Pramana – J. Phys. (2019) 93:98
https://doi.org/10.1007/s12043-019-1858-4

Switching from sub- to superluminal light in a Y-type atomic system using wavelength mismatching

KAVITA YADAV* and AJAY WASAN

Department of Physics, Indian Institute of Technology, Roorkee 247 667, India
*Corresponding author. E-mail: kavitayiitr@gmail.com

MS received 16 January 2019; revised 16 May 2019; accepted 11 July 2019

Abstract. We study the optical properties of a four-level Y system theoretically in ^{87}Rb atoms using density matrix formalism. On account of wavelength mismatching effects in the system transparency window splits and enhanced absorption regions are observed on either side of the line centre. This enhanced absorption is associated with anomalous dispersion resulting in superluminal light propagation. We also explore the dressed state analysis in the system. The effect of mismatching factor on the group index and group velocity is discussed.

Keywords. Sub- and superluminal light propagation; wavelength mismatching; group index; optical switching.

PACS Nos **42.50.−p; 42.50.Gy; 42.65.−k**

1. Introduction

Multilevel atomic systems have been a well acknowledged medium to study atomic coherence and quantum interference. Electromagnetically-induced transparency (EIT), a quantum interference phenomenon, cancels absorption of a weak probe field on the application of a strong control field [1,2]. This transparency is associated with normal dispersion at the line centre and a slow light can be obtained [3]. On the contrary, enhanced absorption results in anomalous dispersion which leads to superluminal light [4–6]. Initially, EIT was studied in simple three-level systems such as Λ, Ξ and V. But later it has been explored in multilevel systems having more than three levels. These multilevel systems result in complex EIT behaviour [7].

Effect of wavelength mismatching in an inhomogeneously broadened Y-type system has been studied and modifications in the absorption features are discussed [8,9]. Lasing without inversion is also discussed in Y-type system with variation in detuning [10]. So far the wavelength mismatching effects are studied to elucidate the modification in the absorption behaviour of EIT. However, there has not been any explicit study of this mismatching on the dispersion and group index. It would be interesting to evaluate both types of light propagation, sub- as well as superluminal, in a single system.

In the present work, we study the light propagation behaviour in a Y-type system for ^{87}Rb atoms using wavelength mismatching effects. EIT transparency can be interpreted in terms of quantum interference as well as formation of dressed states. The dressed state analysis to understand the underlying physics provides a simple explanation of the light–atom interaction. Wavelength matched condition is difficult to achieve in multilevel systems for a real atom, and hence we take into account the effect of wavelength mismatching in our considered system. In most of the EIT studies, the superluminal light propagation is caused by enhanced absorption at the line centre. But in our study, enhanced absorption is not observed at the line centre. However, we observe narrow absorption peaks on both sides of the line centre.

This paper is organised in the following way: Section 2 gives the theoretical model of a Y-type four-level system obtained with the help of density matrix formalism. Section 3 comprises the result and discussion part, and finally in §4 we present the conclusion of this work.

2. Theoretical model of the four-level Y-type system

We show the level structure of a four-level system in ^{87}Rb atom in figure 1. Rb states $5P_{1/2}$ and $5P_{3/2}$ act as ground and intermediate levels ($|1\rangle$ and $|2\rangle$). To study the wavelength mismatching in the system, different Rb states, $5D_{5/2}$, $7S_{1/2}$, $7D_{5/2}$ and $44D_{5/2}$, are taken as

