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Friedmann–Robertson–Walker accelerating Universe with interactive dark energy

G K GOSWAMI[1], ANIRUDH PRADHAN[2],* and A BEESHAM[3]

[1]Department of Mathematics, Kalyan Post Graduate College, Bhilai 490 006, India
[2]Department of Mathematics, Institute of Applied Sciences and Humanities, GLA University, Mathura 281 406, India
[3]Department of Mathematical Sciences, University of Zululand, Kwa-Dlangezwa 3886, South Africa
*Corresponding author. E-mail: pradhan.anirudh@gmail.com

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Abstract. In this work, we study a cosmological model based on the cosmological principle which exhibits a transition from deceleration to acceleration. We consider baryonic matter dark energy (DE), and 'curvature' energy. Both baryonic matter and DE have variable equations of state. It is assumed that DE interacts with and transforms energy to baryonic matter. A Friedmann–Robertson–Walker (FRW) Universe filled with two fluids has been discussed. The model is shown to satisfy current observational constraints. This Universe is at present in a phantom phase after passing through a quintessence phase in the past. Various cosmological parameters regarding the accelerating Universe have been presented. The evolution of DE, Hubble, deceleration parameters, etc. have been described with the aid of figures. Our theoretical results have been compared with the SNe Ia related Union 2.1 compilation 581 data and we have observed that our derived model is in good agreement with the current observational constraints. We have also explored the physical properties of the model.

Keywords. Friedmann–Robertson–Walker Universe; SNe Ia data; observational parameters; accelerating Universe.

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1. Introduction

The cosmological principle (CP), which states that there is no privileged position in the Universe and it is as such spatially homogeneous and isotropic, is the backbone of any cosmological model of the Universe. The Friedmann–Robertson–Walker (FRW) line element fits best with the CP. The FRW model, in the background of a perfect fluid distribution of matter, represents an expanding and decelerating Universe. However, the latest findings on observational grounds during the last three decades by various cosmological missions like observations on type-Ia Supernovae (SNe Ia) [1–5], CMBR fluctuations [6,7], large scale structure (LSS) analysis [8,9], SDSS Collaboration [10,11], WMAP Collaboration [12], Chandra X-ray observatory [13], Hubble space telescope cluster supernova survey *V* [14], BOSS Collaboration [15], WiggleZ dark energy survey [16] and latest Planck Collaboration results [17] all confirm that our Universe is undergoing an accelerating expansion. In ΛCDM cosmology [18,19], the Λ-term is used as a candidate of dark energy (DE) with the equation of state

$$p_\Lambda = \rho_\Lambda = \frac{-\Lambda c^4}{8\pi G}.$$

However, the model suffers from, *inter alia*, fine-tuning, and cosmic coincidence problems [20]. Any acceptable cosmological model must explain the accelerating Universe.

In any cosmological model, we need to find out the rate of expansion of the Universe determined by the Hubble constant. Observationally, we require high-precision measurement observatories to estimate the Hubble constant. In general relativity, the energy conservation equation provides a linear relationship amongst the rate of expansion, pressure, density and temperature. DE negative pressure and density are also included in it. The cosmologists have solved the problem of the equation of state (EoS) for baryonic matter

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by providing the phases of the Universe like stiff matter, radiation-dominated and the present dust-dominated Universe, but the determination of the EoS for DE is an important problem in observational cosmology at present. Carroll and Hoffman [21] presented a DE model in which DE is considered in a conventional manner as a fluid by the EoS parameter $\omega_{de} = p_{de}/\rho_{de}$. Many researchers [22–29] have also developed DE perfect fluid models of the Universe with a variable EoS parameter ω_{de} producing negative pressure. These researchers have considered different parametric forms of ω_{de}. At present, it is nearly equal to -1. So far, the two main theories related to the variable EoS for DE are quintessence and phantom models of DE. In the quintessence model $-1 \leq \omega_{de} < 0$ whereas in the phantom model $\omega_{de} \leq -1$. Latest surveys [30–34] rule out the possibility of $\omega_{de} \ll -1$, but ω_{de} may be a little less than -1. But we are facing fine-tuning and coincidence problems [35]. So we need a dynamical DE with an effective EoS, $\omega_{de} = p_{de}/\rho_{de} < -1/3$. The two types of data [36] and [9] provide limits on ω_{de} as $-1.67 < \omega_{de} < -0.62$ and $-1.33 < \omega_{de} < -0.79$, respectively. Komatsu *et al* [34] and Hinshaw *et al* [37] estimated limits on ω_{de} as $-1.44 < \omega_{de} < -0.92$ at 68% confidence level.

