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# Scattering of neutrons on α-particles in three-dimensional space

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**Abstract.** This letter demonstrates that there are some mistakes in the article of Roshan *et al* (*Pramana – J. Phys.* **90(3)**: 30, 2018) in dealing with the scattering of neutrons on α-particles in three-dimensional space. In fact, there is no one-to-one correspondence between the incident neutron energy and the scattering angle of the α-particle even if they have definite velocities before scattering. Instead, the situation is complex, and all the scattered α-particles will emit in a cone, which is called the velocity cone. At each scattering angle, which is smaller than the apex angle of the velocity cone, there are two groups of α-particles with different kinetic energies.

**Keywords.** Neutron energy; $\alpha$ scattering; three-dimensional space.

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## 1. Introduction

The measurement of the neutron energy spectrum is a long-lasting demand. As the neutron cannot be measured directly, it is a good idea to measure neutron energy through its scattering on the α-particle by detecting the emission angle and the energy of the scattered α-particle. Roshan *et al* [1] conceived an approach to measure the energy of neutrons by means of the scattering of neutrons on α-particles. In their method, a collimated neutron beam (whose energy needs to be measured) collides with the α beam (from an α source with definite energy). The two beams of the particles emit in vertical directions before their scattering. Then the direction and the energy of the scattered α-particles are measured with a position-sensitive charged-particle detector. After deriving four equations (eq. (4) in [1] is wrong), Roshan *et al* [1] claimed that 'From this set of equations, neutron with a particular energy is related to an α-particle with the corresponding scattering angle.', while in this paper we shall demonstrate that this statement is wrong.

The key of the present problem is the conservation of kinetic energy and momentum. However, instead of just writing conservation equations, we shall utilise geometry means by using the concepts of velocity triangle, velocity sphere and velocity cone for clarity. The motion of the system of neutron and α-particle will be decomposed into the motion of the centre-of-mass, and the motions of the neutron and the α-particle in the centre-of-mass reference system.

In the following sections, $E_{\mathrm{n}}$ and $E_{\alpha}$ are used respectively to denote energies of the neutron and the α-particle before scattering. We shall confine the neutron energy $E_{\mathrm{n}} \leq 20\,\mathrm{MeV}$ for discussion. Typically, $E_{\mathrm{n}} = 2.5\,\mathrm{MeV}$ (d–d reaction) and $E_{\alpha} = 5.0\,\mathrm{MeV}$ (polonium α source). Therefore, the relativity effect is small enough to be ignored.

## 2. Physical quantities before scattering

Figure 1 shows the velocity triangle, in which there are five velocities within a common plane describing the movement of the two particles and the whole system before scattering. All quantities will be decided in this section. Specially, the incident velocities of the neutron ($\vec{v}_{\mathrm{n}}$) and the α-particle ($\vec{v}_{\alpha}$) before scattering are perpendicular with respect to each other. The magnitudes of the velocities (i.e. the speed) of the neutron and the α-particle can be calculated from their kinetic energies:

$$v_{\mathrm{n}} = \sqrt{\frac{2E_{\mathrm{n}}}{m}}, \tag{1}$$

$$v_{\alpha} = \sqrt{\frac{2E_{\alpha}}{4m}}, \tag{2}$$

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where $m = 1.67 \times 10^{-27}$ kg which is the mass of the neutron. For 2.5 MeV neutrons and 5.0 MeV α-particles, the corresponding speeds are $2.19 \times 10^7$ and $1.55 \times 10^7$ m/s, respectively.

### 2.1 *The direction of the velocity of the centre-of-mass* ($\vec{v}_C$)

The direction of the velocity of the centre-of-mass $\vec{v}_C$ can be quantified by the angle between the velocity of the centre-of-mass ($\vec{v}_C$) and that of the α-particle ($\vec{v}_\alpha$), denoted as $\theta_0$ in figure 1.

