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A novelty to the nonlinear rotating Rayleigh–Taylor instability

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Abstract. This paper presents a novel approach for studying the nonlinear Rayleigh–Taylor instability (RTI). The system deals with two rotating superposed infinite hydromagnetic Darcian flows through porous media under the influence of a uniform tangential magnetic field. The field allows the presence of currents on the surface of separation. The appropriate linear governing equations are solved and confirmed with the corresponding nonlinear boundary conditions. A nonlinear characteristic of the surface deflection is deducted. Away from the traditional techniques of the stability analysis, the work introduces a new one. The analysis depends mainly on the homotopy perturbation method (HPM). To achieve an analytical approximate periodic solution of the surface deflection, the secular terms are removed. This cancellation resulted in well-known amplitude equations. These equations are utilised to achieve stability criteria of the system. Therefore, the stability configuration is exercised in linear as well as nonlinear approaches. The mathematical procedure adopted here is simple, promising and powerful. The method may be used to treat more complicated nonlinear differential equations that arise in science, physics and engineering applications. A numerical calculation is performed to graph the implication of various parameters on the stability picture. In addition, for more convenience, the surface deflection is depicted.

Keywords. Rayleigh–Taylor instability; rotating flow; porous media; magnetic fluids; homotopy perturbation method.

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1. Introduction

Lewis [1] conducted the first experiment that validates the theory of Rayleigh–Taylor instability (RTI). This phenomenon has great importance in geophysics, controlled fusion, industrial applications and astrophysical process. RTI is one of the hydrodynamic instabilities that can be observed easily. The influence of numerous physical parameters such as surface tension, viscosity and variable density on the RTI has been reported in the pioneering work of Chandrasekhar [2]. RTI finds great applications in many areas of the physics of plasma. Furthermore, it is an important topic that concerns with the inertial confinement fusion [3]. Therefore, Piriz *et al* [4] introduced a novel technique to investigate the RTI and obtained exact solutions for the simplest cases and provides approximate analytical expressions for important and more complex cases involving non-ideal fluids. Their approach depends on Newton's second law. The nonlinear instability of the surface waves, which propagate over the interface between fluids, is established by Nayfeh [5]. He used the technique of the multiple time scales to derive two partial differential equations which determine the stability criterion of the surface elevation. These two equations are coupled together to produce another two alternate Schrödinger equations. Mohamed *et al* [6] developed a weakly nonlinear electrogravitational instability of a horizontal plane interface for two streaming fluids through porous media. Their analysis is based on the viscous potential theory, which assumes that the viscous forces are affecting only the interface between the fluids. In their approach, the linear form of the equation of motion is solved in the light of the nonlinear boundary conditions. The use of the Taylor expansion together with the multiple scale schemes yielded the well-known nonlinear Schrödinger equation. This scheme has been successfully adopted by El-Dib [7] in the case of a plane interface between two strong viscous homogeneous incompressible fluids through porous media. His nonlinear characteristic equation resulted in the well-known nonlinear cubic Schrödinger equation.

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The phenomenon of porous media gained enormous importance in the last few decades. This interest comes in accordance with the wider applications in physics and engineering. These applications involve groundwater hydrology, petroleum production, packed bed reactor in the chemical industry, mechanics of soil, the cooling of electronic systems and chemical engineering. Several applications of the problems of flowing throughout porous media in geophysics are found in the work of Dullien [8]. Bau [9] presented the theory of fully saturated porous media throughout the model of the Kelvin–Helmholtz instability. In this theory, he investigated the Darcian fluid together with the non-Darcian one. El-Dib and Ghaly [10] studied nonlinear instability in porous media. In the absence of the porous medium, they found an increase in stability. Sharma and Kumar [11] investigated the RTI through porous media. They showed that the regulation becomes unstable for viscous–viscoelastic layers in the porous media. Furthermore, the stressed field increases the stability configuration. Moatimid *et al* [12] studied the problem of the ferrofluid of the Kelvin–Helmholtz instability in porous media. Throughout the linear stability approach, a general characteristic equation is deduced and the transition curves are depicted. In view of the nonlinear approach, a Ginzburg–Landau equation that governs the stability criteria is achieved.

Magnetic fluids, sometimes called ferrofluids, are described as suspensions of infinitesimal particles of magnetic property in appropriate carrier liquids. These suspensions contain magnetic particles of size about 10 mm in diameter. The structure of magnetic fluids has great importance in numerous technical applications. The study on this topic is given in Rosensweig's work [13]. Furthermore, Zelazo and Melcher [14] pointed out that the uniform magnetic field, at the interface between two ferrofluids, causes a stabilising influence of the system. In addition, they demonstrated experimentally the instability of the fluid column. The nonlinear stability analysis of the surface waves, in the presence of normal magnetic fields, is investigated by Malik and Singh [15]. Their analysis indicates various instability regions. Simultaneously, the instability cannot be suppressed by the application of strong magnetic field. Kant and Singh [16] investigated the weakly nonlinear propagation of RTI of the ferrofluid. Their stability analysis involves the presence of two instability regions. In addition, the field makes dual role in these regions. Finally, the crossing wave number that divides the stability/instability regions is derived. Elhefnawy [17] studied the problem of nonlinear RTI under the influence of tangential magnetic field. He used multiple time-scale analysis to obtain the stability criteria. Two nonlinear Schrödinger equations are obtained. He showed that the tangential field has a dual role in the stability profile. Finally, the uniform magnetic permeability affects the stability configuration. Consequently, El-Dib [18] investigated the RTI between two viscous magnetic fluids that are influenced by a horizontal magnetic field. The amplitude equations reveal a non-ordinary differential equation with complex coefficients. The Routh–Hurwitz conditions are adopted to achieve the stability criteria in view of the linear approach. He found that stability is enhanced with the increase of the viscosity parameter. The nonlinear RTI in porous media is analysed by Elhnefnawy *et al* [19]. For simplicity, their analysis adopted the potential flow theory. The solvability conditions resulted in a Ginzburg–Landau equation. This equation is employed to judge the criteria of the modulation instability. The numerical calculations identify the stability/instability of regions.

The instability of the classical RTI attracts considerable interest in many astrophysical situations. There is a growing interest when the rotating fluids are affected by a horizontal magnetic field. This aspect is the basis of the investigation of Bhatia and Mathur [20]. They found that the critical wave number is influenced by the rotation. In addition, the field and the rotation play a stabilising effect. Chandrasekhar [21] analysed the influence of uniform magnetic field and steady rotation on the gravitational instability in a static medium. He found that the field together with rotation inhibits the contraction and fragmentation of the interstellar clouds. Scase *et al* [22] studied the influence of rotation in the RTI. It is concluded that the destabilising regions may be suppressed by the rotation. Furthermore, the unstable axisymmetric wave modes may be stabilised by rotating the system above a critical rotation rate. Sharma *et al* [23] investigated the RTI under the influence of a small rotation. They showed that the slight uniform rotation is a stabilising influence on the propagation of RTI. In contrast, the Atwood number has a destabilising influence. Prajapati [24] has investigated the effect of rigid rotation on the RTI at the interface in a magnetised strongly coupled viscoelastic fluid. Tao *et al* [25] studied the nonlinear RTI. They found that the instability may be retarded at arbitrary numbers in rotating the system with the axis of rotation normal to the acceleration of the interface between two uniform inviscid fluids. Simultaneously, the Coriolis force provides an effective restoring force on the disturbed interface. Finally, the uniform rotation has an instability role throughout the nonlinear approach. El-Dib and Mady [26] investigated the nonlinear RTI of two rotating superposed magnetised fluids under the influence of vertical/horizontal magnetic fields. They utilised the concept of an expanded frequency to identify the stability criteria. One can say that their approach is vital, promising and powerful for analysing nonlinear systems.