Of late, it has been discovered that the interaction between DE and dark matter (DM) offers an attractive alternative to the Standard Model of cosmology [38,39]. In these works, the motivation to study interacting DE model arises from high-energy physics. In a recent work, Risaliti and Lusso [40] and Riess *et al* [41] stated that a rigid Λ is ruled out by 4σ and allowing for running vacuum favoured phantom type DE ($\omega < -1$) and ΛCDM is claimed to be ruled out by 4.4σ motivating the study of interactive DE models. Interacting DE models [42–46] lead to the idea that DE and DM do not evolve separately but interact with each other non-gravitationally (see [47] and references therein). Stable solutions corresponding to the accelerated expansion in higher-dimensional FRW cosmology at late times have been obtained in [48] and some dynamical aspects of interacting quintessence models in FRW space–times and phase-space analysis for the 'best-fit Universe' or concordance model have been investigated in [49]. Zhang and Liu [50] have constructed DE models with higher derivative terms. Liang *et al* [51] have investigated two-fluid dilation models of DE. The modified Chaplygin gas with interaction between holographic DE and DM has been discussed by Wang *et al* [52]. Recently, Amirhashchi *et al* [53,54], Pradhan *et al* [55], Saha *et al* [56], Pradhan [57] and Kumar [58] have studied an FRW-based DE model in which they considered interacting and non-interacting fluids, one for the baryonic matter and the other for DE.

In this work, we study a model which exhibits a transition from deceleration to acceleration. We consider the baryonic matter, DE and 'curvature' energy. Both baryonic matter and DE have variable EoS. We assume that DE interacts with and transforms to baryonic matter. The cosmological importance of the two-fluid scenario is discussed in detail. The model is shown to satisfy current observational constraints. As per our model, our Universe is at present in a phantom phase after passing through a quintessence phase in the past. Various cosmological parameters relating to the history of the Universe have been presented.

2. Metric and field equations

Einstein's field equations (EFEs) are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi G}{c^4}T_{ij}, \tag{1}$$

where R_{ij} is the Ricci tensor, R is the scalar curvature and T_{ij} is the stress–energy tensor taken as

$$T_{ij} = T_{ij}(m) + T_{ij}(de), \tag{2}$$

where

$$T_{ij}(m) = (\rho_m + p_m)u_i u_j - p_m g_{ij} \tag{3}$$

and

$$T_{ij}(de) = (\rho_{de} + p_{de})u_i u_j - p_{de} g_{ij}. \tag{4}$$

In a co-moving coordinate system, $u^\alpha = 0$; $\alpha = 1$–3.

The FRW space–time (in units in which $c = 1$) is given by

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a(t)^2\left[\frac{\mathrm{d}r^2}{(1+kr^2)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right], \tag{5}$$

where $a(t)$ stands for the scale factor and k is the curvature parameter, $k = -1$ for closed, $k = 1$ for open and $k = 0$ for a spatially flat Universe.

Solving EFEs (1) for the FRW metric (5), we obtain the following system of equations:

$$2\frac{\ddot{a}}{a} + H^2 = -8\pi G p + \frac{k}{a^2}, \tag{6}$$

$$H^2 = \frac{8\pi G}{3}\rho + \frac{k}{a^2}, \tag{7}$$

where $H = \dot{a}/a$ is the Hubble constant. Here the overdot means differentiation with respect to cosmological time

t. We have deliberately put the curvature term to the right of eqs (6) and (7), as this term is made to act like energy.

