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Analysis of energetic intensity of Cern neutrino conversion types using the theory of covariance functions

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Abstract. In this paper, the changes of energetic intensity of Cern neutrinos during an experiment are analysed. For the analysis of energetic intensity of neutrinos, the theory of covariance functions was applied. The estimates of cross-covariance functions of digital data of energetic intensity of neutrinos or autocovariance functions of single data are calculated according to the random functions formed in the energetic intensity measuring data arrays in the form of vectors. The covariance functions have been calculated by varying the quantisation interval on the energetic scale and applying the computer program developed using MATLAB 7 package of procedures.

Keywords. Cern neutrinos; neutrino conversion-type covariance function; quantisation interval.

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1. Introduction

The Opera experiment using the emission of neutrinos produced by Cern Large Hadron Collider (LHC) gave a huge push in establishing the speed of neutrinos. Neutrino is not a stable elementary particle; its three principal conversion types, namely, electron neutrinos, muon neutrinos and τ neutrinos, $v_e \leftrightarrow v_m \leftrightarrow v_\tau$, are known. Neutrinos of each conversion type have different oscillation parameters, such as wavelength, frequency, energy (amplitude) and possibly different speeds [1–4].

To analyse the measurements data of the three types of neutrinos, the theory of covariance functions was applied [5,6]. We assume that the errors of the measurement of parameters of the neutrino oscillation energy are random and possibly systematic and the average errors $M\Delta = \text{const.} \rightarrow 0$, their disperse is $D\Delta = \text{const.}$ and the covariance function of signals depends on the difference of arguments only, i.e. on the quantisation interval in the energy value scale. Also the trend of each vector of the measurement data was eliminated from the vectors of the neutrino oscillations energy measurement data.

For processing the digital signals, discrete Fourier transformation [7,8] and wavelet theory [9,10] were applied.

2. The covariance model of parameters of the energetic intensity of Cern neutrinos

For the theoretical model, we assume that the errors of parameters of the energetic intensity of Cern neutrinos are random and possibly systematic.

In each vector of the measurement data arrays of the energetic parameters of the neutrinos, the trend of the measuring data of the said vector is eliminated.

We consider that the random function formed according to the measurement data arrays of the energetic parameters of the neutrinos is stationary (in a broad sense), because its average value $M\{\varphi(t)\} \rightarrow \text{const}$, and the covariance function depends only on the difference

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between the arguments – $K_\varphi(\tau)$. An autocovariance function of a single data array or a cross-covariance function of two arrays $K_\varphi(\tau)$ shall be expressed as follows [11]:

$$K_\varphi(\tau) = \frac{1}{T-\tau}\int_0^{T-\tau} \delta\varphi_1(u)\delta\varphi_2(u+\tau)\mathrm{d}u, \quad (1)$$

where $\delta\varphi_1(u)$, $\delta\varphi_1(u+\tau)$ denote the centred values of the energetic parameters of the neutrinos, u is the parameter of oscillation, $\tau = k \cdot \Delta$ is a variable quantisation interval, k is the number of measurement units, Δ is the value of the measurement unit and T indicates time.

The estimate $K'_\varphi(\tau)$ of the covariance function according to the available data of energetic parameters of the neutrinos measurement data shall be calculated as follows:

$$K'_\varphi(\tau) = K'_\varphi(k) = \frac{1}{n-k}\sum_{i=1}^{n-k} \delta\varphi_1(u_i)\delta\varphi_2(u_{i+k}), \quad (2)$$

where n denotes the total number of discrete intervals.

Formula (2) may be applied as an autocovariance function or a cross-covariance function. If the function is an autocovariance function, the arrays $\varphi_1(u)$ and $\varphi_2(u+\tau)$ are parts of single arrays and if the function is cross-covariance, they are two different arrays.

The estimate of the normalised covariance function is

$$R'_\varphi(k) = \frac{K'_\varphi(k)}{K'_\varphi(0)} = \frac{K'_\varphi(k)}{\sigma'^2_\varphi}, \quad (3)$$

where σ'_φ is the estimate of the standard deviation of a random function.