There is a rapid development in the application of nonlinear science and engineering in obtaining analytical forms of their problems. Therefore, there are many perturbation methods to analyse these problems. Unfortunately, these methods have their own limitation. Simultaneously, they are restricted to the assumption that a small parameter must be found in the equation. Otherwise, perturbation methods will be a failure. Therefore, the small parameter restricts the implication of the perturbation methods. Sometimes, an unsuitable choice of this small parameter leads to wrong results. To control these impediments, the homotopy perturbation method (HPM) arises. It is a simple, promising and strong mathematical method for analysing the functional equations [27–29]. The main advantage of this approach is that it does not depend on any small parameter. It only requires a little iteration to obtain accurate solutions. The approach becomes a strong tool. It may be used in analysing linear and nonlinear functional equations. In light of this method, one assumes an artificial parameter that has no physical meaning and ranges from zero to unity. This parameter may be put in any place in the homotopy equation, such that away from this parameter, the governing equation is simplified and yields a primary solution. The HPM gives an extremely rapid convergence of the analytical series in many cases. Further, only a little repetition gives a good approximate solution. Ayati and Biazar [30] elaborated on the convergence of the HPM. Many researchers used HPM for solving their problems. El-Dib [31] suggested a modified version of the homotopy perturbation methodology by combining the multiple scales methodology. However, the mix of the multiple scales methodology with the homotopy perturbation methodology is known as He-multiple-scales method. This technique is successfully used to improve the accuracy and computational efficiency of the nonlinear partial differential equation of the nonlinear Klein–Gordon equation [32]. El-Dib and Moatimid [33] adapted the HPM to obtain exact solutions of linear as well as nonlinear differential equations. The basic idea of their approach is to choose a suitable trial function, usually, in the form of a power series. The cancellation of the first-order approximate solution guarantees that all higher orders are also cancelled. Consequently, the remaining zero-order solution will be supported to become an exact one.

Nayfeh [5], Murakami [34], Mohamed *et al* [6], El-Dib [7], El-Dib and Ghaly [10], Elhefnawy [17], Hemamalini and Anjali Devi [35], Moatimid and Mostapha [36] and other researchers investigated the interaction of finite amplitude waves using multiple scales method. They introduce the time scales $T_n = \varepsilon^n t$ and spatial scales $X_n = \varepsilon^n x, n = 0$–2. They assume a small parameter ε expressing the steepness ratio of the wave. Sometimes the choice of small parameters depends on minimising one or more of the physical parameters. Actually, this procedure gives a restriction to the problem. In addition, the analysis in the foregoing works is lengthy. It does not enable us to obtain a formula of the analytical approximate solution of the amplitude wave. In contrast, the present work is concerned with the nonlinear stability analysis, regardless of using any small parameter. Therefore, this approach is very simple and may be used as a promising and powerful technique. Therefore, the aim of this paper is to introduce a novel approach for studying the nonlinear analysis of the RTI problem under the influence of a uniform magnetic field. The fields permit the presence of free surface currents at the surface of separation. Physically, the problem is interesting because it involves a uniform external rotation. Because of the great interest in the geophysical and engineering applications for stability through porous media, the current work is performed to investigate the influence of rotating fluids throughout the porous media on the stability behaviour. The analysis reveals the linear and nonlinear approaches. The rest of this paper is systematically organised as follows: §2 gives the physical modulation of the problem together with the appropriate nonlinear boundary conditions. The solution to the problem is reported in §3. Section 4 gives the derivation of the transcendental nonlinear characteristic dispersion relation. A modified HPM is added in this section. A numerical calculation, together with the stability profile, is presented in §5. Finally, the final results are given in §6.

2. Statement of the problem

The governing equations of motion may be formulated in two parts as follows. The first part deals with the hydrodynamics. As is well known from the literature, see e.g. the configuration of El-Dib and Mady [26], in a rotating frame of reference, where $\underline{\Omega} = \Omega \underline{e}_z$, the motion of an inviscid incompressible fluid may be governed by the following Euler's equation:

$$\frac{\partial \underline{V}}{\partial t} + \left(\underline{V} \cdot \nabla\right)\underline{V} + 2\left(\underline{\Omega} \times \underline{V}\right) = -\frac{1}{\rho}\nabla\pi - \nu\underline{V} - g\underline{e}_z, \tag{1}$$

where π is the reduced pressure which is given by

$$\pi = p - \frac{1}{2}\rho\left|\underline{\Omega} \times \underline{r}\right|^2 - \frac{1}{2}\mu H^2. \tag{2}$$

The incompressibility condition is given as

$$\nabla \cdot \underline{V} = 0, \tag{3}$$

where ρ is the fluid density, $\underline{V} = (u, v, w)$ represents the velocity field of the fluid particles, the term $2(\underline{\Omega} \times \underline{V})$ refers to the Coriolis force, the term $(1/2)\nabla|\underline{\Omega} \times \underline{r}|^2$ addresses the centrifugal force, $\underline{r}$ is the position vector of any point of the fluid, p is the hydrodynamic pressure, ν is Darcy's coefficient, g is the gravitational force and $\underline{e}_z$ is the unit vector in the z-axis.

It is worthwhile to notice that the Coriolis term is usually inserted in the governing equation, at which the angular velocity Ω has a uniform value in the rotating frame of reference.

The equilibrium configuration of the governing equation (1) yields

$$\pi_{0j}(x, y, z, t) = -\rho_j g z + \lambda_{0j}, \tag{4}$$

where λ_{0j} is the integration time-dependent function.

The second part concerns with the magnetic separation. As usual in the problems of the stability theory, it is convenient to assume a quasistatic approximation [37,38]. The considered system will be subjected to a uniform magnetic field. It is assumed that this field admits the presence of surface currents at the interface. In this case, Maxwell's equations may be formulated as

$$\nabla \cdot (\mu_j \underline{H}_j) = 0 \tag{5}$$

and

$$\nabla \times \underline{H}_j = \underline{J}_{\mathrm{f}}, \quad j = 1, 2, \tag{6}$$

where μ refers to the magnetic permeability of the fluid phase and the superscript to the fluid phase. Superscripts (1) and (2) indicate the upper fluid and lower fluid, respectively, and J_{f} is the free surface current density. Therefore, away from the surface currents, the magnetic field may be defined in terms of the magnetostatic potential $\phi_j(x, y, z, t)$, i.e. $\underline{H}_j = -\nabla\phi_j(x, y, z, t)$, such that the total magnetic fields may be expressed as

$$\underline{H}_j = H_{0j}\underline{e}_x - \nabla\phi_j(x, y, z, t). \tag{7}$$

Equation (5) says that the magnetostatic potential satisfies the following Laplace's equation:

$$\nabla^2\phi_j(x, y, z, t) = 0. \tag{8}$$

As stated above, the aim of this paper is to investigate the nonlinear RTI under a uniform, normal angular velocity together with a uniform tangential magnetic field. Therefore, the description of the considered system is represented by the three-dimensional finite amplitude of capillary-gravity Stocks surface wave [39] that is propagated on a plane surface of separation. For more convenience, the Cartesian coordinates (x, y, z) are utilised. It follows that the interface may be identified by the horizontal plane $z = 0$. This surface of separation is well defined and initially flat and has an infinite extension in the xy-plane. The plane $z = 0$ defines the undisturbed state. This interface separates two infinite incompressible magnetic fluids. The upper fluid occupies the infinite region $\infty > z > \xi$. This region has characteristic ρ_1, μ_1 and ν_1. Simultaneously, the lower liquid fills the infinite region $-\infty < z < \xi$. It has the characteristic ρ_2, μ_2 and ν_2. For simplicity, the porosities in both media are taken as unity. The two fluids are affected by a uniform tangential magnetic field, which acts along the positive x-direction. Therefore, the magnetic field intensity may be addressed as $\underline{H}_0(H_0, 0, 0)$. The magnetic fields are producing free electric surface currents [40,41]. The tension force between the two fluids is assumed to be uniform. It is defined by the coefficient σ_{T}. The system is influenced by uniform gravitational field $\underline{g}$ which acts along the negative z-direction. Furthermore, the system is influenced by the uniform angular velocity $\underline{\Omega}(0, 0, \Omega)$. For more convenience, the upper and lower fluids are assumed to be identical. The surface deflection [2] may be expressed by the following equation:

$$z = \xi(x, y, t). \tag{9}$$

The equation of the interface is identified as a geometric locus of points that satisfies the following relation:

$$S(x, y, z, t) = z - \xi(x, y, t) = 0. \tag{10}$$

Therefore, the unit normal to the interface may be expressed as

$$\underline{n} = \frac{\nabla S}{|\nabla S|} = (-\xi_x\underline{e}_x - \xi_y\underline{e}_y + \underline{e}_z)(1 + \xi_x^2 + \xi_y^2)^{-1/2}, \tag{11}$$

where $\underline{e}_x$ and $\underline{e}_y$ are the unit vectors in the x- and y-directions, respectively.