We define the density and pressure for the curvature energy as

$$\rho_k = \frac{3k}{8\pi G a^2}, \quad p_k = -\frac{k}{8\pi G a^2}. \tag{8}$$

Equations (6) and (7) reduce to

$$2\frac{\ddot{a}}{a} + H^2 = -8\pi G(p + p_k) \tag{9}$$

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_k). \tag{10}$$

The energy density ρ in eq. (7) comprises two types of energy, viz., matter energy (ρ_m) and dark energy (ρ_{de}). The pressure p in eq. (6) comprises pressure due to matter and DE respectively. We can write

$$\rho = \rho_m + \rho_{de} \tag{11}$$

and

$$p = p_m + p_{de}. \tag{12}$$

We use the following energy conservation law to find the pressure and density which is obtained from $T^{ij}_{;j} = 0$:

$$\dot{\rho} + 3H(p + \rho) = 0, \tag{13}$$

where

$$\rho = \rho_m + \rho_{de} + \rho_k$$

and

$$p = p_m + p_{de} + p_k,$$

are the total density and pressure of the Universe, respectively. It is interesting that eq. (13) is satisfied by ρ_k and p_k independently, i.e.,

$$\dot{\rho}_k + 3H(p_k + \rho_k) = 0 \tag{14}$$

so that

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho_m + \rho_{de}) + 3H(p_m + p_{de} + \rho_m + \rho_{de}) = 0.$$

We assume that DE interacts with and transforms energy to baryonic matter. For this, the continuity equations for the dark and baryonic fluids can be written as follows:

$$\dot{\rho}_m + 3H(p_m + \rho_m) = Q \tag{15}$$

and

$$\dot{\rho}_{de} + 3H(p_{de} + \rho_{de}) = -Q \tag{16}$$

The quantity Q represents the energy transfer from DE to baryonic matter, and so we take $Q \geq 0$. We follow Amendola *et al* [59] and Gou *et al* [60], to assume that

$$Q = 3H\sigma\rho_m, \tag{17}$$

where σ is the coupling constant and is positive.

At present, our Universe is dust filled, and so we take $p_m = 0$. Integrating eq. (15) with the help of eq. (17), we get

$$\rho_m = (\rho_m)_0 \left(\frac{a_0}{a}\right)^{3(1-\sigma)}. \tag{18}$$

Clearly DE helps in the expansion of the Universe through energy transfer. The EoS for the curvature energy is obtained as

$$p_k = \omega_k \rho_k, \quad \text{where } \omega_k = -1/3. \tag{19}$$

This gives

$$\rho_k \propto a^{-2} = (\rho_k)_0 \left[\frac{a_0}{a}\right]^2. \tag{20}$$

As is well known, all cosmic observations such as the Hubble constant, deceleration parameter, age of the Universe, luminosity distance, apparent magnitude, etc., are related to the cosmic red-shift. So we use the following well-known relation:

$$\frac{a_0}{a} = 1 + z. \tag{21}$$

The density parameter is defined as $\Omega = 8\pi G\rho/3c^2H^2$. The value of the energy density for which $\Omega = 1$ is called the critical density ρ_c, given by $\rho_c = 3c^2H^2/8\pi G$. The critical density and density parameters for the energy density, DE and curvature density are, respectively, defined by

$$\rho_c = \frac{3H^2}{8\pi G},$$

$$\Omega_m = \frac{\rho_m}{\rho_c},$$

$$\Omega_{de} = \frac{\rho_{de}}{\rho_c},$$

$$\Omega_k = \frac{\rho_k}{\rho_c}, \tag{22}$$

where ρ_c, Ω_m, Ω_{de} and Ω_k are the critical density, matter energy density parameter, DE density parameter and curvature density parameter, respectively.

Therefore the FRW field equations change to

$$H^2(1-\Omega_{de}) = H_0^2\left[(\Omega_m)_0\left(\frac{a_0}{a}\right)^{3(1-\sigma)} + (\Omega_k)_0\left(\frac{a_0}{a}\right)^2\right] \tag{23}$$

and

$$2q = 1 + 3\omega_k(\Omega_k)_0\left(\frac{a_0H_0}{aH}\right)^2 + 3\omega_{de}\Omega_{de}, \tag{24}$$

where q is the deceleration parameter defined by

$$q = -\frac{\ddot{a}}{aH^2}. \tag{25}$$

3. Hybrid scale factor with Plank results

We have only two equations with the scale factor a, pressure p and energy density ρ to be determined. So we have to use a certain ansatz. As motivation for the ansatz, we note some important solutions. The De Sitter Universe has scale factor $a(t) = \exp(\Lambda t)$ where Λ is the positive cosmological constant. Later on, FRW cosmological models were proposed in which Einstein and De Sitter gave the power-law expansion law $a(t) = t^{2/3}$ for flat space–time. Of late, during the last three decades, researchers are working with accelerating expanding models that describe a transition from deceleration to acceleration.