For the elimination of the trend of vectors of the ith digital measurement data array, the following formulas shall be used:

$$\delta\varphi_i = \varphi_i - e \cdot \bar{\varphi}_i^{\mathrm{T}} = (\delta\varphi_{i1}, \ldots, \delta\varphi_{im}). \quad (4)$$

Here $\delta\varphi_i$ is the array of reduced values of the ith digital array where the trend of vectors is eliminated, φ_i is the ith array of oscillation strength, e is the unit vector $(n \times 1)$, n is the number of rows in the ith array, $\bar{\varphi}_i$ is the vector of average values of vectors of the ith array and $\delta\varphi_{ij}$ is the jth vector of the ith array of reduced values.

The vector of average values of vectors of the ith array shall be calculated according to the following formulas:

$$\bar{\varphi}_i = \frac{1}{n}\varphi_i^{\mathrm{T}} \cdot e,$$
$$\bar{\varphi}_i^{\mathrm{T}} = \frac{1}{n}e^{\mathrm{T}} \cdot \varphi_i. \quad (5)$$

The realisation of the random function of the jth vector of the ith field force array in the form of a vector shall be expressed as follows:

$$\delta\varphi_{ij} = \left(\delta\varphi_{ij,1}, \ldots, \delta\varphi_{ij,m}\right). \quad (6)$$

The estimate of the covariance matrix of the ith array of the neutrinos energetic parameters is expressed as follows:

$$K'(\delta\varphi_i) = \frac{1}{n-1}\delta\varphi_i^{\mathrm{T}}\delta\varphi_i. \quad (7)$$

The estimate of a covariance matrix of two arrays of neutrinos energetic parameters is expressed as follows:

$$K'\left(\delta\varphi_i, \delta\varphi_j\right) = \frac{1}{n-1}\delta\varphi_i^{\mathrm{T}}\delta\varphi_j, \quad (8)$$

where the sizes of arrays $\delta\varphi_i$, $\delta\varphi_j$ should be the same.

The estimates of covariance matrices $K'(\delta\varphi_i)$ and $K'\left(\delta\varphi_i, \delta\varphi_j\right)$ are reduced to estimates of the matrices of correlation coefficients $R'(\delta\varphi_i)$ and $R'\left(\delta\varphi_i, \delta\varphi_j\right)$ [12]:

$$R'(\delta\varphi_i) = D_i^{-1/2}K'(\delta\varphi_i)D_i^{-1/2}, \quad (9)$$
$$R'\left(\delta\varphi_i, \delta\varphi_j\right) = D_{ij}^{-1/2}K'\left(\delta\varphi_i, \delta\varphi_j\right)D_{ij}^{-1/2}, \quad (10)$$

where D_i and D_{ij} indicate the diagonal matrices of the basic diagonal terms of the estimates of covariance matrices $K'(\delta\varphi_i)$ and $K'\left(\delta\varphi_i, \delta\varphi_j\right)$, respectively.

The accuracy of the calculated correlation coefficients is defined by the standard deviation σ_r, and the value of the latter is found from the following formula:

$$\sigma_r = \frac{1}{\sqrt{k}}\left(1 - r^2\right),$$

where $k \to 200$ and r is the correlation coefficient. The maximum value of the estimate of the standard deviation is obtained when r is close to zero and in this case, $\sigma'_r \approx 0.07$. When $r \approx 0.5$, the deviation $\sigma'_r \approx 0.05$.

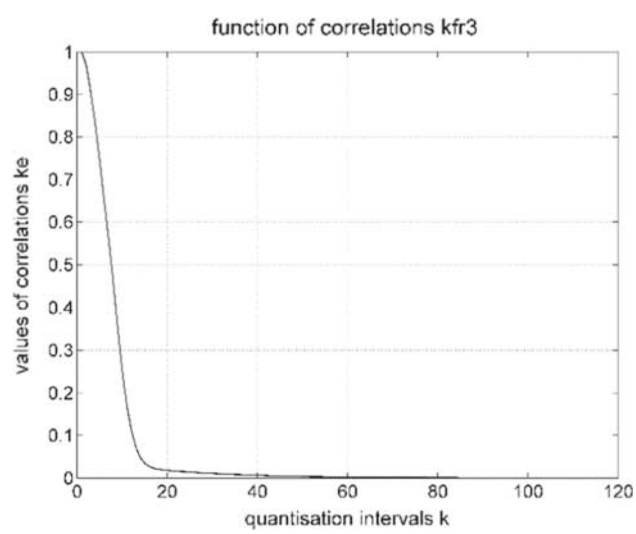

Figure 1. The normalised autocovariance function of the neutrino parameter of nm.