As stated previously, the equations of motion are classified into two categories: The first category comes from the hydrodynamic part and the second one stands for Maxwell's equations. As seen, these equations are the partial differential equations applicable at every point in the fluid. The integration of these equations resulted in some arbitrary constants/functions. To evaluate these constants, an additional statement is needed, which defines the boundary conditions. For more details in these aspects, the reader is encouraged to consult Batchelor [39] and Melcher [37]. For this purpose, as stated before, it is convenient to identify the function $\phi(x, y, z, t)$ as a finite function to be added due to the perturbed interface. Away from the interface, its influence disappears. Therefore, the derivatives of

$\phi(x, y, z, t)$ with respect to x, y and z must vanish. At the dividing surface, the following appropriate nonlinear boundary conditions must be verified.

In accordance with the hydrodynamic contribution:

(i) At the dividing surface, the velocity components u, v and w are subjected to the following kinematic boundary condition:

$$w_j = \xi_t + u_j \xi_x + v_j \xi_y, \quad z = \xi. \tag{12}$$

This equation resulted in the material character of the dividing surface between the two fluids.

Away from the interface, the fluid velocities must be neglected. In other words, one gets

$$\underline{V}_j(x, y, \pm\infty, t) = 0. \tag{13}$$

In accordance with the magnetic contribution, the surface magnetic forces $\underline{F}_{\mathrm{m}}$ [38] may be formulated as

$$\underline{F}_{\mathrm{m}} = \underline{J}_{\mathrm{f}} \wedge (\mu \underline{H}) - \frac{1}{2}\underline{H} \cdot \underline{H} \nabla \mu + \nabla\left(\frac{1}{2}\underline{H} \cdot \underline{H} \frac{\partial \mu}{\partial \rho} \rho\right). \tag{14}$$

The first term in eq. (14) concerns with the presence of interface currents. The second term occurs due to the inhomogeneity of the magnetic material. In the current case, this term should be ignored. Simultaneously, the third one must be omitted in view of the incompressibility criterion.

In the steady configuration, two cases must be addressed: (1) when the surface currents are absent at the interface. In this case, the jump of the tangential magnetic fields across the interface must be zero. It follows that $H_{01} = H_{02} = H_0$ and (2) when the surface currents are present. In this case, the horizontal components of the magnetic field should be discontinuous. Therefore, the tangential components of the magnetic field, keeping in mind $H_{01} \neq H_{02}$, give $H_{01} - H_{02} = J_{\mathrm{f}}$.

Due to the presence of surface currents, the boundary condition of the continuity of vertical component of the magnetic field occurs. In contrast, the continuity of the tangential components of the tangential magnetic field is not valid. Simultaneously, as shown by many researchers, see Melcher [37], the tangential component of the stress tensor should be continuous. As summarised in the previous magnetic boundary conditions, one finds:

(i) The continuity of the normal component of the magnetic displacement of the interface gives

$$\underline{n} \cdot \left(\mu_1 \underline{H}_1 - \mu_2 \underline{H}_2\right) = 0, \quad z = \xi. \tag{15}$$

This implies that

$$\xi_x(\mu_1\phi_{x1} - \mu_2\phi_{x2}) + \xi_y\left(\mu_1\phi_{y1} - \mu_2\phi_{y2}\right) - (\mu_1\phi_{z1} - \mu_2\phi_{z2}) = 0, \quad z = \xi. \tag{16}$$

(ii) The continuity of the tangential component of the stress tensor at the interface $z = \xi$ yields

$$\underline{n} \times \left(\underline{F}_1 - \underline{F}_2\right) = 0, \quad z = \xi, \tag{17}$$

where $\underline{F}$ is the total force acting at the interface, which is defined as

$$\underline{F} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \times \begin{pmatrix} -\xi_x \\ -\xi_y \\ 1 \end{pmatrix} \left(1 + \xi_x^2 + \xi_y^2\right)^{-1/2}. \tag{18}$$

Here the stress tensor is defined as

$$\sigma_{ij} = -\pi\delta_{ij} + \mu H_i H_j - \frac{1}{2}\mu H^2 \delta_{ij}, \tag{19}$$

where δ_{ij} is the Kronecker's delta.

(iii) The residual boundary condition, at the boundary between the two fluids, comes from the balance of hydrodynamics and magnetic stresses and the surface tension. These stresses include the pressure, surface tension forces and magnetic effects. In other words, the vertical component of the stress tensor σ_{ij} is intermittent at the interface by the amount of the surface tension. These stresses resulted in the following conservation of the momentum balance. This condition may be formulated as follows:

$$\underline{n} \cdot \left(\underline{F}_1 - \underline{F}_2\right) = \sigma_{\mathrm{T}} \nabla \cdot \underline{n}, \quad \text{at } z = \xi. \tag{20}$$

The set of previous boundary conditions is evaluated at the interface $z = \xi(x, y, t)$ as the interface is deformed; all variables have slight variations from their equilibrium values. Considering a small displacement, it follows that the boundary conditions of all perturbed variables need to be estimated at the equilibrium position rather than at the interface. Therefore, all physical quantities are to be expanded by means of the Maclaurin series at the equilibrium state $z = 0$.

The zero order of the normal stress tensor at the interface leads to

$$\lambda_{02} - \lambda_{01} = (\rho_1 - \rho_2) g z + \frac{1}{2}\left(\mu_2 H_{02}^2 - \mu_1 H_{01}^{2'}\right). \tag{21}$$

As a consequence, the zero order of the pressure, which is defined in (4), has a modification by the magnetic force. At this end, the equations of motion together with the corresponding boundary conditions completely define the considered boundary-value problem. Therefore, special solutions will be given in the following section.

3. Method of solution

As a usual analysis in the temporal hydrodynamic stability examination [2], all perturbed quantities are performed as a uniform travelling wave train. In view of the normal mode analysis, one gets

$$\underline{V}(x, y, z, t) = \underline{V}(z)\mathrm{e}^{\mathrm{i}k\zeta - \mathrm{i}\omega_0 t}, \tag{22}$$

$$\pi(x, y, z, t) = \hat{\pi}(z)\mathrm{e}^{\mathrm{i}k\zeta - \mathrm{i}\omega_0 t} \tag{23}$$

and

$$\phi(x, y, z, t) = \hat{\phi}(z)\mathrm{e}^{\mathrm{i}k\zeta - \mathrm{i}\omega_0 t}, \tag{24}$$

where the spatial variable ζ is defined as

$$\zeta = \frac{1}{k}\left(k_x x + k_y y\right), \quad k = \sqrt{k_x^2 + k_y^2}, \tag{25}$$

k may be real and positive and ω_0 is the frequency of the produced surface waves.

Suppose that there is a uniform travelling wave train propagating along the surface of separation, such that

$$\xi(x, y, t) = \gamma\mathrm{e}^{\mathrm{i}k\zeta - \mathrm{i}\omega_0 t} + \text{c.c.}, \tag{26}$$

where c.c. refers to the complex conjugate of the preceding terms and the amplitude γ is constant and real.

Employing eqs (22) and (23) into the governing equations of motion that are given by eqs (1) and (3), one gets

$$\hat{\pi}(z, t) = -\frac{1}{k^2}\left(1 - \frac{4\Omega^2}{\omega_0^2}\right)(-\mathrm{i}\rho\omega_0 + \nu)\frac{\mathrm{d}\hat{w}(z)}{\mathrm{d}z}, \tag{27}$$

$$\frac{\mathrm{d}^2\hat{w}(z)}{\mathrm{d}z^2} - q^2\hat{w}(z) = 0, \tag{28}$$

where the quantity q is given by

$$q = k^2\left(1 - \frac{4\Omega^2}{\omega_0^2}\right)^{-1/2}. \tag{29}$$

At a slowly rotating fluid, one may assume that $\Omega < \frac{1}{2}\omega_0$, so that $4\Omega^2/\omega_0^2$ is a sufficiently small quantity. Therefore, the linear part of the quantity q can be obtained from the binomial theorem as

$$q \cong k\left(1 + \frac{2\Omega^2}{\omega_0^2}\right). \tag{30}$$

Using the boundary condition (12) and continuity equation (3) and then applying the Maclaurin series at $z = 0$, we obtain the solution of eq. (28). This solution gives the distribution of normal velocity components in the following forms:

$$w_1(x, y, z, t) = -\mathrm{i}\omega_0(1 - \xi q)^{-1}\xi\mathrm{e}^{-qz}, \quad z > 0 \tag{31a}$$

and

$$w_2(x, y, z, t) = -\mathrm{i}\omega_0(1 + \xi q)^{-1}\xi\mathrm{e}^{qz}, \quad z < 0. \tag{31b}$$