In the literature, a constant deceleration parameter ([61–64] and references therein), has been used to give power or exponential law. As has been discussed in the Introduction, in view of the recent observations of type-Ia supernova [1–5], WMAP Collaboration [12,65,66] and Planck Collaboration [17], there is a need for a time-dependent deceleration parameter which describes decelerated expansion in the past, and accelerating expansion at present. So there must be a transition from deceleration to acceleration. The deceleration parameter must show a change in signature [67–69].

Abdusattar and Prajapati [70] proposed the following functional form for the deceleration parameter q:

$$q = \frac{kn}{(k+t)^2} - 1, \tag{26}$$

where they considered $k > 0$ and $n > 0$ as constants. This choice of the scale factor provides $q = 0$ when $t = \sqrt{kn} - k$, $q > 0$ (decelerated expansion) for $t < \sqrt{kn} - k$ and $q < 0$ (accelerated expansion) for $t > \sqrt{kn} - k$. On solving eq. (25), we get the following integral:

$$a(t) = c_2 \exp \int \frac{\mathrm{d}t}{\int(1+q)\,\mathrm{d}t + c_1}, \tag{27}$$

where c_1 and c_2 are constants of integration.

Integrating eq. (27) with the help of eq. (26) and making appropriate choices for the arbitrary constants c_1 and c_2, $a(t)$ is obtained as

$$a(t) = t^{\alpha}\exp(\beta t). \tag{28}$$

Akarsu *et al* [71] also used hybrid expansion law (HEL) with scalar field reconstruction of observational constraints and cosmic history. Avile's *et al* [72] used HEL with integrating cosmic fluid. Several researchers [73–80] have considered the HEL for solving different cosmological problems in general relativity and $f(R, T)$ gravity theories. Some work is done by Moraes [81] and Moraes *et al* [82]. Recently, Moraes and Sahoo [83] investigated non-minimal matter geometry coupling in the $f(R, T)$ gravity by using HEL.

The hybrid scale factor has a transition behaviour from deceleration to acceleration. Capozziello *et al* [84] studied the cosmographic bounds on cosmological deceleration–acceleration transition red-shift in $f(R)$ gravity. Capozziello considered a Tailor expansion of $f(z)$ in term of $a(t) = 1/(1+z)$ which for Friedmann equations, comes in the range $z \le 2$. Capozziello *et al* [85] also extract constraints on the transition red-shift z_{tr} in the framework of $f(T)$ gravity which becomes compatible with the constraints predicted by ΛCDM model at the $1-\sigma$ confidence level. Their values seem to be slightly smaller than the theoretical expectation, i.e., $z_{\mathrm{tr}} = 0.74$ according to [86]. Recently, Farooq *et al* [87] compiled updated list of 38 measurements of Hubble parameter $H(z)$ between red-shifts $0 \le z \le 2.36$ and used them to put constraints on model parameters of constant and time-varying DE cosmological models, both spatially flat and curved.

Now we determine the constants α and β on the basis of the latest observational findings due to Planck [17]. The values of the cosmological parameters at present are as follows:

$$(\Omega_m)_0 = 0.30,\quad (\Omega_k)_0 = \pm 0.005,\quad (\omega_{de})_0 = -1,$$
$$(\Omega_{de})_0 = 0.70 \pm 0.005,\quad H_0 = 0.07\,\mathrm{Gyr}^{-1}$$
$$q_0 \simeq 0.55,\quad t_0 = 13.72\,\mathrm{Gyr}. \tag{29}$$

From eq. (28), we get the following equation:

$$\alpha(1+z)HH_z = \alpha(q+1)H^2 = (H-\beta)^2 = \frac{\alpha^2}{t^2}. \tag{30}$$

From eq. (30) and Planck's results [eq. (29)], we get the value of constants α and β as

$$\beta = 0.0397474 \sim 0.04,\quad \alpha = 0.415066 \sim 0.415. \tag{31}$$

4. Physical properties of the model

4.1 *Hubble constant H*

The determination of the two physical quantities, H_0 and q, plays an important role in describing the evolution of the Universe. H_0 provides us the rate of expansion of the Universe which in turn helps in estimating the age of the Universe, whereas the deceleration parameter q describes the decelerating or accelerating phases during the evolution of the Universe. From the last two decades, many attempts [88–92] have been made to estimate the value of the Hubble constant as $H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $69.7^{+4.9}_{-5.0}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $71 \pm 2.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $70.4^{+1.3}_{-1.4}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $73.8 \pm 2.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $67 \pm 3.2\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ respectively. For detailed discussions, readers are referred to Kumar [93].