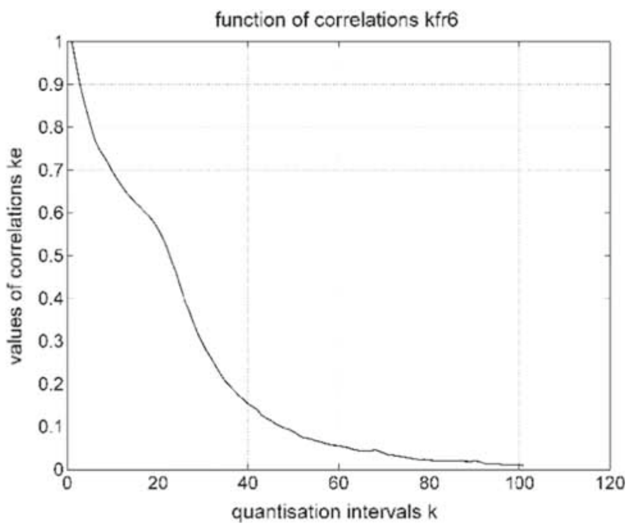

Figure 2. The normalised autocovariance function of the neutrino parameter of anm.

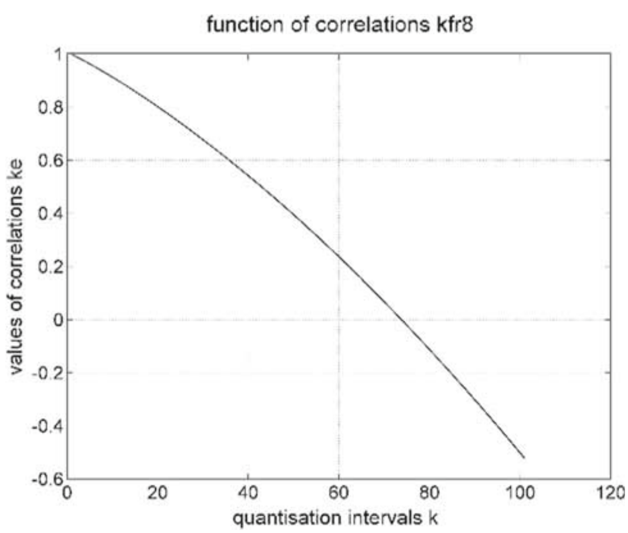

Figure 3. The normalised autocovariance function of the neutrino parameter of τ.

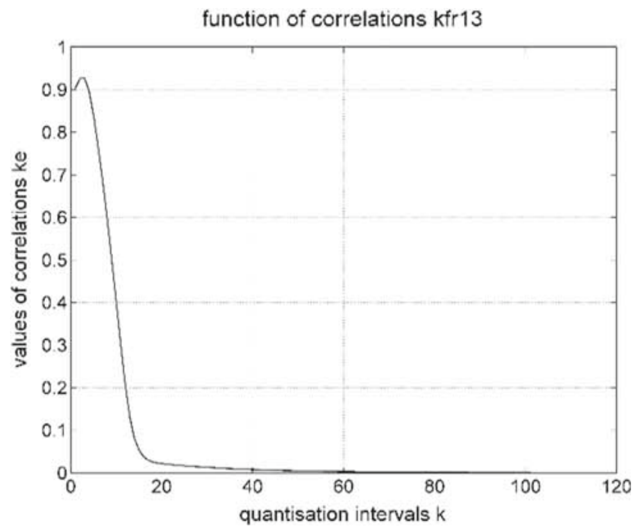

Figure 4. The normalised cross-covariance function of the neutrino parameters of ne (2004) and nm (2004).