Employing the distributions of eq. (31) into eq. (27), the pressure distribution in two fluids may be written in the following forms:

$$\pi_1(x, y, z, t) = -\frac{(\rho_1\omega_0^2 + \mathrm{i}\omega_0\nu_1)}{q} \times(1 - \xi q)^{-1}\xi\mathrm{e}^{-qz}, \quad z > 0 \tag{32a}$$

and

$$\pi_2(x, y, z, t) = \frac{(\rho_2\omega_0^2 + \mathrm{i}\omega_0\nu_2)}{q} \times(1 + \xi q)^{-1}\xi\mathrm{e}^{qz}, \quad z < 0. \tag{32b}$$

To derive the solution of the magnetic potential function $\phi(x, y, z, t)$, inserting eq. (24) into the Laplace eq. (8), after using the boundary conditions that are given in eqs (16) and (19), the resulting solutions become

$$\phi_1(x, y, z, t) = \frac{\mathrm{i}kH}{H - 1}J_{\mathrm{f}}\frac{\xi}{(1 - k\xi)}\mathrm{e}^{-kz}, \quad z > 0, \tag{33a}$$

$$\phi_2(x, y, z, t) = \frac{\mathrm{i}k}{H - 1}J_{\mathrm{f}}\frac{\xi}{(1 + k\xi)}\mathrm{e}^{kz}, \quad z < 0, \tag{33b}$$

where $H = H_0^{(1)}/H_0^{(2)}$ is used for deriving eqs (33a) and (33b).

As the nonlinear terms are ignored, the linear profile emerges and it is identical to those acquired previously by El-Dib [40] for interface supporting free electric surface current and Chandrasekhar [2] for unadulterated fluids.

4. The transcendental characteristic equation

At the boundary between the two fluids, the fluid and the magnetic stresses must be balanced. The segments of these burdens rely upon hydrodynamic weight, surface pressure stresses and attractive anxieties. In what follows, the nonlinear condition overseeing the interracial relocation will be evaluated. Utilising the normal part of the speed circulation (31), the weight dissemination (32) and the attractive potential dispersion (33) to the typical pressure tensor (21), the subsequent is the nonlinear dispersion relation in the displacement ξ:

$$\begin{aligned}&\frac{1}{q}(1-\xi q)^{-1}(\rho_1\omega_0^2+\mathrm{i}\omega_0\nu_1)\xi\\&\quad+\frac{1}{q}(1+\xi q)^{-1}(\rho_2\omega_0^2+\mathrm{i}\omega_0\nu_2)\xi-(\rho_1-\rho_2)g\xi\\&\quad+k^2\sigma_{\mathrm{T}}\xi(1-k^2\xi^2)^{-3/2}+\frac{J_{\mathrm{f}}^2}{(H-1)^2}(1-k^2\xi^2)^{-1}\\&\quad\times[(\mu_1H^2+\mu_2)k\xi+(\mu_1H^2-\mu^{(2)})k^2\xi^2]\\&\quad=0.\end{aligned}\tag{34}$$

Equation (34) is the characteristic equation which represents the generalised form of the linear characteristic equation that is given in Chandrasekhar's book [2] for rotating fluids. In order to simplify its complicated form, the two parts may be separated, so that one finds

$$\begin{aligned}&\frac{\omega_0^2}{q}[\rho_1(1-\xi q)^{-1}+\rho_2(1+\xi q)^{-1}]\xi\\&\quad-(\rho_1-\rho_2)g\xi+k^2\sigma_{\mathrm{T}}\xi(1-k^2\xi^2)^{-3/2}\\&\quad+\frac{J_{\mathrm{f}}^2}{(H-1)^2}(1-k^2\xi^2)^{-1}[(\mu_1H^2+\mu_2)k\xi\\&\quad+(\mu_1H^2-\mu_2)k^2\xi^2]=0\end{aligned}\tag{35}$$

and

$$(\nu_1+\nu_2)+(\nu_1-\nu_2)\xi q=0.\tag{36}$$

Using eq. (36) to remove ξq from eq. (35), one gets

$$\begin{aligned}&\frac{\omega_0^2}{2q\eta}(\nu-1)(\rho_1-\rho_2\eta)\left(1-k^2\xi^2\right)\xi\\&\quad-(\rho_1-\rho_2)g\xi\left(1-k^2\xi^2\right)\\&\quad+k^2\sigma_{\mathrm{T}}\xi\left(1-k^2\xi^2\right)^{-1/2}+\frac{kJ_{\mathrm{f}}^2}{(H-1)^2}\\&\quad\times\left[\left(\mu_1H^2+\mu_2\right)\xi+\left(\mu_1H^2-\mu_2\right)k\xi^2\right]=0,\end{aligned}\tag{37}$$

where $\nu=\nu_1/\nu_2$ is used. For all physical parameters, except the surface tension coefficient, it is seen that the highest power for the elevation ξ is of the cubic order. Therefore, in view of the binomial theory, we kept only the primary terms in the series of the square root. In view of a slight rotation of the fluids, eq. (37) is arranged in the form

$$\omega^2(\omega_0,k)\xi+\alpha\xi^2+\beta\xi^3=0,\tag{38}$$

where

$$\begin{aligned}\omega^2(\omega_0,k)&=\frac{\left(\omega_0^2-2\Omega^2\right)}{2\nu}(\nu-1)(\rho_1-\rho_2\nu)\\&\quad-k(\rho_1-\rho_2)g+k^3\sigma_{\mathrm{T}}\\&\quad+\frac{k^2J_{\mathrm{f}}^2}{(H-1)^2}(\mu_1H^2+\mu_2),\end{aligned}\tag{39}$$

$$\alpha=\frac{k^3J_{\mathrm{f}}^2}{(H-1)^2}\left(\mu_1H^2-\mu_2\right)\tag{40}$$

and

$$\beta=-k^2\left[\frac{(\omega_0^2-2\Omega^2)}{2\nu}(\nu-1)(\rho_1-\rho_2\nu)-k(\rho_1-\rho_2)g-\frac{1}{2}k^3\sigma_{\mathrm{T}}\right].\tag{41}$$

Throughout the linear stability theory, the linear equations of motion together with appropriate boundary conditions are coupled, dropping the nonlinear terms. This approach leads to the well-known linear dispersion relation given in Chandrasekhar's book [2]. In the present approach, the nonlinear parameters are not cancelled from the boundary conditions (see refs [6,7]). The thought of the nonlinear idea may be a slight departure from the linear viewpoint. At this stage, the dispersion equation sought to be extended to incorporate nonlinear terms within the surface elevation. The conclusion of the nonlinear terms in the dispersion the equation is extremely within the elevation parameter. Since $\xi\neq0$, eq. (38) gives

$$\omega^2(\omega_0,k)+\alpha\xi+\beta\xi^2=0.\tag{42}$$

Equation (42) may be viewed as an expansion of the linear frequency in terms of the undetermined variable ξ. This variable needs to be determined. Relation (42) refers to a nonlinear correction due to the nonlinear physical problem. It contains the linear frequency ω_0 and the primary elevation $\xi_0(x,y,t)$ which are defined in (26).