The exact solution of eq. (30) is obtained for the Hubble constant (H) as a function of red-shift (z) as follows:

$$(H-\beta)^{\alpha} = A\ \exp\left(\frac{\alpha\beta}{H-\beta}\right)(1+z), \tag{32}$$

where the constant of integration (A) is obtained as $A = 0.134$ on the basis of the present value of $H(H_0 = 0.07\ \mathrm{Gyr^{-1}})$. A numerical solution of eq. (32) shows that the Hubble constant is an increasing function of the red-shift. We present figures 1 and 2 to illustrate the solution.

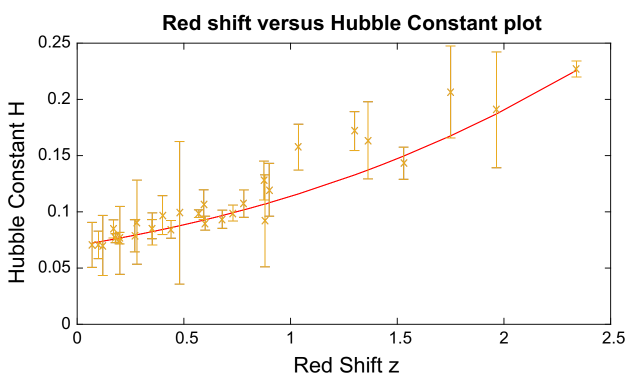

Figure 1. Plot of Hubble constant (H) vs. red-shift (z).

As is clear from the figures, the Hubble constant is, in fact, not a constant. It varies slowly over the red-shift and time. Various researchers [15,16,94–97] have estimated values of the Hubble constant at different red-shifts using a differential age approach and galaxy clustering method. They have described various observed values of the Hubble constant H_{ob} along with corrections in the range $0 \le z \le 2$. It is found that both the observed and theoretical values tally considerably and support our model.

From figure 1, we observe that H increases with the increase of red-shift. In this figure, cross signs are 31 observed values of the Hubble constant (H_{ob}) with corrections, whereas the linear curve is the theoretical graph of the Hubble constant (H) as per our model. Figure 2 plots the variation of red-shift (z) with time (t), which shows that the red-shift was more in the early Universe than at present.

4.2 *Transition from deceleration to acceleration*

Now we can obtain the deceleration parameter (q) in terms of red-shift (z) by using eqs (30) and (31). We present figure 3 to illustrate the solution. This describes the transition from deceleration to acceleration.

At $z = 0.9557$ and 0.9558, our model gives the following values of H, q and the corresponding time.

$H(0.9557) \to 0.111206$,
$q(0.9557) \to -0.0000124355$,
$t(0.9557) \to 5.81124$

and

$H(0.9558) \to 0.111212$, $q(0.9558) \to 0.0000450098$,
$t(0.9558) \to 5.81078$.

This means that the acceleration had begun at $z \to 0.95575$, $t \to 5.81104$ Gyr and $H \to 0.111209\ \mathrm{Gyr^{-1}}$.

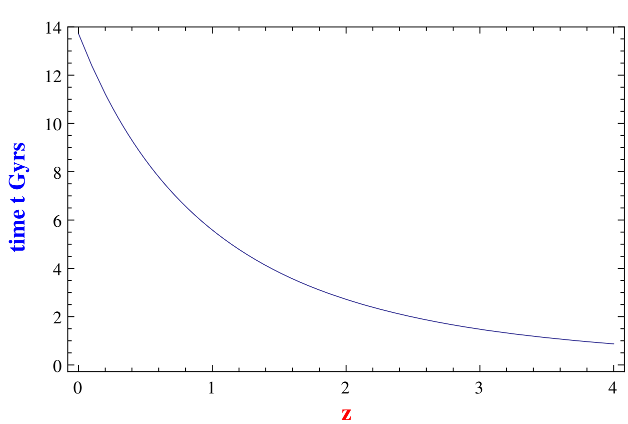

Figure 2. Variation of z vs. t.