Published online: 04 October 2019

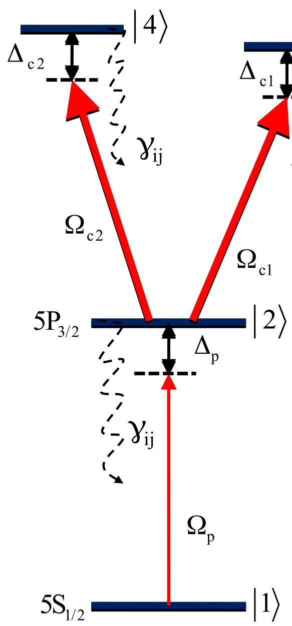

Figure 1. Composite Y-type system formed by the linkage of two Ξ systems. In the four-level model, $5D_{5/2}$, $7S_{1/2}$, $7D_{5/2}$ and $44D_{5/2}$ are taken as excited states to show wavelength mismatching. The dotted zig-zag lines represent decay from the intermediate and excited states with decohrence rate γ_{ij}.

excited levels $|3\rangle$ and $|4\rangle$. A weak probe field Ω_p links the levels $|1\rangle$ and $|2\rangle$ and two strong coupling fields Ω_{c1} and Ω_{c2} couple the levels $|2\rangle - |3\rangle$ and $|3\rangle - |4\rangle$, respectively.

The total Hamiltonian of the considered system using the rotating wave approximation is

$$\hat{H} = \hbar[(\Delta_p + k_p v)|2\rangle\langle 2| + (\Delta_p + k_p v + \Delta_{c1} - k_{c1} v) + |3\rangle\langle 3| + (\Delta_p + k_p v + \Delta_{c2} - k_{c2} v)|4\rangle\langle 4|] + \frac{\hbar}{2}(\Omega_p|1\rangle\langle 2| + \Omega_{c1}|2\rangle\langle 3| + \Omega_{c2}|2\rangle\langle 4| + \text{H.c.}), \quad (1)$$

where H.c. is the Hermitian conjugate of the precursory off-diagonal elements and k_p, k_{c1} and k_{c2} are the wave vectors of the probe and the control fields, respectively. The control and the probe fields are taken counterpropagating to each other in Ξ configuration to reduce the effect of residual Doppler width [11]. The detuning terms for these transitions are given as Δ_p, Δ_{c1} and Δ_{c2} such that $\Delta_p = \omega_{12} - \omega_p$, $\Delta_{c1} = \omega_{23} - \omega_{c1}$, $\Delta_{c2} = \omega_{24} - \omega_{c2}$, where ω_{ij} is the $|i\rangle \to |j\rangle$ atomic transition frequency. The decay rates from the intermediate level $|2\rangle$ and excited levels $|3\rangle$, $|4\rangle$ are Γ_2, Γ_3 and Γ_4, respectively.

Table 1. Decay rates and wavelength mismatching factors (α) for different Rb states.

States	Wavelength (nm)	$\Gamma_4/2\pi$ (MHz) [16,17]	Mismatching factor (α)
$5D_{5/2}$	776	0.68	−0.005
$7S_{1/2}$	741	1.8	−0.0526
$7D_{5/2}$	572	0.48	−0.3636
$44D_{5/2}$	480	0.3	−0.625

The density matrix equations are obtained in the semiclassical regime with the help of the Liouville equation incorporating the relaxation terms as follows:

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - \frac{1}{2}\{\Gamma, \rho\}, \quad (2)$$

where ρ is the density matrix and Γ is the relaxation matrix.

The coupled density matrix equations for the system can be obtained by substituting eq. (1) in eq. (3). The solution of these equations helps to obtain the analytical solution under the weak probe approximation [12].

$$\rho_{12} = \frac{(i\Omega_p/2)}{(\gamma_2/2) - i\delta_1 + \frac{(\Omega_{c1}/2)^2}{(\gamma_3/2) - i\delta_2} + \frac{(\Omega_{c2}/2)^2}{(\gamma_4/2) - i\delta_3}}, \quad (3)$$

where

$$\delta_1 = \Delta_p - k_p v,$$
$$\delta_2 = \Delta_p - k_p v + \Delta_{c1} + k_{c1} v,$$
$$\delta_3 = \Delta_p - k_p v + \Delta_{c2} + k_{c2} v. \quad (4)$$

The optical decoherence in the system is caused by decays from the intermediate and excited states, $\gamma_{ij} = (\Gamma_i + \Gamma_j)/2$, where $\Gamma_1 = 0$. The effective decoherence rate ($\gamma_{k=2,3,4}$) also includes the laser linewidth (γ_l), i.e., $\gamma_{k=2,3,4} = \gamma_{ij} + \gamma_l$ [6,13]. The two-photon coherence terms for the constituent Ξ subsystems are ρ_{13} and ρ_{14}, respectively. We consider the transit time (τ_d) of atoms with the optical field a few hundred of μs [9,14].