The linear dispersion relation of the slowly rotating Darcian magnetic Rayleigh–Taylor flow in the presence of free surface currents is obtained by ignoring higher powers of ξ in eq. (38). Therefore, one gets

$$\omega^2(\omega_0,k)\xi=0.\tag{43}$$

Equation (43) may be written as

$$\omega_0^2 = \Omega^2 - \frac{\nu}{(\nu-1)(\rho_1-\rho_2\nu)} \times \left[k^3\sigma_T - k(\rho_1-\rho_2)g + \frac{k^2 J_f^2}{(H-1)^2}\left(\mu_1 H^2 + \mu_2\right)\right]. \quad (44)$$

The stability criterion needs $\omega_0^2 > 0$, and therefore the linear stability requires

$$\Omega^2 > \frac{\nu}{(\nu-1)(\rho_1-\rho_2\nu)} \times \left[k^3\sigma_T - k(\rho_1-\rho_2)g + \frac{k^2 J_f^2}{(H-1)^2}\left(\mu_1 H^2 + \mu_2\right)\right]. \quad (45)$$

As seen from eq. (26), the uniform travelling wave train solution requires that the parameter ω_0 must be of real value. This implies that ω_0^2 should be positive. It follows that the present linear stability approach needs the validation of inequality (45). First of all, it is clear from eq. (45) that the rotation parameter Ω has a stabilising effect [22]. It is seen that the contribution of porous media produces the term $\nu/((\nu-1)(\rho_1-\rho_2\nu))$. Actually, the stability/instability, which depends mainly on the significance of this term, is given as follows: If this term is positive, the magnetic field has a stabilising effect and vice versa. This influence is in contrast with the case of non-porous media. Since the third term in inequality (45) has always been positive, it follows that the role of the magnetic field does not depend on the ratio H or the surface current J_f. Therefore, the porous media allow the magnetic field to play a dual role in the stability criterion. In addition, the role of gravity (g) depends on the sign of $(\rho_1-\rho_2)$. Moreover, the surface tension has a stabilising effect. These are early results first obtained by many researchers [2]. The classical stability criterion is substantially modified in accordance with the rotation parameter Ω. In the absence of the rotation and porous media, the above stability criterion is similar to that obtained early by Chandrasekhar [2] and Rosensweig [13]. In the linear stability of the limiting case, in the absence of both rotation and Darcian parameter, El-Dib [40] demonstrated that the free surface current density has a stabilising influence on the magnetic flow. Moatimid *et al* [42] examined Darcy's coefficient through the linear stability analysis of the cylindrical interface and demonstrated the stabilising influence. Baldwin *et al* [43] studied the stabilising influence of rigid fluid rotation on the RTI. Dolai and Prajapati [44] showed that both the magnetic field and the rotation parameter have strong correlation effects to stabilise the growth rate of linear RTI.

It is useful to investigate a numerical estimation for linear stability of the wave propagating at the interface in the presence of both rotation and the Darcian parameter. In order to introduce this examination, numerical calculations for stability condition (45) have been performed as cited in §5.

The implication of nonlinearity in the stability profile will be discussed in the following subsection.

4.1 *The modified HPM with a conformable operator*

The HPM, which is a combination of homotopy in topology [45] and classic perturbation techniques, provides us a convenient procedure to obtain analytic or approximate solutions to numerous problems arising in different fields. It can convert any nonlinear problem into finite linear problems. It requires little iteration to obtain accurate solutions. It is worthwhile to note that the method has flexibility in adding or subtracting any term in the homotopy equation to obtain a uniform zero-order solution.

For incorporating the contribution of the nonlinear parts in the characteristic equation given by eq. (38) in the stability picture, it is more convenient to rewrite this equation as follows:

$$\xi_{\theta\theta} + \omega^2\xi - \xi_{\theta\theta} + \alpha\xi^2 + \beta\xi^3 = 0, \quad (46)$$

where θ represents the wave parameter and is given by

$$\theta = k\zeta - \omega_0 t. \quad (47)$$

Construct the following conformal modified homotopy equation:

$$\xi_{\theta\theta} + \omega^2\xi + \delta\left(-\xi_{\theta\theta} + \alpha\xi^2 + \beta\xi^3\right) = 0, \quad (48)$$

where δ is the homotopy parameter which is defined as $\delta \in [0, 1]$. In the HPM, an embedding parameter δ is introduced in an artificial manner. It is dimensionless and has no physical meaning. Equation (48) will turn to eq. (38) when $\delta \to 1$. In the zero-order state, as $\delta \to 0$, eq. (48) gives a linear harmonic differential equation having ω as the natural frequency.

Consider a homotopy expansion of the function ξ as a power series in the parameter δ, and one gets

$$\xi(\theta;\delta) = \xi_0(\theta) + \delta\xi_1(\theta) + \delta^2\xi_2(\theta) + \cdots . \quad (49)$$

As usual, inserting eq. (49) into the homotopy equation (48), and then equating the indicial powers of δ, one finds

$$\xi_{0\theta\theta} + \omega^2\xi_0 = 0, \quad (50)$$

$$\xi_{1\theta\theta} + \omega^2\xi_1 = \xi_{0\theta\theta} - \alpha\xi_0^2 - \beta\xi_0^3 \quad (51)$$

and

$$\xi_{2\theta\theta} + \omega^2 \xi_2 = \xi_{1\theta\theta} - 2\alpha\xi_0\xi_1 - 3\beta\xi_0^2\xi_1. \quad (52)$$

The solution of eq. (50) may be written as

$$\xi_0 = A e^{i\omega\theta} + \bar{A} e^{-i\omega\theta}, \quad (53)$$

where the amplitude A, in general, is a complex constant and $\bar{A}$ refers to its complex conjugate. Inserting the primary solution (53) into eq. (51), the condition for obtaining a uniform solution requires

$$\omega^2 = 3\beta A\bar{A}. \quad (54)$$

Accordingly, the uniform solution of eq. (51) yields

$$\xi_1 = \frac{\alpha}{3\omega^2}(A^2 e^{2i\omega\theta} - 6A\bar{A} + \bar{A}^2 e^{-2i\omega\theta}) + \frac{\beta}{8\omega^2}(A^3 e^{3i\omega\theta} + \bar{A}^3 e^{-3i\omega\theta}). \quad (55)$$

Substituting solutions (53) and (55) into eq. (52), the cancellation of the secular terms requires

$$|A|^2 = \frac{80}{9}\frac{\alpha^2}{\beta^2}. \quad (56)$$

The particular solution of eq. (52) yields

$$\begin{aligned}\xi_2 = {} & \frac{4\alpha}{9\omega^2}(A^2 e^{2i\omega\theta} + \bar{A}^2 e^{-2i\omega\theta}) \\ & + \frac{9\beta}{64\omega^2}(A^3 e^{3i\omega\theta} + \bar{A}^3 e^{-3i\omega\theta}) \\ & + \frac{2\alpha^2}{24\omega^4}(A^3 e^{3i\omega\theta} + \bar{A}^3 e^{-3i\omega\theta}) \\ & + \frac{\alpha\beta}{12\omega^4}(A^4 e^{4i\omega\theta} - 15A^3\bar{A} e^{2i\omega\theta} \\ & + 120A^2\bar{A}^2 - 15A\bar{A}^3 e^{-2i\omega\theta} + \bar{A}^4 e^{-4i\omega\theta}) \\ & + \frac{\beta^2}{64\omega^4}(A^5 e^{5i\omega\theta} + 6A^4\bar{A} e^{3i\omega\theta} \\ & + 6\bar{A}^4 A e^{-3i\omega\theta} + \bar{A}^5 e^{-5i\omega\theta}). \end{aligned} \quad (57)$$

Combining the solvability conditions that are given in eqs (54) and (56), the dispersion relation in view of the nonlinear approach becomes

$$\omega^2 = \frac{80\alpha^2}{3\beta}. \quad (58)$$

Substituting eqs (39)–(41) into eq. (58), the above nonlinear dispersion relation may be arranged in the form

$$\omega_0^4 + 2B\omega_0^2 + 4C = 0, \quad (59)$$

where

$$B = 2\Omega^2 + \frac{\nu}{(\nu-1)(\rho_1 - \rho_2\nu)} \times \left[\frac{1}{2}k^3\sigma_T - 2k(\rho_1-\rho_2)g + \frac{k^2 J_f^2}{(H-1)^2}(\mu_1 H^2 + \mu_2)\right], \quad (60)$$

$$\begin{aligned} C = {} & \Omega^4 + 2\Omega^2 \frac{\nu}{(\nu-1)(\rho_1-\rho_2\nu)} \\ & \times \left[\frac{1}{2}k^3\sigma_T - 2k(\rho_1-\rho_2)g + \frac{k^2 J_f^2}{(H-1)^2}(\mu_1 H^2 + \mu_2)\right] \\ & + \frac{k^2\nu^2[(\rho_1-\rho_2)g + \frac{1}{2}k^2\sigma_T]}{(\nu-1)^2(\rho_1-\rho_2\nu)^2} \\ & \times \left[(\rho_1-\rho_2)g - k^2\sigma_T - \frac{k J_f^2}{(H-1)^2}(\mu_1 H^2 + \mu_2)\right] \\ & + \frac{80k^2\nu^2}{3(\nu-1)^2(\rho_1-\rho_2\nu)^2} \\ & \times \frac{k^2 J_f^4}{(H-1)^4}(\mu_1 H^2 - \mu_2)^2. \end{aligned} \quad (61)$$

Clearly, the stability criteria along the following inequalities are

$$B < 0, \quad C > 0 \quad \text{and} \quad B^2 - 4C > 0. \quad (62)$$

As seen before, the transition curve in the linear stability approach, as given in eq. (43), represents a quadratic equation in ω_0. Consequently, there is only one criterion in the linear stability concept, as given by inequality (45). In contrast, the nonlinear approach yields a quartic equation in ω_0, as given in eq. (59). Therefore, there are three criteria of the nonlinear stability that are given in (62). This shows the accuracy in handling the nonlinear stability theory.