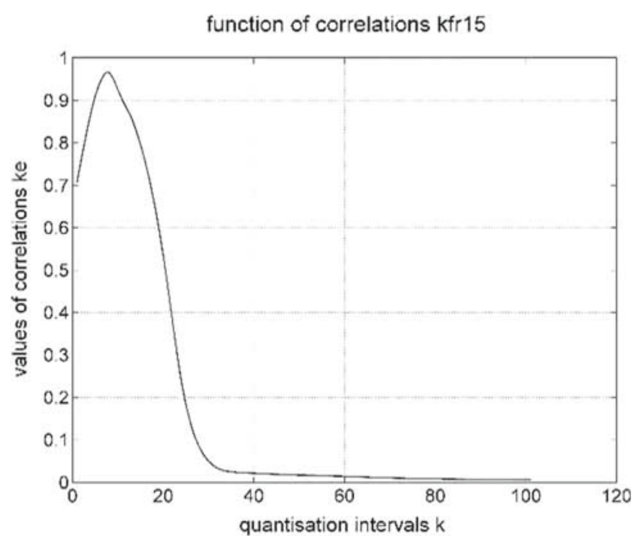

Figure 5. The normalised cross-covariance function of the neutrino parameters of ne (2004) and nm (2012).

3. The results of the experiment and the covariance analysis

For the analysis of covariance functions, the measurement data arrays of neutrinos produced by Cern LHC, including electron neutrinos, muon neutrinos and τ neutrinos ($v_e \leftrightarrow v_m \leftrightarrow v_\tau$), obtained in 2004 and 2012 [13] were used.

The number of measurement units of the quantisation interval of normalised covariance function varies from 1 to $n/2$, where $n \to 200$ – the number of rows (elements) of vectors of neutrino array. The value of measurement unit of the quantisation interval $\Delta = 2$ GeV. The neutrino array was formed by vectors of eight neutrino fluences:

1. ne fluence $\left[\mathrm{GeV}^{-1}\,\mathrm{m}^{-2}\,(\mathrm{pot})^{-1}\right]$ electron neutrinos (2004),
2. ane fluence $\left[\mathrm{GeV}^{-1}\,\mathrm{m}^{-2}\,(\mathrm{pot})^{-1}\right]$ antielectron neutrinos (2004),
3. nm fluence $\left[\mathrm{GeV}^{-1}\,\mathrm{m}^{-2}\,(\mathrm{pot})^{-1}\right]$ muon neutrinos (2004),
4. anm fluence $\left[\mathrm{GeV}^{-1}\,\mathrm{m}^{-2}\,(\mathrm{pot})^{-1}\right]$ antimuon neutrinos (2004),
5. nm fluence $\left[\mathrm{GeV}^{-1}\,\mathrm{m}^{-2}\,(\mathrm{pot})^{-1}\right]$ muon neutrinos (2012),

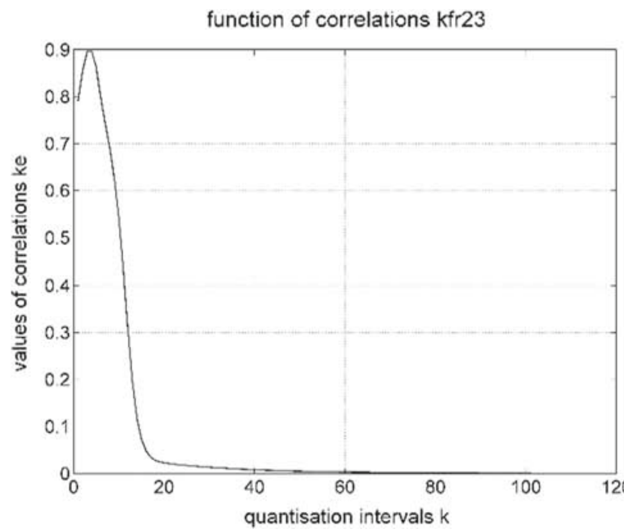

Figure 6. The normalised cross-covariance function of the neutrino parameters of ane and nm (2004).