The probe response is studied in terms of imaginary and real parts of the coherence term ρ_{12} [15]. The real part of the coherence term ρ_{12} gives the dispersion properties and the nature of dispersion slope at the line centre speculates the light propagation behaviour in the medium.

We study EIT with various excited states including Rydberg state to see the effect of wavelength

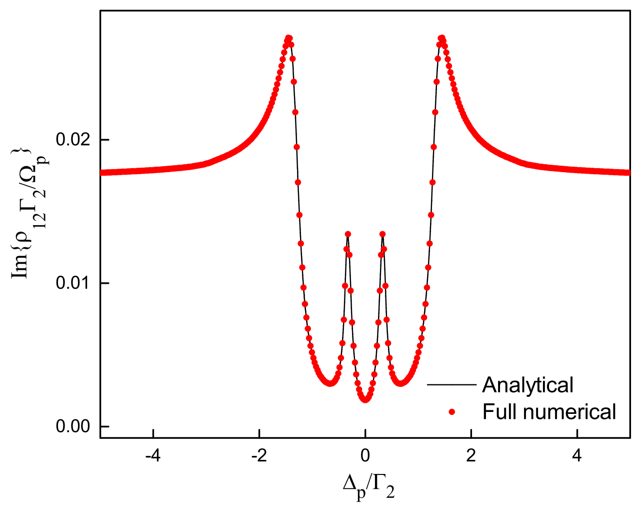

Figure 2. Comparison of analytical and full numerical solution for the Y-type system for room-temperature atomic vapours. In the calculation, the parameters are: $\Omega_p = 0.001 \times \Gamma_2$, $\Delta_{c1} = \Delta_{c2} = 0$ and $\Omega_{c1} = \Omega_{c2} = 4\Gamma_2$.

mismatching in the Y-type system. The wavelength mismatching factor (α) is obtained as follows [8]:

$$\alpha = \frac{k_p - k_{c1/c2}}{k_p}. \tag{5}$$

Wavelength mismatching factors for different excited states of Rb atom are listed in table 1.

To ensure the exactness of the analytical solution, in figure 2 we plot the comparative absorption behaviour of analytical and full numerical solution for room-temperature atomic vapours. The results show that our analytical solution is in good agreement with the full numerical results.

3. Results and discussion

We study the Y-type system for room-temperature ^{87}Rb atomic vapours. We also discuss the group index variation associated with dispersion properties of the system. For the wavelength matched condition splitting is not observed in the transparency window [9]. But when complete wavelength mismatch condition is taken such that $\lambda_p \neq \lambda_{c1} \neq \lambda_{c2}$, complex EIT behaviour is observed. To show the wavelength mismatching effect in our considered system, we fix λ_{c1} and vary λ_{c2}. Thus, three possible combinations of λ_{c1} and λ_{c2} are discussed in increasing order of mismatching factor (α).