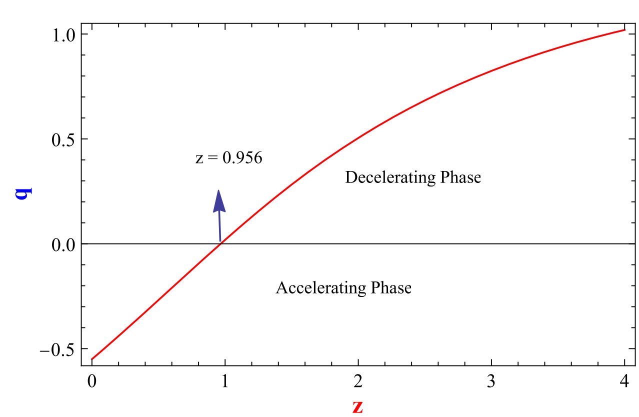

Figure 3. Variation of q with z.

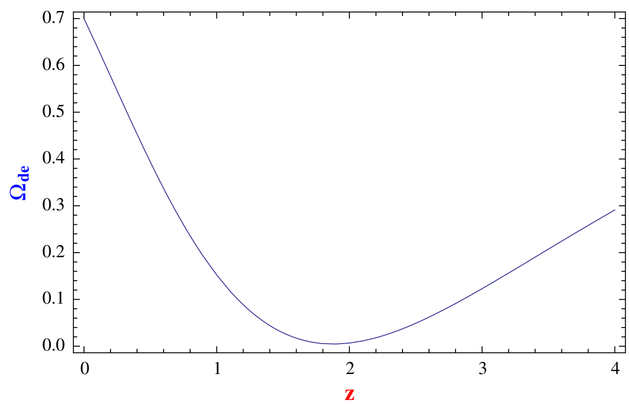

Figure 4. Plot of Ω_{de} vs. z.

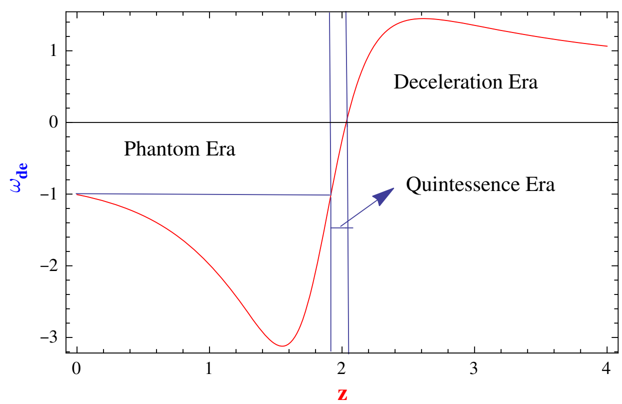

Figure 5. Plot of ω_{de} vs. z. $0 \leq z \leq 1.99$: phantom phase, $1.99 \leq z \leq 2.0315$: quintessence phase and $z \geq 1.99$: deceleration phase.

4.3 *DE parameter Ω_{de} and EoS ω_{de}*

Now, from eqs (21)–(24), (30) and (32), the density parameter (Ω_{de}) and EoS parameter (ω_{de}) for DE are given by the following equations and are solved numerically.

$$H^2\Omega_{de} = H^2 - (\Omega_m)_0 H_0^2 (1+z)^{3(1-\sigma)} \tag{33}$$

$$\omega_{de} = \frac{(2-3\alpha)H^2 - 4\beta H + 2\beta^2}{3\alpha[H^2 - H_0^2(\Omega_m)_0(1+z)^{3(1-\sigma)}]}, \tag{34}$$

where we have taken $(\Omega_k)_0 = 0$ for the present dust-filled spatially flat Universe. We would take $\sigma = 0.04$ for numerical solutions to match with latest observations. We solve eqs (33) and (34) with the help of eqs (30) and (31) and present figures 3 and 4 to illustrate the solution.

Figure 5 depicts the variation of ω_{de} with respect to red-shift z. From this figure our model envisages that at present we are living in a phantom phase $\omega_{(de)} \leq -1$. In the past, at $z = 1.549$, $\omega_{(de)} = -3.12191$ was minimum, then it started increasing. This phase remains for the period $0 \leq z \leq 1.99$. Our Universe entered into a quintessence phase at $z = 1.99$ where ω_{de} comes up to -0.333123. As per our model, the period for the quintessence phase is

$1.99 \leq z \leq 2.0315$.