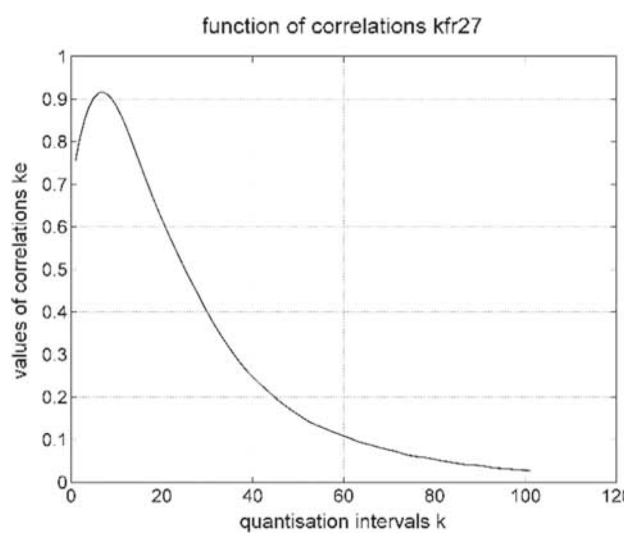

Figure 7. The normalised cross-covariance function of the neutrino parameters of ane and nm (2012).

6. anm fluence $\left[\mathrm{GeV}^{-1}\,\mathrm{m}^{-2}\,(\mathrm{pot})^{-1}\right]$ antimuon neutrinos (2012),
7. ne fluence $\left[\mathrm{GeV}^{-1}\,\mathrm{m}^{-2}\,(\mathrm{pot})^{-1}\right]$ electron neutrinos (2012),
8. nτ fluence $\left[\mathrm{GeV}^{-1}\,\mathrm{m}^{-2}\,(\mathrm{pot})^{-1}\right]$ τ neutrinos total (2012).

For neutrino fluence data from 2004 and 2012 epoch, please visit http://www.mi.infn.it/~psala/icarus.cngs.html.

The unit of neutrino energetic intensity parameters is $\left[\mathrm{n/m}^2\,/\mathrm{GeV}/10^{19}\right]$. The values of neutrino energy are varied from 0.5 to 400 GeV [14,15].

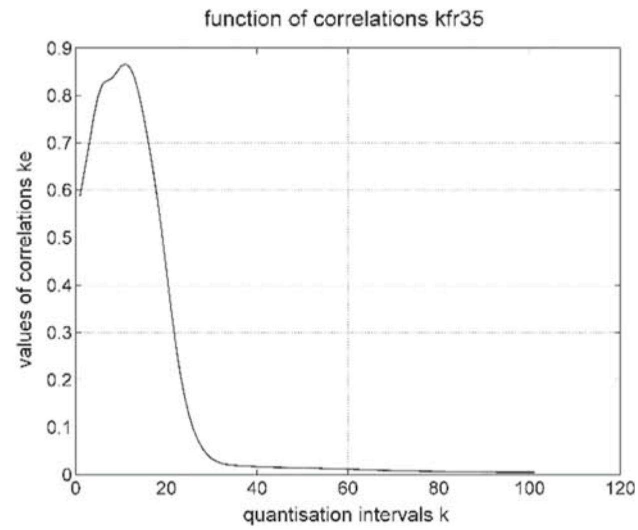

Figure 8. The normalised cross-covariance function of the neutrino parameters of nm (2004) and nm (2012).

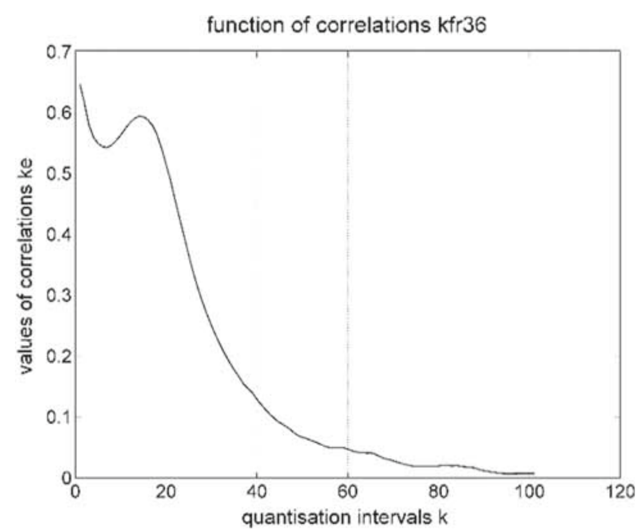

Figure 9. The normalised cross-covariance function of the neutrino parameters of nm (2004) and anm (2012).

