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Solitons in conformable time-fractional Wu–Zhang system arising in coastal design

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Abstract. The modified $\exp(-\Omega(\zeta))$-expansion function method is applied to the Wu–Zhang system with conformable time-fractional derivative to construct new analytical solutions. We have obtained some soliton-type solutions such as dark, singular and combined soliton solutions. We have seen that all solutions have provided the mentioned equation system. For the suitable value of the solutions, the 2D–3D and contour surfaces have been plotted.

Keywords. Soliton solutions; nonlinear dynamics; conformable fractional derivative; Wu–Zhang system; modified $\exp(-\Omega(\zeta))$-expansion function method.

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1. Introduction

In nature, many phenomena are modelled using differential equations. Nonlinear differential equations have an important place in areas of physics such as solid state, fluid dynamics, viscoelasticity, plasma physics and optics. Many mathematicians and physicists have intensely studied nonlinear differential equations to find analytical solutions to understand the behaviours of these structures better.

Fractional calculus has taken on an important role in nonlinear dynamics. In the literature [1–13], it is seen that so many applications use fractional calculus. The study on fractional calculus started with L'Hospital's question 'what does $\mathrm{d}^n f/\mathrm{d}x^n$ mean if $n = 1/2$' in 1695. Since then, fractional calculus has attracted the attention of both mathematicians and applied scientists. Many mathematicians have produced definitions of the fractional derivative such as Riemann–Liouville, Caputo, Grünwald–Letnikov and Jumarie. These definitions are based on integral form. At least some of the properties of classic calculus, such as the derivative of the quotient of two functions, the chain rule, the product of two functions, have not been provided by these definitions. Due to this, Khalila *et al* [14] have presented a new definition of the fractional derivative which is a conformable fractional derivative using the basic limit definition of the derivative. In this paper, we use the modified $\exp(-\Omega(\zeta))$-expansion function method with the definition of a conformable fractional derivative to the nonlinear conformable time-fractional Wu–Zhang system.

We have considered the following equation system which is the nonlinear conformable time-fractional Wu–Zhang system [15]:

$$u_t^{\gamma}(x,t) = -u(x,t)u_x(x,t) - v_x(x,t),$$
$$v_t^{\gamma}(x,t) = -v(x,t)u_x(x,t) - u(x,t)v_x(x,t) - \frac{1}{3}u_{xxx}(x,t), \quad 0 < \gamma \leq 1, \tag{1}$$

where γ denotes the conformable derivative with respect to t, v is the elevation of the water wave and u is the surface velocity of water along the x-direction. Equation (1) is the generalised form of the classical Wu–Zhang system.

In 1996, Wu and Zhang [16] propounded three equation sets to model nonlinear and dispersive long gravity waves which described waves travelling in two horizontal directions on shallow water of uniform depth. After doing some transformation and reduction, $(1+1)$-dimensional dispersive long wave equation, known as the Wu–Zhang system has been obtained. These equations [17] are very useful for harbour and coastal designs in coastal and civil engineering.

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Solitary waves (or solitons) are widely used and have an important role in the engineering field, especially coastal engineering. These waves arise as deep water waves and shallow water waves. Researchers have employed solitary waves to model tsunami-like events, recently. It is important for civil and coastal engineers to understand the kinematic and dynamic structures of soliton waves to prevent or minimise disasters caused by tsunamis after major earthquakes in the ocean coasts [18–21].

The classical Wu–Zhang system has been investigated to obtain soliton solutions by many researchers. In [22], solitary wave, shock wave and singular solitary wave solutions are obtained by using the extended trial equation method, Lie symmetry analysis and the mapping method. Triki *et al* [23] used the ansatz method to the three-component Wu–Zhang equations, and then the 1-soliton solution has been presented. Recently, different methods such as Backlund transformation method [24], Darboux transformation [25–27], asymptotic analysis method [28], Painleve analysis method [29] and extended Painleve expansion [30] have been used to improve these equations.

However, there is no comprehensive study on the conformable time-fractional Wu–Zhang system as a classical one in the literature. In [15], periodic and solitary wave solutions are obtained by the first integral method. Eslami and Rezazadeh [15] have found only some trigonometric and hyperbolic function solutions. In this paper, we shall present more general solutions which are dark, singular and combined soliton solutions.

This paper is outlined as follows: In §2, we give some important definitions of the conformable fractional derivative. We construct the modified $\exp(-\Omega(\zeta))$-expansion function method in §3. The application of the proposed method to the time-fractional Wu–Zhang system with conformable sense is given in §4. In the last section, we submit some conclusions about the obtained solutions.

2. The facts of the conformable derivative

DEFINITION
Suppose that $f\colon [0,\infty) \to \mathrm{R}$, the conformable fractional derivative of f of order α is defined as

$$T_\alpha(f)(t) = \lim_{\varepsilon\to 0} \frac{f\left(t+\varepsilon t^{1-\alpha}\right) - f(t)}{\varepsilon}$$

for all $t > 0, \alpha \in (0,1)$ [14].

