/2n}} \mathrm{e}^{i[\chi(\xi) - \Omega t]} \tag{20}$$

and the consequent chirping is given by

$$\delta\omega(t,x) = -\left(\frac{\sigma r^{2n}}{1+\phi\cosh^2(\mu\xi)} + \eta \right), \tag{21}$$

where μ, k and ϕ are given by eqs (17)–(19).

The dynamical behaviour of this solution is shown in figure 1b for $\mu = 2.5$, $k = 2$, $\phi = 1.678$, $\Omega = 0.5$, $n = 1$ and $\beta = -1.5$.

Another form of the optical soliton is dark soliton in the field of nonlinear optics. Now, the dark soliton to eq. (1) is extracted by considering the following relation:

$$\rho(\xi) = [N(1 \pm \tanh(\mu\xi))]^{1/2n}, \tag{22}$$

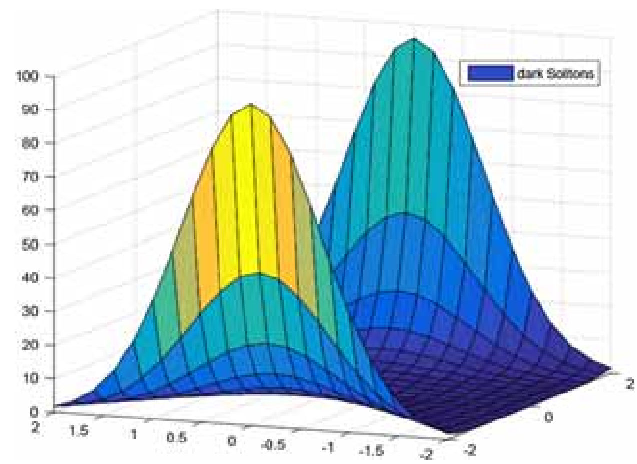

Figure 2. The dynamical behaviour of solution (25) for different values of parameters.

where μ and N are defined by the following relations:

$$\mu = \left[-\frac{C_3^2(1-12n)}{20nC_2C_4} \right]^{1/2} \tag{23}$$

and

$$N = \left[\frac{C_3(1-12n)}{20nC_4} \right]. \tag{24}$$

If the wave parameter μ is real, then we get the resulting chirped dark soliton solution for eq. (1) as

$$q(x,t) = [N(1 \pm \tanh(\mu\xi))]^{1/2n} \mathrm{e}^{i[\chi(\xi) - \Omega t]} \tag{25}$$

to consequent chirping given by

$$\delta\omega(t,x) = \sigma N(1 \pm \tanh(\mu\xi) - \eta) \tag{26}$$

if we suppose μ and N as in eqs (23) and (24).