According to the definition of the velocity of the centre-of-mass,

$$\tan\theta_0 = \frac{m v_n}{4m v_\alpha}. \quad (3)$$

From eqs (1) to (3), we have

$$\tan\theta_0 = \frac{1}{2}\sqrt{\frac{E_n}{E_\alpha}}. \quad (4)$$

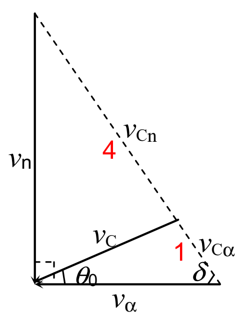


**Figure 1.** Quantities before scattering (the velocity triangles).

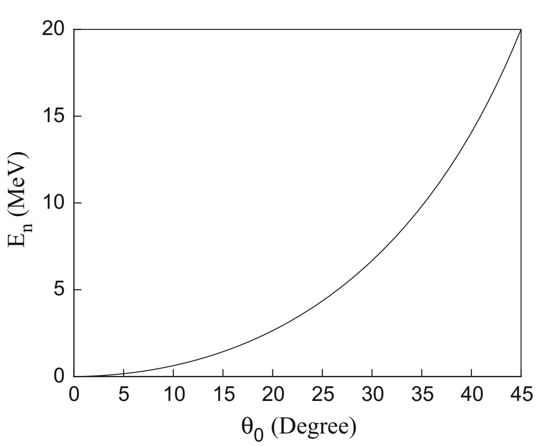


**Figure 2.** The relationship between $\theta_0$ (the angle between the velocity of the centre-of-mass ($\vec{v}_C$) and the velocity of the α-particle ($\vec{v}_\alpha$)) and the neutron energy ($E_n$) (for $E_\alpha = 5.0$ MeV).

For definite $E_\alpha$, there is a one-to-one correspondence between $\theta_0$ and $E_n$. For example, when $E_\alpha = 5.0$ MeV, $\theta_0$ increases from 0 to 45° as the neutron energy $E_n$ increases from 0 to 20 MeV, as figure 2 shows.

### 2.2 *The speed of the centre-of-mass* $v_C$

The relative speed between the neutron and the α-particle can be calculated as

$$v_{n\alpha} = \sqrt{v_n^2 + v_\alpha^2} = \sqrt{\frac{2E_n}{m} + \frac{2E_\alpha}{4m}}. \quad (5)$$

Thus, the speed of the α-particle in the centre-of-mass reference system can be calculated as

$$v_{C\alpha} = \frac{1}{5} v_{n\alpha} = \frac{1}{5}\sqrt{\frac{2E_n}{m} + \frac{2E_\alpha}{4m}}. \quad (6)$$

The speed of the neutron in the centre-of-mass reference system ($v_{Cn}$) can be calculated by a similar approach.

Figure 1 indicates that

$$\tan\delta = \frac{v_n}{v_\alpha} = \sqrt{\frac{4E_n}{E_\alpha}}. \quad (7)$$

The angle $\delta$ is defined in figure 1. Therefore,

$$\cos\delta = \frac{1}{\sqrt{1+\tan^2\delta}} = \sqrt{\frac{E_\alpha}{E_\alpha + 4E_n}}. \quad (8)$$

Then the speed of the centre-of-mass ($v_C$) can be calculated according to the following equation:

$$\begin{aligned} v_C^2 &= v_\alpha^2 + v_{C\alpha}^2 - 2 v_\alpha v_{C\alpha}\cos\delta \\ &= \frac{2E_\alpha}{4m} + \frac{1}{25}\left(\frac{2E_n}{m} + \frac{2E_\alpha}{4m}\right) \\ &\quad -2\sqrt{\frac{2E_\alpha}{4m}} \cdot \frac{1}{5}\sqrt{\frac{2E_n}{m} + \frac{2E_\alpha}{4m}}\sqrt{\frac{E_\alpha}{E_\alpha + 4E_n}}. \end{aligned} \quad (9)$$

So far, both the direction and the magnitude of the velocity of the centre-of-mass ($\vec{v}_C$) are obtained. Furthermore, all physical quantities before scattering are decided.