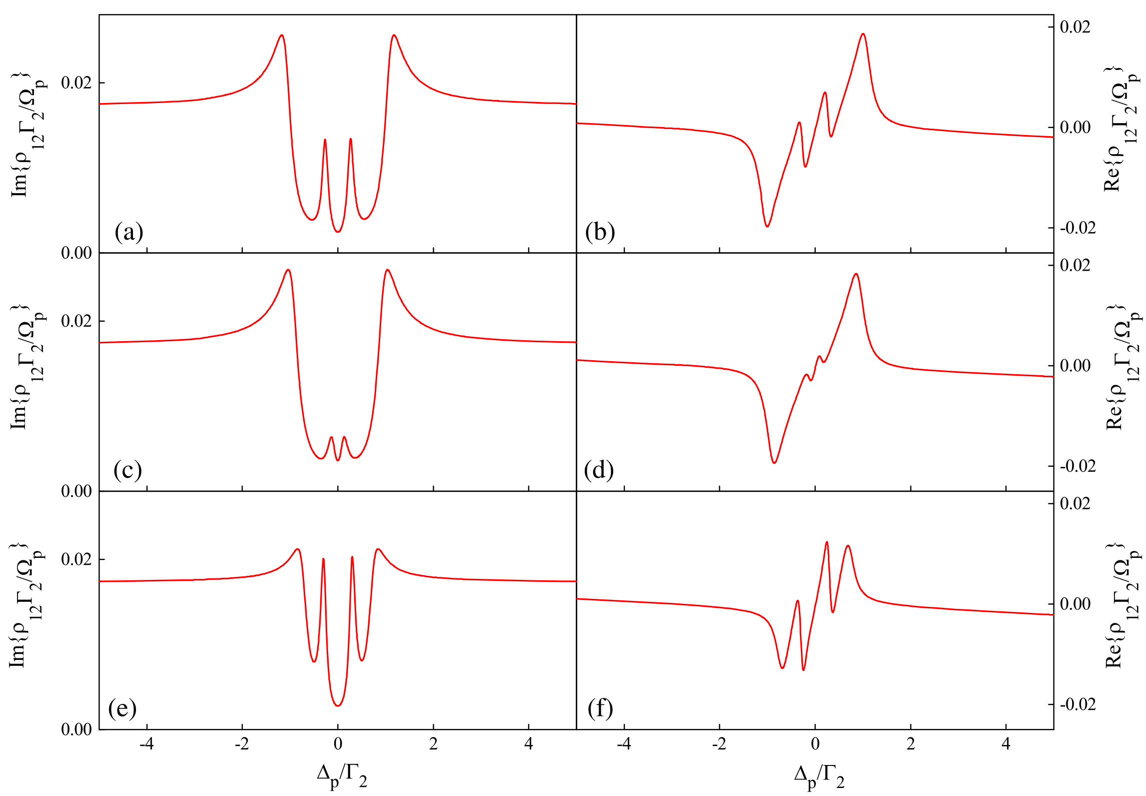

Figure 3. Imaginary (left column) and real (right column) parts of the probe response vs. probe detuning for the Y-type system for room-temperature atomic vapours. In the calculation, the parameters are $\Omega_p = 0.001 \times \Gamma_2$ and $\Delta_{c1} = \Delta_{c2} = 0$ with $\lambda_{c1} = 776$ nm and $\lambda_{c2} = 741$ nm. (**a**), (**b**) $\Omega_{c1} = \Omega_{c2} = 4\Gamma_2$, (**c**), (**d**) $\Omega_{c1} = 2\Gamma_2$, $\Omega_{c2} = 4\Gamma_2$ and (**e**), (**f**) $\Omega_{c1} = 4\Gamma_2$, $\Omega_{c2} = 2\Gamma_2$.