DE favours deceleration at $z \geq 1.99$. The recent supernovae SNI 997ff at $z \simeq 1.7$ is consistent with a decelerated expansion at the epoch of high emission (Riess *et al* [69], Benitez *et al* [98], Turner and Riess [99]).

As per our model, the present value of DE is 0.7. It decreases over the past, attains a minimum value $\Omega_{de} = 0.0368568$ at $z = 1.834$, and then it again increases with red-shift. The DE density is approximately 29% at red-shift 4. As DE density is significant at this red-shift, it might have strong implications on structure formation, but at $z = 4$, EoS parameter $\omega_{de} = 1.0532$ is positive, and so it will favour deceleration and hence structure formation.

4.4 *Luminosity distance*

The red-shift–luminosity distance relation [100] is an important observational tool to study the evolution of the Universe. The expression for the luminosity distance ($D_{\rm L}$) is obtained in terms of red-shift as the light coming out of a distant luminous body gets red-shifted due to the expansion of the Universe. We determine the flux of a source with the help of luminosity distance. It is given as

$$D_{\rm L} = a_0 r(1+z), \tag{35}$$

where r is the radial coordinate of the source. We consider a ray of light having initially

$$\frac{d\theta}{ds} = 0 \text{ and } \frac{d\phi}{ds} = 0. \tag{36}$$

Then the geodesic for the metric (5) will determine

$$\frac{d^2\theta}{ds^2} = 0 \text{ and } \frac{d^2\phi}{ds^2} = 0. \tag{37}$$

So if we pick up a light ray in a radial direction, then it continues to move along the r-direction always, and we get the following equation for the path of the light ray:

$$ds^2 = c^2\,dt^2 - \frac{a^2}{1+kr^2}\,dr^2 = 0. \tag{38}$$

As we have seen, the effect of curvature is very small at present, $(\Omega_k)_0 = 0.005$. So for the sake of simplification, we take $k = 0$. From this we obtain

$$r = \int_0^r dr = \int_0^t \frac{c\,dt}{a(t)} = \frac{1}{a_0 H_0}\int_0^z \frac{c\,dz}{h(z)}, \tag{39}$$

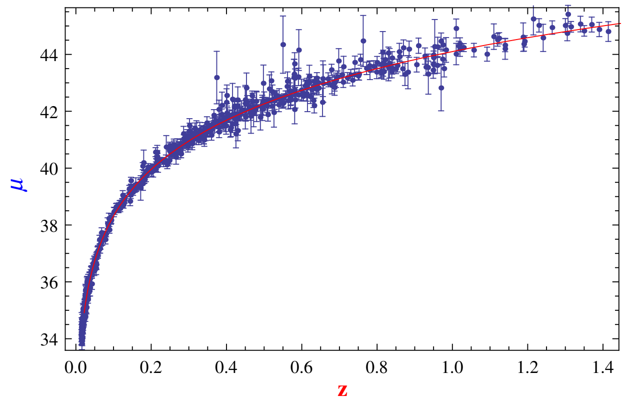

Figure 6. Plot of the distance modulus ($\mu = M - m_b$) vs. red-shift z. Crosses are SNe Ia related Union 2.1 compilation 581 data with possible corrections.

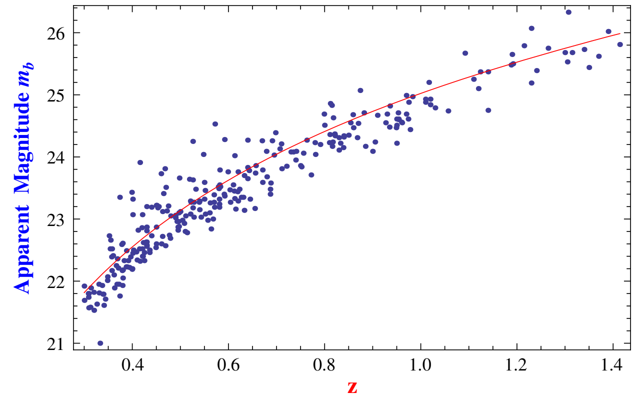

Figure 7. Plot of apparent magnitude (m_b) vs. z. Dots are SNe Ia related Union 2.1 compilation 287 data.

where we have used $\mathrm{d}t = \mathrm{d}z/\dot{z}, \dot{z} = -H(1+z)$ and $h(z) = H/H_0$.