The perturbed solution of the characteristic equation given in eq. (48), up to three perturbed terms, is obtained by combining eqs (53), (55) and (57) into the perturbed solution given in eq. (49) and setting $\delta \to 1$, one finds

$$\xi(\theta) = -\frac{2\alpha}{\omega^2}A\bar{A} + \frac{10\alpha\beta}{\omega^4}A^2\bar{A}^2 + (A\mathrm{e}^{\mathrm{i}\omega\theta} + \bar{A}\mathrm{e}^{-\mathrm{i}\omega\theta}) + \left(\frac{7\alpha}{9\omega^2} - \frac{5\alpha\beta}{4\omega^4}|A|^2\right)(A^2\mathrm{e}^{2\mathrm{i}\omega\theta} + \bar{A}^2\mathrm{e}^{-2\mathrm{i}\omega\theta}) + \left(\frac{17\beta}{64\omega^2} + \frac{2\alpha^2}{24\omega^4} + \frac{3\beta^2}{32\omega^4}|A|^2\right) \times (A^3\mathrm{e}^{3\mathrm{i}\omega\theta} + \bar{A}^3\mathrm{e}^{-3\mathrm{i}\omega\theta}) + \frac{\alpha\beta}{12\omega^4}(A^4\mathrm{e}^{4\mathrm{i}\omega\theta} + \bar{A}^4\mathrm{e}^{-4\mathrm{i}\omega\theta}) + \frac{\beta^2}{64\omega^4}(A^5\mathrm{e}^{5\mathrm{i}\omega\theta} + \bar{A}^5\mathrm{e}^{-5\mathrm{i}\omega\theta}). \quad (63)$$

The perturbed solution given in eq. (63) is simplified by using eqs (56) and (58) to get

$$\xi(\theta) = \frac{4}{9}\frac{\alpha}{\beta} + (A\mathrm{e}^{\mathrm{i}\omega\theta} + \bar{A}\mathrm{e}^{-\mathrm{i}\omega\theta}) + \frac{13\beta}{6\times 160\alpha}(A^2\mathrm{e}^{2\mathrm{i}\omega\theta} + \bar{A}^2\mathrm{e}^{-2\mathrm{i}\omega\theta}) + \frac{9\beta^2}{800\alpha^2}(A^3\mathrm{e}^{3\mathrm{i}\omega\theta} + \bar{A}^3\mathrm{e}^{-3\mathrm{i}\omega\theta}) + \frac{3\beta^3}{4\times 80\times 80\alpha^3}(A^4\mathrm{e}^{4\mathrm{i}\omega\theta} + \bar{A}^4\mathrm{e}^{-4\mathrm{i}\omega\theta}) + \frac{9\beta^4}{64\times 80\times 80\alpha^4}(A^5\mathrm{e}^{5\mathrm{i}\omega\theta} + \bar{A}^5\mathrm{e}^{-5\mathrm{i}\omega\theta}). \quad (64)$$

In light of the wave train considered in (26), the comparison with (56) reveals that the amplitude γ has the form

$$\gamma = \pm\sqrt{\frac{80}{9}\frac{\alpha}{\beta}}. \quad (65)$$

At this stage, the above solution may be written in the following form:

$$\xi(\theta) = \frac{4\alpha}{9\beta}\left(1 + 6\sqrt{5}\cos\omega\theta + \frac{13}{24}\cos 2\omega\theta + \frac{3}{5}\sqrt{5}\cos 3\omega\theta + \frac{1}{24}\cos 4\omega\theta + \frac{1}{96}\sqrt{5}\cos 5\omega\theta\right). \quad (66)$$

5. Numerical discussion

In what follows, for more convenience, the numerical calculations will be made to illustrate graphically the implications of various physical parameters on the stability picture. Therefore, the stability criteria will be plotted together with the perturbed surface deflection. For the suitability of calculations, the physical parameters must be identified in a non-dimensional form. This may be done by several ways depending mainly on the choice of the characteristics of length, time and mass. For this purpose, consider the following characteristics: $(\sigma_\mathrm{T}/\rho_2\omega_0^2)^{1/3}$, $1/\omega_0$ and $\sigma_\mathrm{T}/\omega_0^2$ to refer to the length, time and mass, respectively. The other non-dimensional quantities are given by: $k = k^*\omega_0^2/g$, $\Omega = \Omega^*\omega_0$ and $J_\mathrm{f}^2 = J_\mathrm{f}^{*2}\omega_0^2\sigma_\mathrm{T}/\mu_2 g$. For simplicity, the '$*$' mark may be ignored in the coming analysis.

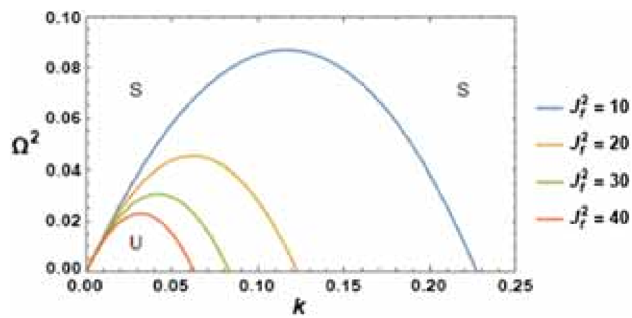

Figure 1. Linear stability diagram of the inequality given in (45) for various values of J_f^2.

In view of the linear stability theory, the stability criterion given in eq. (45) is displayed to sketch the square of the angular frequency Ω^2 vs. the wave number k. Therefore, figure 1 is plotted for a system having the particulars $\rho = 1.5$, $H = 10$, $\mu = 0.15$ and $\nu = 0.7$. The parameter ρ refers to the ratio ρ_1/ρ_2. This means that the system is statically unstable. This figure shows the influence of surface current in the stability diagram. As shown, the increase in J_f^2 increases the stability region. Therefore, the presence of surface currents has a stabilising influence. This mechanism was shown in the previous work of El-Dib [40]. Figure 2 concerns with the same particulars of the same system as given in figure 1 except that $J_\mathrm{f}^2 = 10$. It is sketched to display the destabilising influence of the ratio of the magnetic field intensities H on the linear stability analysis. Actually, this effect occurs due to the presence of surface currents at the interface. In the absence of surface currents, it is shown previously that the tangential magnetic field has a stabilising influence (see Rosensweig [13], Melcher [37] and El-Dib and Moatimid [46]). In contrast to the previous mechanism, this figure shows a destabilising influence in accordance with the tangential magnetic field. The implication of Darcy's coefficients on the stability picture is displayed in figure 3. This figure has the same particulars as given in figure 1 except that $J_\mathrm{f}^2 = 6$

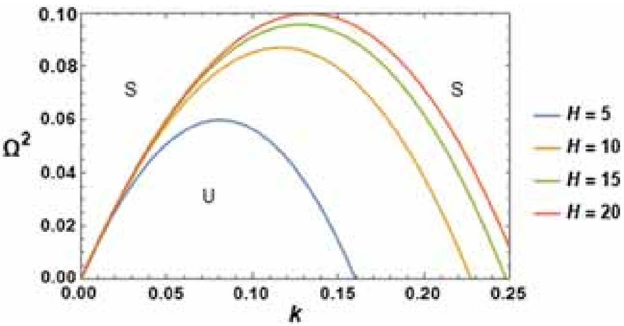

Figure 2. Linear stability diagram of the inequality given in (45) for various values of H.

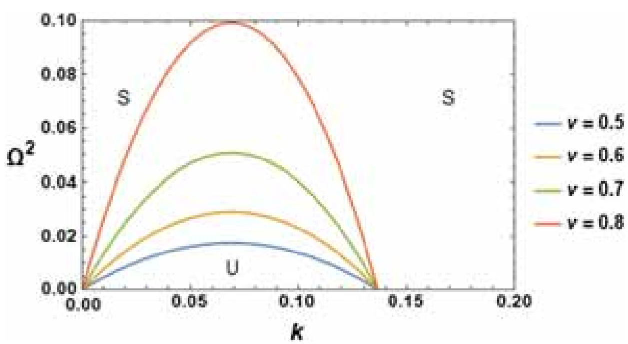

Figure 3. Linear stability diagram of the inequality given in (45) for various values of ν.