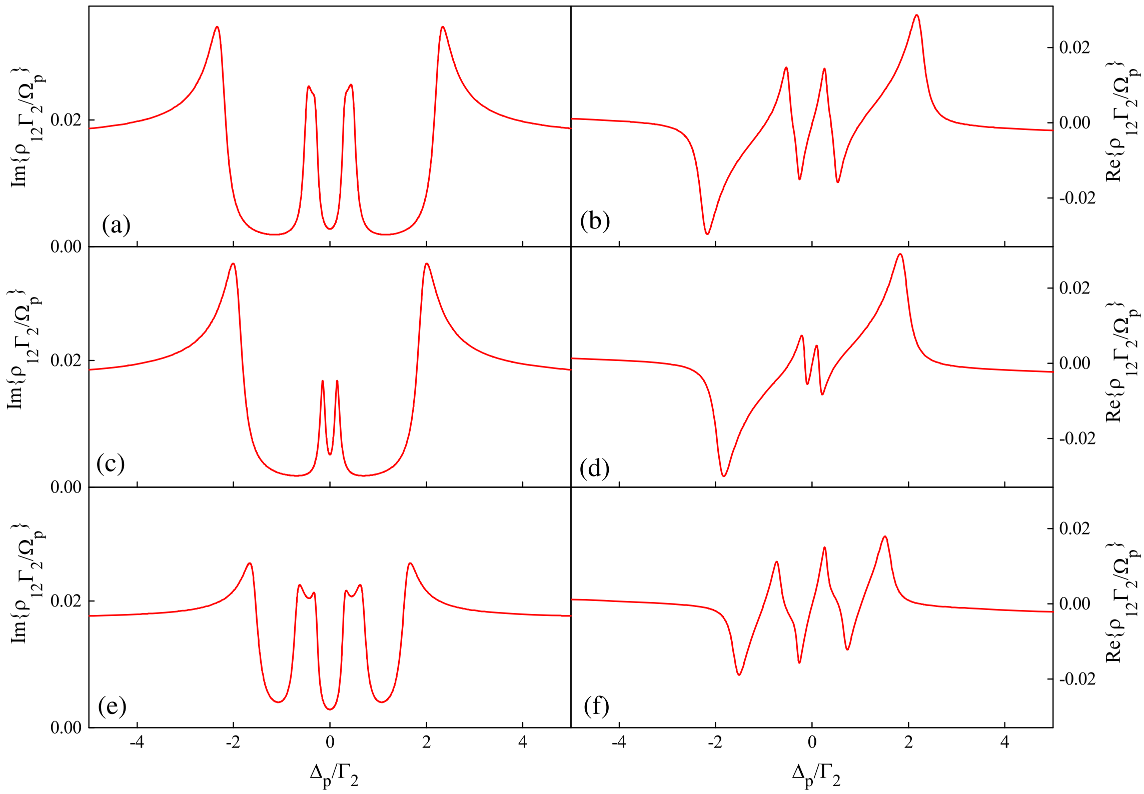

Figure 4. Imaginary (left column) and real (right column) parts of the probe response vs. probe detuning for the Y-type system for room-temperature atomic vapours. In the calculation, the parameters are $\Omega_p = 0.001 \times \Gamma_2$ and $\Delta_{c1} = \Delta_{c2} = 0$ with $\lambda_{c1} = 776$ nm and $\lambda_{c2} = 572$ nm. (**a**), (**b**) $\Omega_{c1} = \Omega_{c2} = 4\Gamma_2$, (**c**), (**d**) $\Omega_{c1} = 2\Gamma_2$, $\Omega_{c2} = 4\Gamma_2$ and (**e**), (**f**) $\Omega_{c1} = 4\Gamma_2$, $\Omega_{c2} = 2\Gamma_2$.

3.1 *Dressed state analysis*

We present a simple analysis of the dressed state picture for the four-level system [18,19]. The eigenvalues of the Hamiltonian (eq. (1)) are 0, 0 and $(kv/2) \pm [\sqrt{((kv)^2 + \Omega_{c1}^2 + \Omega_{c2}^2)}/2]$. The dressed in terms of the dark ($|0\rangle$) and bright ($|\pm\rangle$) states for the system at the resonance condition ($\Delta_p = \Delta_{c1} = \Delta_{c2} = 0$) are

$$|0\rangle = \frac{1}{\sqrt{\Omega_{c1}^2 + \Omega_{c2}^2}}(\Omega_{c2}|3\rangle - \Omega_{c1}|4\rangle),$$

$$|\pm\rangle = \frac{1}{2\sqrt{C}}\left(|2\rangle + \frac{\Omega_{c1}}{\sqrt{C} \pm kv}(|3\rangle + |4\rangle)\right), \quad (6)$$

where $C = (kv)^2 + \Omega_{c1}^2 + \Omega_{c2}^2$. The dark state is a combination of the ground and excited states with no contribution from the intermediate state and when the probe field is driven a transparency is observed instead of an absorption. The state $|1\rangle$ cannot interact with $|0\rangle$, and therefore the absorption is vanished at the line centre.