## 3. Physical quantities after scattering

After scattering, both the direction and the magnitude of $\vec{v}_C$ remain. In the centre-of-mass system, the α-particle (as well as the neutron) will emit in all directions in three-dimensional space, as shown in figure 3. The neutron and the corresponding α-particle will emit in opposite directions, while their speeds (magnitudes of

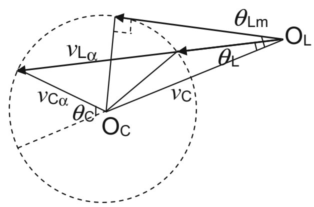


**Figure 3.** Quantities after scattering (the velocity sphere and velocity cone).

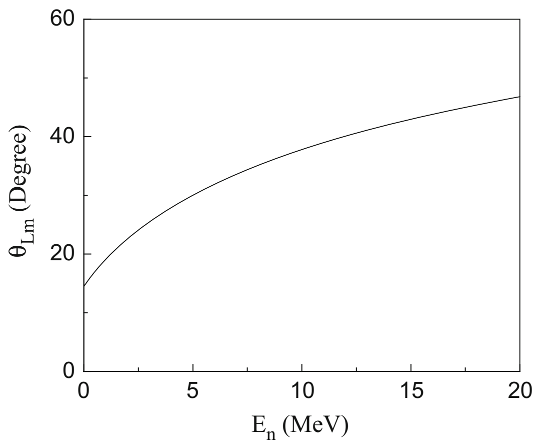


**Figure 4.** The relationship between the apex angle of the velocity cone ($\theta_{Lm}$) and neutron energy ($E_n$) (when $E_\alpha = 5.0$ MeV).

velocities) after scattering remain unchanged. Therefore, in the centre-of-mass system, all the possible velocities of α-particles (as well as the neutrons) should start from the same point (the centre-of-mass $O_C$ in figure 3) and end at a sphere (the velocity sphere of the α-particle). In the laboratory reference system, however, velocities of α-particles should start from another point outside the sphere ($O_L$ in figure 3, with $\overrightarrow{O_C O_L} = -\vec{v}_C$) and end at the same sphere, which means that all the α-particles can emit only in a cone (the velocity cone).

### 3.1 *The relationship between the neutron energy* ($E_n$) *and the apex angle of the velocity cone* ($\theta_{Lm}$)

The apex angle of the velocity cone ($\theta_{Lm}$), as shown in figure 3, can be quantified through

$$\sin\theta_{Lm} = \frac{v_{C\alpha}}{v_C}. \tag{10}$$

By using eqs (6), (9) and (10), the relationship between $E_n$ and $\theta_{Lm}$ can be determined for definite $E_\alpha$, as shown in figure 4 ($E_\alpha = 5.0$ MeV). One can see from figure 4 that $\theta_{Lm}$ increases along with $E_n$.

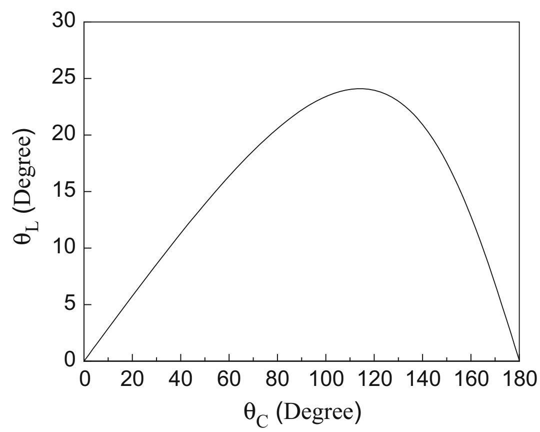


**Figure 5.** The relationship between $\theta_C$ and $\theta_L$ ($E_n$ = 2.5 MeV and $E_\alpha = 5.0$ MeV).

### 3.2 *The relationship between the scattering angle of the α-particle in the centre-of-mass system* ($\theta_C$) *and in the laboratory system* ($\theta_L$)