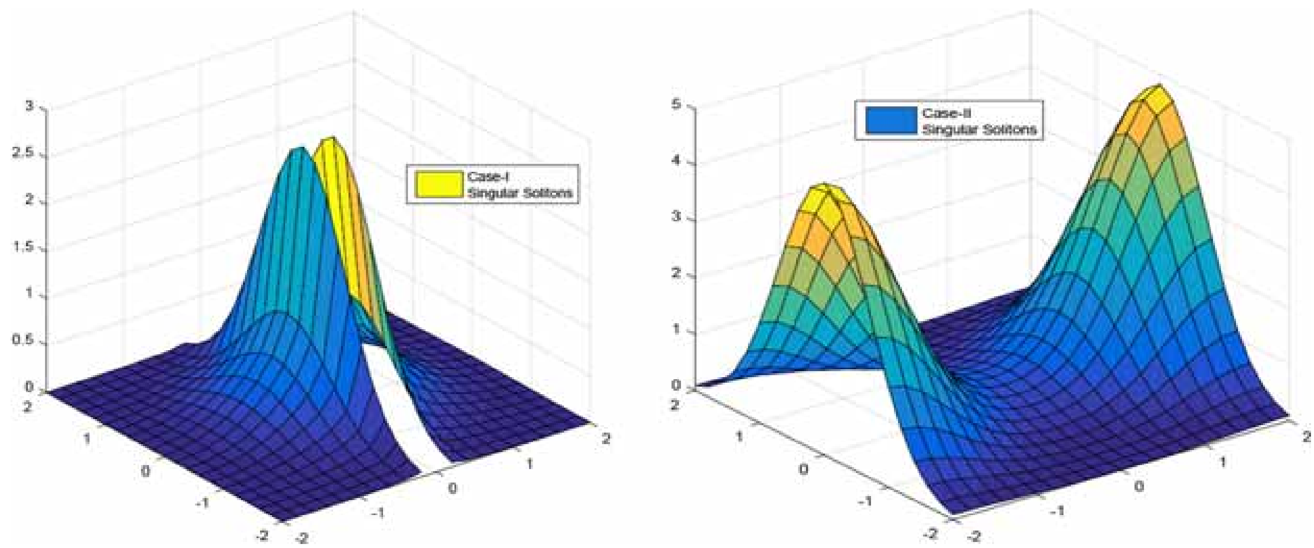

Figure 3. The dynamical behaviours of solutions (30) and (36) for different values of parameters.

The dynamical behaviour of this solution is shown in figure 2 for $N = 2.2$, $\mu = 2$, $E = 0.707$, $\Omega = 2$, $n = 1$, $\xi = 2$ and $\beta = -1.5$.

The singular solitons are derived for this when we consider the form of solutions and their associated chirping conditions as described in eqs (31) and (37).

Type-I: Let the first form of singular soliton be given as

$$\rho(\xi) = [D(1 \pm \coth(\mu\xi))]^{1/2n}, \tag{27}$$

where μ and D are defined by the following relations:

$$\mu = \left[\frac{2n^2 C_3}{C_2(1-2n)}\right]^{1/2} \tag{28}$$

and

$$D = \left[\frac{2n(C_3 - C_1) + C_1}{C_3(1-2n)}\right]. \tag{29}$$

If μ, which is a wave parmeter, is real then we get the resulting chirped dark soliton solution for eq. (1) as

$$q(x,t) = [D(1 \pm \coth(\mu\xi))]^{1/2n} e^{i[\chi(\xi) - \Omega t]}, \tag{30}$$

to the consequent chirping set by

$$\delta\omega(t,x) = -\sigma D(1 \pm \coth(\mu\xi) - \eta) \tag{31}$$

if we suppose μ and D as given in eqs (28) and (29).

The dynamical behaviour of this solution is shown in figure 3a for $D = 3$, $\mu = 2.4$, $E = 0.707$, $\Omega = 2$, $n = 2$, $\xi = 4$ and $\beta = -1.3$.

Type-II: Let the second form of the singular soliton is given by

$$\rho(\xi) = \frac{A}{[1 + R\sinh(\mu\xi)]^{1/2n}}, \tag{32}$$

where μ, A and R are defined by the following relations:

$$\mu = \left[-\frac{4n^2 C_1}{C_2}\right]^{1/2}, \tag{33}$$

$$A = \left[-\frac{2(n+1)C_1}{C_3}\right]^{1/2n} \tag{34}$$

and

$$R = \left[-1 + \frac{4C_1 C_4}{C_3^2(2n+1)}\right]. \tag{35}$$

Based on these findings, we set a second chirped singular soliton solution of eq. (1) of the system.

$$q(x,t) = \frac{A}{[1 + R\sinh(\mu\xi)]^{1/2n}} e^{i[\chi(\xi) - \Omega t]} \tag{36}$$

and its consequent chirping is given as

$$\delta\omega(t,x) = -\left(\frac{\sigma A^{2n}}{1 + R\sinh(\mu\xi)} + \eta\right), \tag{37}$$

where μ, A and R are given by the relations given in eqs (33)–(35).

The dynamical behaviour of this solution is shown in figure 3b for the following values: $\mu = 2.6$, $A = 2$, $R = 2.5$, $\phi = 1.78$, $\Omega = 0.55$, $n = 2$ and $\beta = -1.5$.

4. Conclusion

In this paper, the dynamics of chirped solitons is studied in a medium with dual power law of nonlinearity in optical monomode fibres with CLLE. The dark, bright and singular nonlinear chirp solitons are derived. The conforming integrability criteria are considered by providing the conditions, certainly arising from the analysis, and which exist with certain relations of the parameters of GVD and self-steepening nonlinearity. The paper, consequently, gives a lot of encouragement, motivation and inspiration to do future research in the area of chirp solitons.

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