3.2 *Thermal averaging*

When we take into account the effect of atomic motion, Doppler broadening comes into play. Counter-intuitively, Doppler broadening helps in narrowing the EIT resonances due to thermal averaging of all the velocity classes [20,21]. In general, at the room temperature the velocity of the atomic vapours is explained by Maxwell–Boltzmann distribution. EIT study involves at least two laser beams, and so the direction of beam propagation plays an important role. The beam geometry should be taken such that the Doppler shifts for the probe and coupling fields cancel each other to circumvent the residual Doppler effect. For a Ξ configuration, residual Doppler width can be minimised by using counterpropagating fields. For performing thermal averaging of all the atomic velocity classes, the most probable velocity of the atoms is considered [9]. In our case, for the ^{87}Rb atoms, we take the velocities from −250 to +250 m/s and this range has been divided into 500 points.

First, we take $5D_{5/2}$ and $7S_{1/2}$ as the excited states such that $\lambda_{c1} = 776$ nm and $\lambda_{c2} = 741$ nm. Figure 3 shows the absorption and dispersion properties with

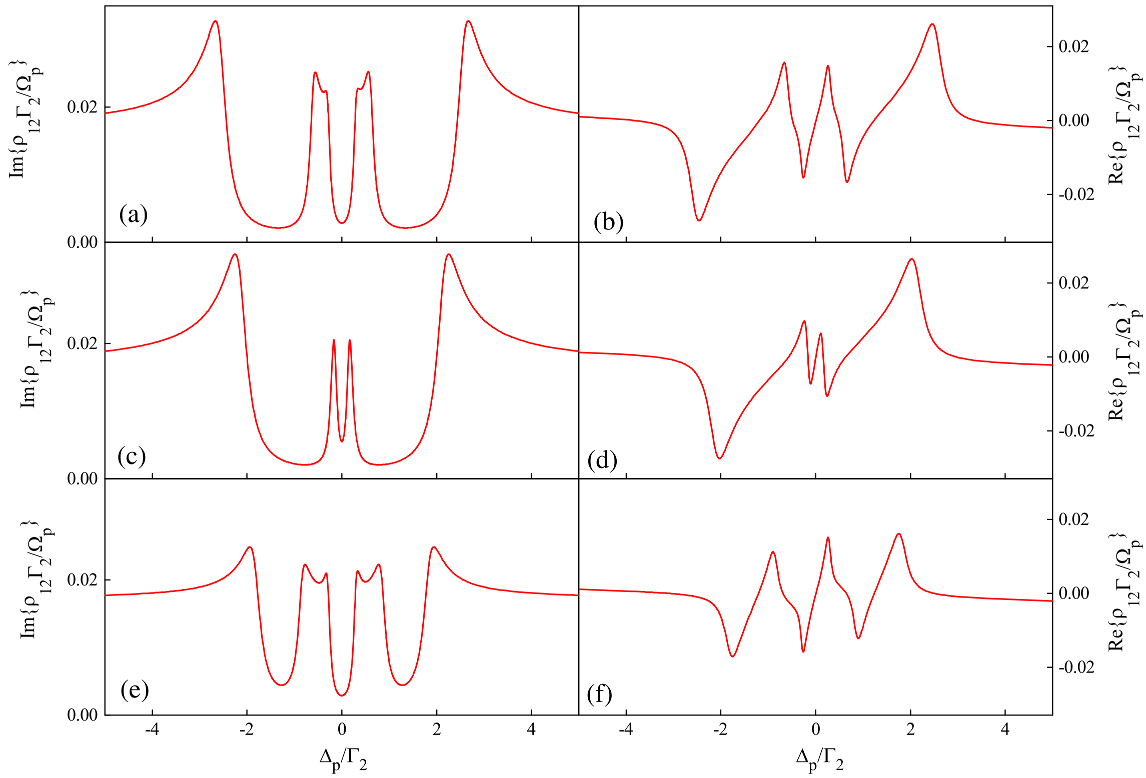

Figure 5. Imaginary (left column) and real (right column) parts of the probe response vs. probe detuning for the Y-type system for room-temperature atomic vapours. In the calculation, the parameters are $\Omega_p = 0.001 \times \Gamma_2$ and $\Delta_{c1} = \Delta_{c2} = 0$ with $\lambda_{c1} = 776$ nm and $\lambda_{c2} = 480$ nm. (**a**), (**b**) $\Omega_{c1} = \Omega_{c2} = 4\Gamma_2$, (**c**), (**d**) $\Omega_{c1} = 2\Gamma_2$, $\Omega_{c2} = 4\Gamma_2$ and (**e**), (**f**) $\Omega_{c1} = 4\Gamma_2$, $\Omega_{c2} = 2\Gamma_2$.