First, for simplicity, we define

$$\gamma \equiv \frac{v_C}{v_{C\alpha}}. \tag{11}$$

From figure 3, we have the following equations:

$$v_{L\alpha}\cos\theta_L = v_C + v_{C\alpha}\cos\theta_C, \tag{12}$$

$$v_{L\alpha}^2 = v_C^2 + v_{C\alpha}^2 - 2v_C v_{C\alpha}\cos(\pi - \theta_C). \tag{13}$$

By using eqs (11)–(13), we can get

$$\cos\theta_L = \frac{\gamma + \cos\theta_C}{\sqrt{1 + \gamma^2 + 2\gamma\cos\theta_C}}. \tag{14}$$

When $\theta_L < \theta_{Lm}$, each $\theta_L$ value corresponds to two $\theta_C$ values for definite $E_n$ and $E_\alpha$, as eq. (14) and figure 5 show (for $E_n = 2.5$ MeV and $E_\alpha = 5.0$ MeV).

### 3.3 *Speed* (*and energy*) *of the α-particle in the laboratory system* $v_{L\alpha}$ (*and* $E_{L\alpha}$)

From figure 3, one can write

$$v_{C\alpha}^2 = v_C^2 + v_{L\alpha}^2 - 2v_C v_{L\alpha}\cos\theta_L. \tag{15}$$

Thus,

$$v_{L\alpha} = v_C\cos\theta_L \pm \sqrt{v_C^2\cos^2\theta_L + v_{C\alpha}^2 - v_C^2}, \tag{16}$$

which is double valued.

Specially, when $\theta_L = 0$, eq. (16) becomes

$$v_{L\alpha} = v_C \pm v_{C\alpha}$$
$$(\text{i.e. } v_{L\alpha\,\max} = v_C + v_{C\alpha}, \quad v_{L\alpha\,\min} = v_C - v_{C\alpha}). \tag{17}$$

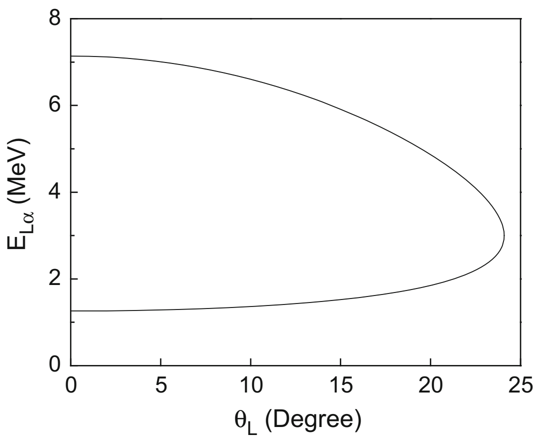


**Figure 6.** α-particle energy ($E_{L\alpha}$) as a function of the emission angle ($\theta_L$) (in the laboratory reference system for $E_n = 2.5$ MeV and $E_\alpha = 5.0$ MeV).

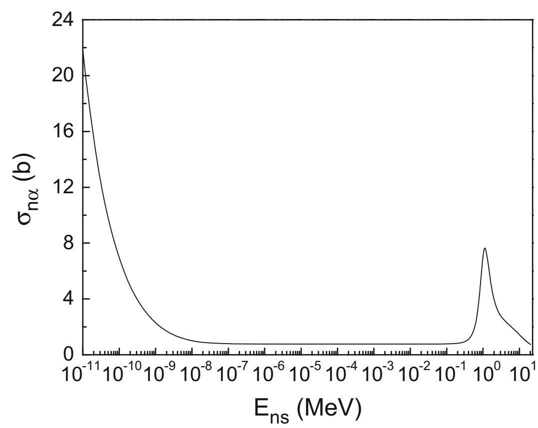


**Figure 7.** Elastic cross-sections of neutron on α-particle below $E_{ns} = 20$ MeV. Data are obtained from the ENDF library [2].