The estimate $K'_{\varphi}(\tau)$ of the normalised autocovariance function $K_{\varphi}(\tau)$ was calculated for each neutrino conversion type vector and the graphical expressions of the normalised autocovariance functions for eight vectors were obtained. In addition, the estimates $K'_{\varphi}(\tau)$ of the normalised cross-covariance functions for combinations of eight vectors were calculated and 28 graphical expressions were obtained.

The value r of the normalised autocovariance function varies in the range from 0 to 1 at the value of the quantisation interval $k \to 0$ to $r \to 0$ at $k \to$ 20–100 (40$-$200 GeV). The values of autocorrelation of muon neutrinos decrease more rapidly, compared

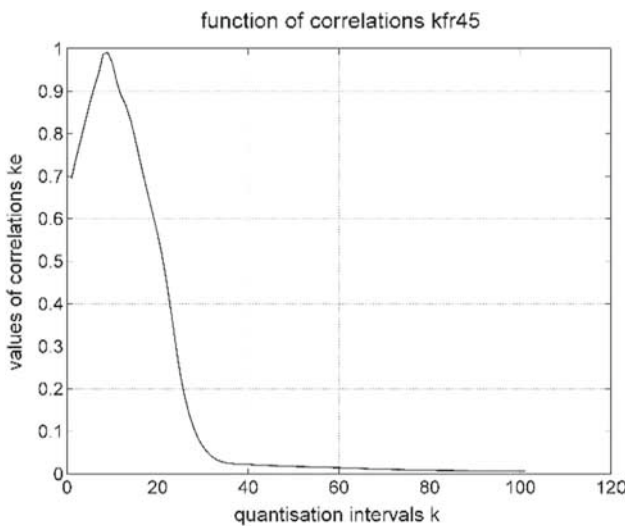

Figure 10. The normalised cross-covariance function of the neutrino parameters of anm (2004) and nm (2012).

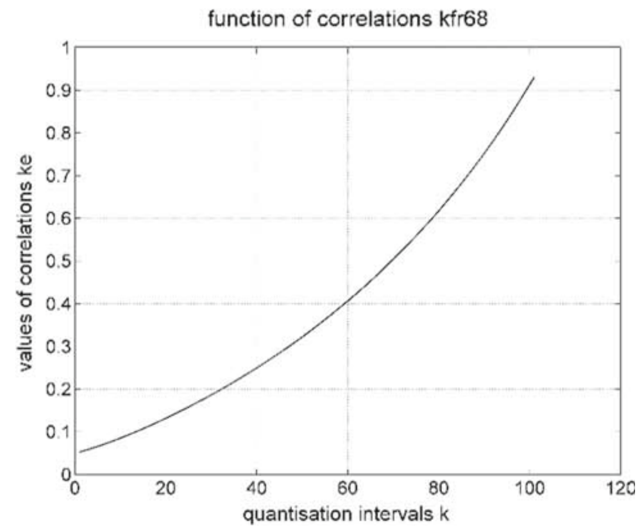

Figure 12. The normalised cross-covariance function of the neutrino parameters of anm (2012) and τ (2012).

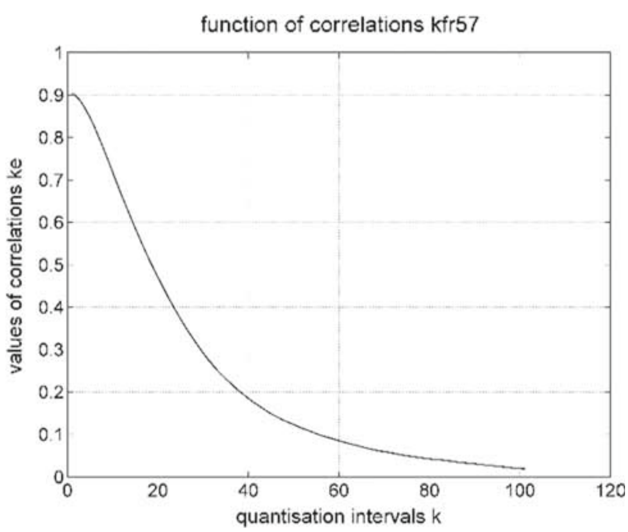

Figure 11. The normalised cross-covariance function of the neutrino parameters of nm (2012) and ne (2012).