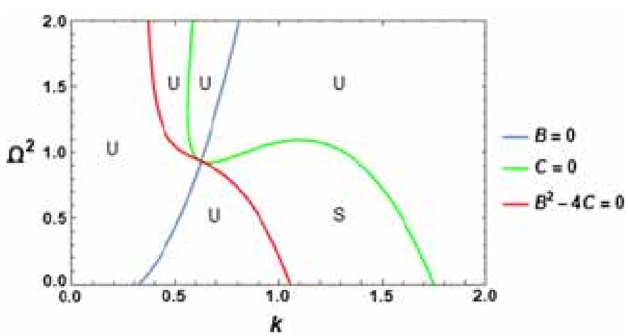

Figure 4. Nonlinear stability diagram of inequalities (62).

and $H = 3$. As seen from the behaviour of the transition curves in this figure, the parameter ν increases the unstable region. Therefore, it plays a destabilising influence due to the presence of surface currents at the interface between the two uniform magnetic fluids. The same sense occurs in the previous work of Moatimid and El-Dib [47]. The investigation of the above linear stability diagrams reveals that the rotation of the fluid has a stabilising influence. This conclusion is similar to those demonstrated in the previous works of Scase *et al* [22], Sharma *et al* [23], Dolai and Prajapati [44] and Prajapati [24].

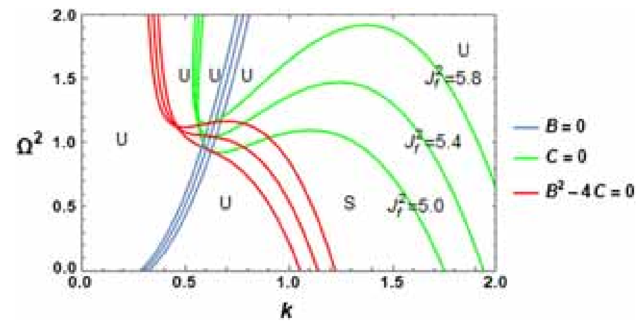

Figure 5. Nonlinear stability diagram of the inequalities given in (62) for various values of J_{f}^2.

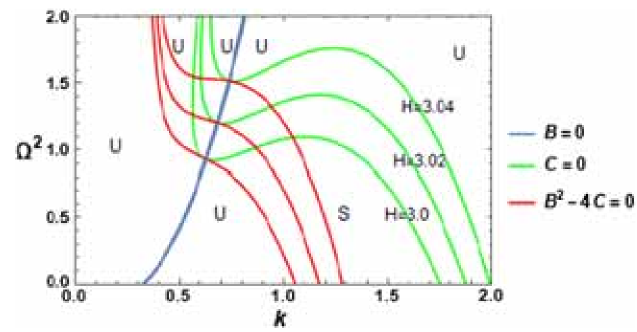

Figure 6. Nonlinear stability diagram of the inequalities given in (62) for various values of H.

As usual, in contrast with the linear approach, the nonlinear one indicates several stability criteria. These criteria are given in (62). Figure 4 is plotted to indicate these criteria. As before, this figure plots the square of the angular frequency Ω^2 vs. the wave number k. The particulars of this figure are chosen as $\rho = 1.5$, $H = 3$, $\mu = 0.15$, $J_{\mathrm{f}}^2 = 5$ and $\nu = 0.7$. As seen, there is a small stable region that satisfies the three conditions $B < 0$, $C > 0$ and $B^2 - 4C > 0$. This stable region occurs at small values of the angular velocity Ω^2. This region lies between the lower red and green curves. The implication of the surface currents J_{f}^2 on the nonlinear approach is depicted in figure 5. This figure has the same particulars as given in figure 4. It is seen that small deviations in the free surface current J_{f}^2 enhances the stability region. The same stabilising role is observed throughout the previous linear stability [40]. In addition, the influence of the ratio of the magnetic field H is displayed in figure 6. This figure has the same particulars as given in figure 4. This figure shows that very small deviation in the ratio of the tangential magnetic field causes more increase in the stable region. Therefore, the field has a stabilising effect on the nonlinear stability picture. This role is in contrast with the role that is already observed in the linear stability (see figure 2). Finally, the implication of the ratio among the Darcy's coefficients is depicted

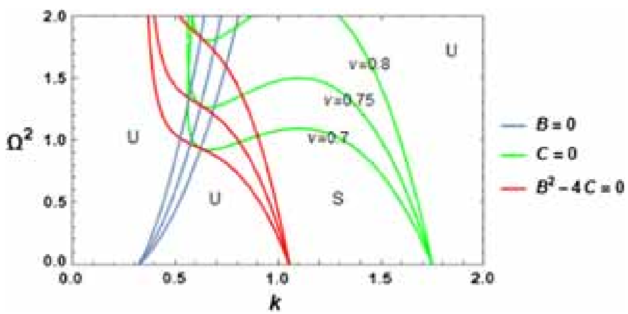

Figure 7. Nonlinear stability diagram of the inequalities given in (62) for various values of ν.

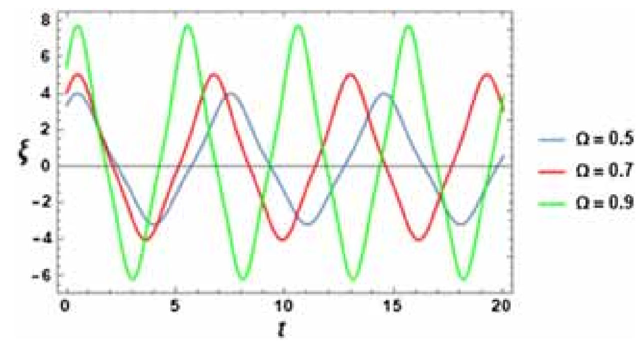

Figure 9. The surface evolution profile given in eq. (66) for various values of Ω.

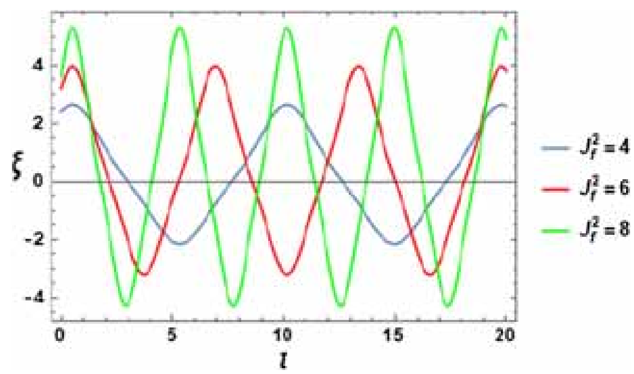

Figure 8. The surface evolution profile given in eq. (66) for various values of J_{f}^2.

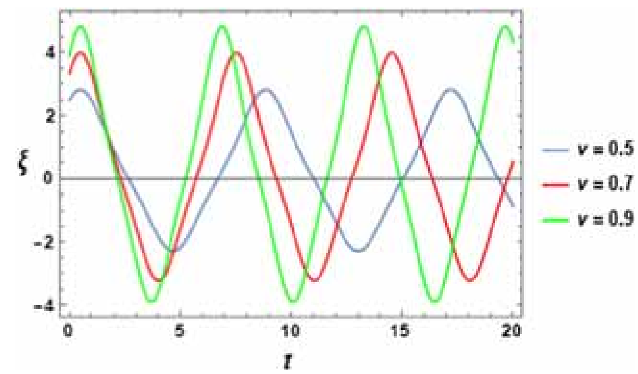

Figure 10. The surface evolution profile given in eq. (66) for various values of ν.

in figure 7. This figure has the same particulars as given in figure 4. A stabilising role is observed in the nonlinear stability. As seen in the previous figure, an opposite influence has been recorded in the linear stability, as shown in figure 3. A general observation throughout figures 5, 6 and 8 is that the increase in the stable region occurs due to a small shift in the red curves, which represents the transition curve of the discriminant and large deviation in the green curves.