Then

$$E_{L\alpha} = \frac{4m}{2} v_{L\alpha}^2 = 2mv_{L\alpha}^2. \tag{18}$$

$E_{L\alpha}$ is double valued as a function of $\theta_L$ because $v_{L\alpha}$ is double valued for definite $E_n$ and $E_\alpha$. The relationship between $E_{L\alpha}$ and $\theta_L$ is plotted in figure 6 for $E_n = 2.5$ MeV and $E_\alpha = 5.0$ MeV. For each scattering angle $\theta_L$ smaller than $\theta_{Lm}$, there are two groups of α-particles: the higher energy group and the lower energy group. As $\theta_L$ increases, the energy of the higher energy group decreases, while the energy of the lower energy group increases.

## 4. Estimation of the scattering rate of one neutron on a beam of α-particles

Data of elastic cross-sections of neutrons on α-particles are available in ENDF library [2]. The elastic cross-sections ($\sigma_{n\alpha}$) as a function of neutron energy ($E_{ns}$) is shown in figure 7. However, these cross-sections are measured in the static-target system.

For the present problem, however, the target nucleus (the α-particle) is moving. The energy of the neutron relative to the α-particle (in the target system) is

$$E_{ns} = \frac{1}{2} m v_{n\alpha}^2. \tag{19}$$

By using eq. (5),

$$E_{ns} = \frac{1}{2} m v_{n\alpha}^2 = \frac{1}{2} m \left( \frac{2E_n}{m} + \frac{2E_\alpha}{4m} \right) = E_n + \frac{E_\alpha}{4}. \tag{20}$$

Take $E_n = 2.5$ MeV and $E_\alpha = 5.0$ MeV for example. Then, $E_{ns} = 3.75$ MeV from eq. (20). The corresponding elastic cross-section from figure 7 (ENDF library [2]) is about 2.5 b.

For a rough estimation, suppose α activity in the collimated beam is in the order of 1 Ci ($3.7 \times 10^{10}$ Bq), while the beam diameter is in the order of 1 $\mu$m. The speed of 5 MeV $\alpha$-particle is $1.55 \times 10^7$ m/s. When the elastic cross-section is 2.5 b ($2.5 \times 10^{-28}$ m$^2$), the estimated scattering rate for one neutron is about $7 \times 10^{-19}$ s$^{-1}$, which means that in 1 s, only one neutron out of $1.5 \times 10^{18}$ neutrons is scattered. Therefore, in principle, we can use the method of neutron α-particle scattering for measuring neutron energy. However, this method is not 'practical' for ordinary intensities of neutron and α beams, because the count rate of the scattered α-particle is too low to be measured. It can only be used when the intensities of both the neutron and the α beams are extremely high.

## 5. Conclusion and discussion

In this paper, we analysed the process of scattering of neutrons on α-particles in three-dimensional space. Generally speaking, there is no one-to-one correspondence between the incident neutron energy and the scattering angle of the scattered α-particle even if they have definite velocities before scattering. By decomposing the motion of the system and using the concepts of velocity triangle, velocity sphere and velocity cone to emphasise the essence of energy conservation and momentum conservation, we derived all the physical quantities before and after scattering. In the laboratory reference system, all α-particles will emit in a cone. For each direction ($\theta_L$) inside the velocity cone, there are two groups of α-particles with different energies. Using the elastic cross-section data obtained from the ENFD library, the scattering rate of neutron on α-particle is estimated for ideal situations.

The physics of this letter is also valid for the scattering of other types of particles; the angle between their initial velocities may change from 90° to other values. Furthermore, the present analysis can also be extended from scatterings to reactions. Those reactions or scatterings with extremely huge cross-sections may find practical applications.

## References

[1] M V Roshan, H Sadeghi, M Ghasabian and A Mazandarani, *Pramana – J. Phys.* **90(3)**: 30 (2018)
[2] ENDF: Evaluated Nuclear Data File, https://www-nds.iaea.org/exfor/endf.htm