Therefore, the luminosity distance is obtained as

$$D_{\mathrm{L}} = \frac{c(1+z)}{H_0} \int_0^z \frac{\mathrm{d}z}{h(z)}. \tag{40}$$

4.5 *Distance modulus μ and apparent magnitude m_b*

The distance modulus (μ) is derived as [18]

$$\begin{aligned} \mu &= m_b - M \\ &= 5 \log_{10}\left(\frac{D_{\mathrm{L}}}{\mathrm{Mpc}}\right) + 25 \\ &= 25 + 5 \log_{10}\left[\frac{c(1+z)}{H_0} \int_0^z \frac{\mathrm{d}z}{h(z)}\right]. \end{aligned} \tag{41}$$

The absolute magnitude M of a supernova [101,102] is

$$M = 16.08 - 25 + 5 \log_{10}(H_0/0.026c). \tag{42}$$

Equations (41) and (42) produce the following expression for the apparent magnitude m_b:

$$m_b = 16.08 + 5 \log_{10}\left[\frac{1+z}{0.026} \int_0^z \frac{\mathrm{d}z}{h(z)}\right]. \tag{43}$$

We solve eqs (41)–(43) with the help of eq. (30). Our theoretical results have been compared with SNe Ia related Union 2.1 compilation 581 data [14] and the derived model was found to be in good agreement with the current observational constraints. Figures 6 and 7 depict the closeness of the observational and theoretical results, thereby justifying our model.

4.6 *χ^2 for distance modulus (μ)*

Our theoretical results have been compared with SNe Ia related 581 data from Union 2.1 compilation [17] with possible error in the range $0 \leq z \leq 1.3$ and the derived model was found to be in good agreement with the current observational constraints. To get quantitative closeness of theory and observation, we obtain χ^2 from the following formula:

$$\chi^2 = \frac{(\mu_{\mathrm{ob}} - \mu_{\mathrm{th}})^2}{\mu_{\mathrm{err}}^2}.$$

It comes to 567.765 which is 97.72% over 581 data, which shows the best fit in theory and observation.

5. Conclusions

In this work, efforts were made to develop a cosmological model which satisfies the cosmological principle and incorporates the latest developments which envisaged that our Universe is accelerating due to DE. We have also proposed a variable EoS for DE in our model. We studied a model with radiation, dust and DE, which shows a transition from deceleration to acceleration. We have successfully subjected our model to various observational tests. The main findings of our model are itemised point-wise as follows.

- The expansion of the Universe is governed by a hybrid expansion law $a(t) = t^\alpha \exp(\beta t)$, where $\alpha = 0.415$, $\beta = 0.04$. This describes the transition from deceleration to acceleration.
- Our model is based on the latest observational findings due to the Planck results [17]. The model agrees with the present cosmological parameters. $(\Omega_m)_0 = 0.30$, $(\Omega_k)_0 = \pm 0.005$, $(\omega_{de})_0 = -1$, $(\Omega_{de})_0 = 0.70 \pm 0.005$, $H_0 = 0.07\ \mathrm{Gyr}^{-1}$, $q_0 = 0.055$ and the present age $t_0 = 13.72$ Gyr.
- Our model has a variable EoS (ω_{de}) for the DE density. Our model envisages that at present we are living in the phantom phase $\omega_{(de)} \leq -1$. In the past

at $z = 1.549, \omega_{(de)} = -3.12191$ was minimum, then it started increasing. This phase remains for the period ($0 \le z \le 1.99$). Our Universe entered into a quintessence phase at $z = 1.99$ where ω_{de} comes up to -0.333123. As per our model, the period for the quintessence phase is

$1.99 \le z \le 2.0315.$

DE favours deceleration at $z \ge 1.99$.

- As per our model, the present value of DE is 0.7. It decreases over the past, attains a minimum value $\Omega_{de} = 0.0368568$ at $z = 1.834$, and then it again increases with red-shift.
- We have calculated the time at which acceleration had begun. The acceleration had begun at $z \to 0.95575, t \to 5.81104$ Gyr and $H \to 0.111209$ Gyr^{-1}. At this time $\Omega_{de} = 0.220369$ and $\omega_{de} = -1.54715$.

In a nutshell, we believe that our study will pave the way to more research in the future, in particular, in the area of the early Universe, inflation and galaxy formation, etc. The proposed hybrid expansion law may help in investigating hidden matter like DM, DE and black holes.

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