For more convenience, it is better to sketch the distribution of the surface deflection of the interface. In accordance with the previous procedure, the cancellation of the secular terms reveals an oscillatory behaviour. Therefore, figure 8 is plotted for a system having the particulars $\rho = 1.5$, $\nu = 0.7$, $H = 3.0$, $\mu = 0.15$, $k = 0.5$, $\Omega = 0.6$ and $\zeta = 1.0$. The diagram is displayed for different values of J_{f}^2. This figure plots surface deviation ξ vs. time t. It is shown that the wavelength increases while the amplitude of the elevation decreases as the parameter J_{f}^2 decreases. Figure 9 displays the profile of the surface deflection for the same system as in figure 8 except that the fixed parameter $J_{\mathrm{f}}^2 = 5$. The surface deflections are depicted for different values of frequency Ω. Similar behaviour, as in figure 8, is observed. Finally, figure 10 is plotted to illustrate the influence of the ratio of Darcy's coefficient in the profile of the surface deflection. This figure has the same particulars as given in figure 8 except that the fixed parameter $J_{\mathrm{f}}^2 = 5$. It is seen that similar behaviour can be observed in figures 8–10. One can conclude that both the parameters, Ω and ν, play dual roles in the stability behaviour.

6. Concluding remarks

This paper presents a novel approach for studying the nonlinear RTI. As usual, the problem is modulated throughout the Cartesian coordinate system. A simplified formulation is made for the problem by considering two infinite incompressible magnetic fluids. Because of the great importance of the porous media, the problem considered a small representation of this phenomenon. The system is acted upon by a uniform tangential magnetic field. This field allows the presence of surface currents at the interface. The analysis of the problem is achieved in three dimensions. A uniform angular velocity is considered along the normal axis. The problem meets its practical importance of a geophysical point of view. The nonlinear analysis is based on the solution of the linear equation of motion together with the implication of the appropriate nonlinear boundary conditions. A transcendental characteristic dispersion equation of the surface deflection is derived. Some special cases are reported upon appropriate data choices. The stability analysis is analysed using linear and nonlinear approaches. To defeat the obstacle in the obtained transcendental equation, a small angular velocity is only taken into account. The stability analysis adapted the HPM. The main results of the problem may be summarised in the following points:

- Throughout the linear stability approach:
  - Only one stability criterion is given by inequality (45).
  - The stability diagram plots the square of the angular frequency Ω^2 vs. the wave number k.
  - To relax the mathematical calculations, a non-dimensional analysis is performed. The numerical calculations show that the surface currents have a stabilising effect. In contrast, the ratios of the magnetic fields as well as the Darcy's coefficients have a destabilising influence.
  - The magnetic field plays a dual role in the stability configuration depending on the sign of the term $\nu/((\nu-1)(\rho_1-\rho_2\nu))$.
- Throughout the nonlinear approach:
  - Several stability criteria have been obtained as given in inequalities (62).
  - As shown in the linear theory, the stability diagram depicts the square of the angular frequency Ω^2 vs. the wave number k.
  - The numerical calculations show that all the parameters H, J_{f}^2, and ν have a stabilising influence.
  - The distribution of the surface deflection is deduced in eq. (66). In addition, its profile is sketched throughout three figures.
- The profile of the surface deflection reveals that:
  - The parameters J_{f}^2, Ω and ν play dual roles in the stability behaviour.

References

[1] D J Lewis, *Proc. R. Soc. London dynamic A* **202**(**1086**), 81 (1950)
[2] S Chandrasekhar, *Hydrodynamic and hydromagnetic stability* (Oxford University Press, Oxford, 1961)
[3] A R Piriz, J Sanz and L F Ibañez, *Phys. Plasmas* **4**, 1117 (1997)
[4] A R Piriz, O D Cortăzar and J J Lŏpez Cela, *Am. J. Phys.* **74**(**12**), 1095 (2006)
[5] A H Nayfeh, *J. Appl. Mech.* **98**, 584 (1976)
[6] A A Mohamed, Y O El-Dib and A A Mady, *Chaos Solitons Fractals* **14**, 1027 (2002)
[7] Y O El-Dib, *J. Magn. Magn. Mater.* **260**, 1 (2003)
[8] F A I Dullien, *Porous media* (Academic Press, New York, 1992)
[9] H H Bau, *Phys. Fluids* **25**(**10**), 1719 (1982)
[10] Y O El-Dib and A Y Ghaly, *Chaos Solitons Fractals* **18**(**1**), 55 (2003)
[11] R C Sharma and P Kumar, *Indian J. Pure Appl. Math.* **24**(**9**), 563 (1993)
[12] G M Moatimid, M H Obied Allah and M A Hassan, *Phys. Plasmas* **20**(**10**), 102111 (2013)
[13] R E Rosensweig, *Ferrohydrodynamics* (Cambridge University Press, Cambridge, 1985)
[14] R E Zelazo and J R Melcher, *J. Fluid Mech.* **39**(**1**), 1 (1969)
[15] S K Malik and M Singh, *Q. Appl. Math.* **47**(**1**), 59 (1989)
[16] R Kant and S K Singh, *Phys. Fluids* **28**(**2**), 3534 (1985)
[17] A R Elhefnawy, *Int. J. Theor. Phys.* **31**(**8**), 1505 (1992)
[18] Y O El-Dib, *J. Plasma Phys.* **51**(**1**), 1 (1994)
[19] A R F Elhefnawy, M A Mahmoud, M A A Mahmoud and G M Khedr, *Can. Appl. Math. Q.* **12**(**3**), 323 (2004)
[20] P K Bhatia and R P Mathur, *Z. Naturforsch.* **60a**, 484 (2005)
[21] S Chandrasekhar, *Astrophys. J.* **119**, 7 (1954), https://doi.org/10.1086/145790
[22] M M Scase, K A Baldwin and R J A Hill, *Phys. Rev. Fluids* **2**(**2**), 024801 (2017)
[23] P K Sharma, R P Prajapti and R K Chhajlani, *Acta Phys. Pol. A* **118**(**4**), 576 (2010)
[24] R P Prajapati, *Phys. Plasmas* **23**, 022106 (2016)
[25] J J Tao, X T He and W H Ye, *Phys. Rev. E* **87**, 013001 (2013)
[26] Y O El-Dib and A A Mady, *J. Comput. Appl. Mech.* **49**(**2**), 50 (2018)
[27] J H He, *Comput. Method. Appl. Mech. Eng.* **178**, 257 (1999)
[28] J H He, *Appl. Math. Comput.* **151**(**1**), 287 (2004)
[29] J H He, *Int. J. Nonlinear Sci. Numer. Simul.* **6**(**2**), 207 (2005)
[30] Z Ayati and J Biazar, *J. Egypt. Math. Soc.* **23**(**2**), 424 (2015)
[31] Y O El-Dib, *Nonlinear Sci. Lett. A* **8**(**4**), 352 (2017)
[32] Y O El-Dib, *Pramana – J. Phys.* **92**: 7 (2019)
[33] Y O El-Dib and G M Moatimid, *Nonlinear Sci. Lett. A* **9**(**3**), 220 (2018)
[34] Y Murakami, *Phys. Lett. A* **131**, 368 (1988)
[35] P T Hemamalini and S P Anjali Devi, *Fluid Dyn. Mater. Process.* **10**(**4**), 491 (2014)
[36] G M Moatimid and D R Mostapha, *AIP Adv.* **9**(**5**), 055302 (2019)
[37] J R Melcher, *Field coupled surface waves* (MIT Press, Cambridge, 1963)
[38] H H Woodson and J R Melcher, *Electromechanical dynamics* (John Wiley and Sons, New York, 1968)
[39] G K Batchelor, *An introduction to fluid dynamics* (Cambridge University Press, Cambridge, 1967)

[40] Y O El-Dib, *J. Plasma Phys.* **65**(**1**), 1 (2001)
[41] Y O El-Dib, *J. Colloid Interface Sci.* **259**, 309 (2003)
[42] G M Moatimid, Y O El-Dib and M H Zekry, *Pramana – J. Phys.* **92**: 22 (2019)
[43] K A Baldwin, M M Scase and R J A Hill, *Sci. Rep.* **5**, 11706 (2015)
[44] B Dolai and R P Prajapati, *Phys. Plasmas* **25**, 083708 (2018)
[45] C Nash and S Sen, *Topology and geometry for physicists* (Academic Press, London, 1983)
[46] Y O El-Dib and G M Moatimid, *Z. Naturforsch.* **57a**, 159 (2002)
[47] G M Moatimid and Y O El-Dib, *Physica A* **333**, 41 (2004)