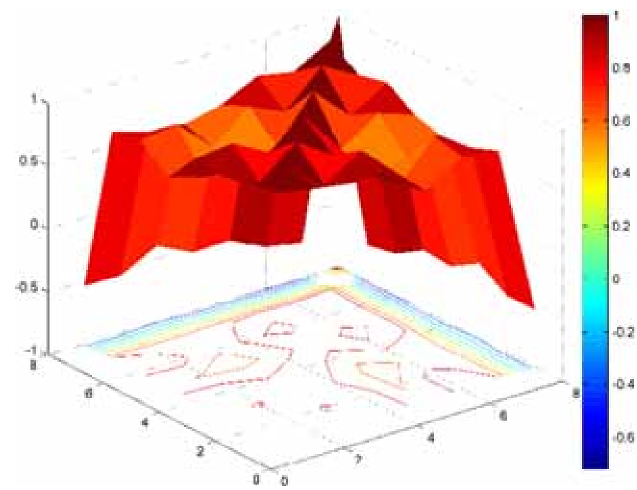

Figure 13. The graphical image of the resumptive (spatial) correlation matrix of the array of eight vectors of neutrinos.

to other types of neutrinos, when $r \rightarrow 0$ at $k \rightarrow 20$ (40 GeV) and 30 (60 GeV).

Autocovariance of τ neutrinos varies from $r \rightarrow 1.0$ at $k \rightarrow 0$ to $r \rightarrow -0.55$ at $k \rightarrow 100$ (200 GeV). So the parameters of τ neutrinos remain correlated across the whole quantisation interval, i.e. in the whole range of energetic values. The expression of τ neutrinos is of polynomial character that is not typical of covariance functions.

The values of normalised cross-covariance functions for oscillations of all neutrino types vary in the range from $r \rightarrow 0.6 : 0.95$ at quantisation interval $k \rightarrow 0$ to $r \rightarrow 0$ at $k \rightarrow 5$ (10 GeV) : 100 (200 GeV). So, the cross-correlation of parameters of all types of neutrinos is strong. However, the characters of the change of correlation differ for different types of neutrinos. 'Damping' of correlation of antimuon neutrinos (2012) and electron neutrinos (2012) with neutrinos of all other types is the slowest. The normalised cross-covariance functions of τ neutrinos with neutrinos of all other types are of polynomial character (that is not typical for covariance functions) and their cross-correlation grows with the growth of the quantisation interval.

The most important graphical expressions of normalised autocovariance and cross-covariance functions are provided in figures 1–12. In figure 13, the graphical image of the resumptive (spatial) correlation matrix of eight vectors of neutrino oscillations is presented. The graphical expression of the correlation matrix is a block of eight pyramids where the values of the correlation coefficients are shown by different colours of the spectrum. In the horizontal plane, the chromatic projection of the pyramids is shown.

4. Conclusions

1. The autocovariance and cross-covariance functions of the energetic parameters of the neutrinos enable the establishment of a correlation between neutrino energetic parameters according to the quantisation interval of their energetic values.
2. The maximum values $r \to 1.0$ of autocovariance functions of the vectors of energetic parameters of the neutrinos are obtained at the quantisation interval $k \to 0$. Then, in the course of the growth of quantisation interval, the rates of decrease of the probabilistic dependence between the self energetic parameters of neutrinos differ and depend on the conversion types of neutrinos. The rate of decrease of probabilistic dependence between muon neutrino energetic parameters in vectors is more rapid, compared to vectors of parameters of other conversion types of neutrinos. The self energetic parameters of τ neutrinos remain correlated in the whole quantisation interval, i.e. the range of energetic values. The expression of normalised autocovariance function of τ neutrinos is of polynomial character (not typical for covariance functions) and its values may be positive or negative.
3. The cross-correlation of the energetic parameters of all types of neutrinos is strong, when $r \to 0.6 : 0.95$ at the quantisation interval $k \to 0$. However, the characters of change of correlation differ for different conversion types of neutrinos. The cross-correlation of the parameters of τ neutrino oscillations grows with the growth of the quantisation interval and its functional expression is not typical for covariance functions.

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