various coupling field values. In figure 3a, for equal values of both the control fields, multiple subwindows are observed within the single transparency window of a Ξ configuration. Nevertheless, a transparency is still observed at the line centre. The Y-type system can be considered as a combination of two Ξ subsystems. Different wavelengths of the control fields are taken to see the effect of mixing of differently broadened Ξ subsystems. This complex EIT behaviour can be attributed to the interference effect produced due to the convolution of EIT in both the Ξ subsystems. Therefore, for the composite system, the two-photon resonance positions are deviated for various velocity groups due to the complete wavelength mismatching. As a result, we observe transparency as well as absorption in a single system leading to sub- and superluminal light propagation, respectively. Further, depending upon the value of the coupling fields, the absorption and dispersion properties of the system are modified.

Now, we replace the $7S_{1/2}$ state with the $7D_5/2$ state as the excited states such that $\lambda_{c2} = 572$ nm and the mismatching factor is enhanced (figure 4). In this case, due to the increasing mismatching, more broadened composite EIT window is observed with enhanced absorption peaks on either side of the line centre. Subsequently, the dispersion properties show widened dispersion wings with negative dispersion slopes around the line centre. Further, we take $44D_{5/2}$ as

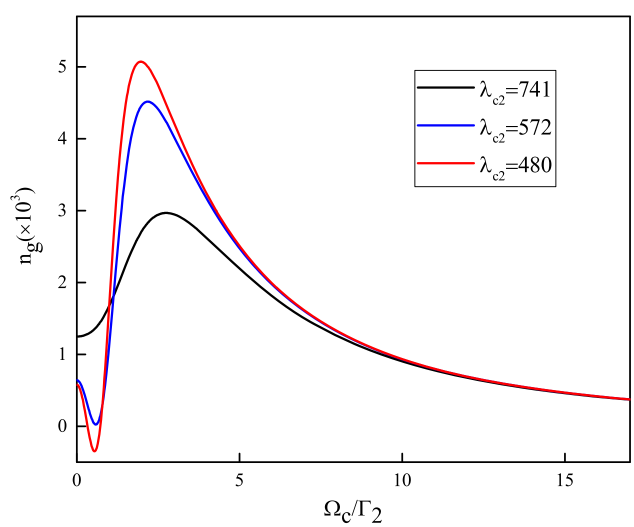

Figure 6. Group index vs. coupling field after thermal averaging for room-temperature atomic vapours. In the calculation, the parameters are $\Omega_p = 0.001 \times \Gamma_2$, $\Delta_{c1} = \Delta_p = \Delta_{c2} = 0$, $\lambda_{c1} = 776$ nm and $\Omega_{c2} = 4\Gamma_2$.

the excited states such that $\lambda_{c2} = 480$ nm. In this case, wavelength mismatching factor increases among the two Ξ subsystems by an order of two. With increase in mismatching, broadening is further increased (figure 5).

3.3 *Group index variation*

Figure 6 shows the group index variation with Ω_{c1}, at fixed λ_{c1} and various λ_{c2}. Group index first increases with Ω_{c1} and then starts decreasing but remains positive throughout when α changes by an order of one such that the excited states are $5D_{5/2}$ and $7S_{1/2}$. Switching from positive to negative n_g is observed when α changes by an order of two for the excited states $7D_{5/2}$ and $44D_{5/2}$ states. This switching is more prominent for the Rydberg state with the highest mismatching factor.

4. Conclusion

We theoretically studied wavelength mismatching effects in a four-level Y-type system for the ^{87}Rb atom. We observe sub- as well as superluminal light propagation concurrently in this system as a result of mismatched wavelengths. The mismatching effects in the system also cause broadening of the EIT transparency window. Thus, on mixing Doppler-free and Doppler-broadened three-level systems, EIT behaviour modifies dramatically. Further, the wavelength mismatching can be utilised for EIT-based optical switching.

Acknowledgements

KY would like to thank Dr Vineet Bharti for his valuable and constructive suggestions. This work is supported by the SERB, India, Grant No. SB/S2/LOP/009/2014.

References

[1] K-J Boller, A Imamoğlu and S E Harris, *Phys. Rev. Lett.* **66**, 2593 (1991)
[2] M Fleischhauer, A Imamoğlu and J P Marangos, *Rev. Mod. Phys.* **77**, 633 (2005)
[3] L V Hau, S E Harris, Z Dutton and C H Behroozi, *Nature* **397**, 594 (1999)
[4] L J Wang, A Kuzmich and A Dogariu, *Nature* **406**, 277 (2000)
[5] V Bharti and V Natarajan, *Opt. Commun.* **392**, 180 (2017)
[6] K Yadav and A Wasan, *Phys. Lett. A* **381**, 3246 (2017)
[7] A B Mirza and S Singh, *Pramana – J. Phys.* **82**, 295 (2014)
[8] A B Mirza and S Singh, *Phys. Rev. A* **85**, 053837 (2012)
[9] V Bharti and A Wasan, *Opt. Commun.* **324**, 238 (2014)
[10] B Al-Nashy, M Abdullah, A G Al-Shatravi and A H AL-Khursan, *Pramana – J. Phys.* **91**: 74 (2018)
[11] S Shepherd, D J Fulton and M H Dunn, *Phys. Rev. A* **54**, 5394 (1996)
[12] S Khan, V Bharti and V Natarajan, *Phys. Lett. A* **48**, 4100 (2016)
[13] J Gea-Banacloche, Y Li, S Jin and M Xiao, *Phys. Rev. A* **51**, 576 (1995)
[14] V Bharti and A Wasan, *J. Phys. B* **45**, 185501 (2012)
[15] G Vemuri, G S Agarwal and B D Nageshwara Rao, *Phys. Rev. A* **53**, 2842 (1996)
[16] M S Safronova and U I Safronova, *Phys. Rev. A* **83**, 052508 (2011)
[17] A K Mohapatra, T R Jackson and C S Adams, *Phys. Rev. Lett.* **98**, 113003 (2007)
[18] C Y Ye, A S Zibrov, Y V Rostovtsev and M O Scully, *Phys. Rev. A* **65**, 043805 (2002)
[19] L B Kong, X H Tu, J Wang, Y Zhu and M S Zhan, *Opt. Commun.* **269**, 362 (2007)
[20] A Javan, O Kocharovskya, H Lee and M O Scully, *Phys. Rev. A* **66**, 013805 (2002)
[21] S M Iftiquar, G R Karve and V Natarajan, *Phys. Rev. A* **77**, 063807 (2008)