Theorem. *Let T_α be the fractional derivative operator with order α and $\alpha \in (0,1]$, f, g be the α-differentiable at point $t > 0$. Then* [14,31]

- $T_\alpha(af+bg) = aT_\alpha(f) + bT_\alpha(g), \forall a, b \in \mathbb{R}$;
- $T_\alpha(t^p) = pt^{p-\alpha}, \forall p \in \mathbb{R}$;
- $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$;
- $T_\alpha(f/g) = (gT_\alpha(f) - fT_\alpha(g))/g^2$;
- $T_\alpha(\lambda) = 0$, *for all constant functions* $f(t) = \lambda$;
- *if f is differentiable, then* $T_\alpha(f)(t) = t^{1-\alpha}(\mathrm{d}f/\mathrm{d}t)(t)$.

3. General structures of the method

In this section, the general structure of the modified $\exp(-\Omega(\zeta))$-expansion function method is organised. This method [32,33] is the improved form of the $\exp(-\Omega(\zeta))$-expansion function method.

Let us consider the nonlinear partial differential equations to apply this method as follows:

$$P(u, u_x, u_t, u_{xx}, u_{tt}, u_{tx}, \ldots) = 0, \tag{2}$$

where $u = u(x,t)$ is the unknown function, P is a polynomial that has $u(x,t)$ function and its partial derivatives with respect to x and t, $\gamma \in (0,1]$ is the order of the conformable derivative.

Step 1. Let us suppose the travelling wave transformation as

$$u(x,t) = U(\zeta), \quad \zeta = x - \frac{lt^\gamma}{\gamma}, \tag{3}$$

where l is a non-zero constant that can be determined later. Using partial derivatives of eq. (3) into eq. (2), eq. (2) is converted to a nonlinear ordinary differential equation defined as

$$N\left(U, U', U'', U''', \ldots\right) = 0, \tag{4}$$

where N is a polynomial depending on U.

Step 2. We suppose that the travelling wave solution of eq. (4) can be expressed as follows:

$$U(\zeta) = \frac{\sum_{i=0}^{N} A_i\left[\exp(-\Omega(\zeta))\right]^i}{\sum_{j=0}^{M} B_j\left[\exp(-\Omega(\zeta))\right]^j} = \frac{A_0 + A_1\exp(-\Omega) + \cdots + A_N\exp(N(-\Omega))}{B_0 + B_1\exp(-\Omega) + \cdots + B_M\exp(M(-\Omega))}, \tag{5}$$

where A_i, B_j $(0 \le i \le N, 0 \le j \le M)$ are constants that can be determined later, $A_N \neq 0$, $B_M \neq 0$ and $\Omega = \Omega(\zeta)$ solve the following ordinary differential equation:

$$\Omega'(\zeta) = \exp(-\Omega(\zeta)) + \mu \exp(\Omega(\zeta)) + \lambda. \tag{6}$$

When we solve eq. (6), we reach five solution families as follows [32–34]:

Family 1. When $\mu \neq 0$, $\lambda^2 - 4\mu > 0$,

$$\Omega(\zeta) = \ln\left(\frac{-\sqrt{\lambda^2-4\mu}}{2\mu} \times \tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\zeta+E)\right) - \frac{\lambda}{2\mu}\right). \tag{7}$$

Family 2. When $\mu \neq 0$, $\lambda^2 - 4\mu < 0$,

$$\Omega(\zeta) = \ln\left(\frac{\sqrt{-\lambda^2+4\mu}}{2\mu} \times \tan\left(\frac{\sqrt{-\lambda^2+4\mu}}{2}(\zeta+E)\right) - \frac{\lambda}{2\mu}\right). \tag{8}$$

Family 3. When $\mu = 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu > 0$,

$$\Omega(\zeta) = -\ln\left(\frac{\lambda}{\exp(\lambda(\zeta+E)) - 1}\right). \tag{9}$$

Family 4. When $\mu \neq 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu = 0$,

$$\Omega(\zeta) = \ln\left(-\frac{2\lambda(\zeta+E)+4}{\lambda^2(\zeta+E)}\right). \tag{10}$$

Family 5. When $\mu = 0$, $\lambda = 0$ and $\lambda^2 - 4\mu = 0$,

$$\Omega(\zeta) = \ln(\zeta + E), \tag{11}$$

where $A_0, A_1, \ldots, A_N, B_0, B_1, \ldots, B_M, E, \lambda, \mu$ are constants and can be determined later. Using the homogeneous balance principle between the highest nonlinear terms and the highest-order derivatives of U in eq. (5) a relationship between N and M can be found.

Step 3. On substituting eq. (6) along with solution families into eq. (5), we have a polynomial of $\exp(\Omega(\zeta))$. After all the coefficients of the similar power of $\exp(\Omega(\zeta))$ are equated to zero, it yields an algebraic equation system in terms of $A_0, A_1, \ldots, A_N$, $B_0, B_1, \ldots, B_M, E, \lambda$. At the end of this procedure, the obtained values of coefficients are substituted into eq. (5), to get the travelling wave solutions of eq. (2).

4. Application

Let us use eq. (3) along with eq. (5) into eq. (1). In this case, we have a nonlinear ordinary differential equation system as follows:

$$lU' = UU' + V', \tag{12}$$

$$lV' = VU' + UV' + \frac{1}{3}U'''. \tag{13}$$

When we arrange eq. (12) after integration with respect to ζ, we obtain

$$V = lU - \frac{1}{2}U^2. \tag{14}$$

Substituting eq. (14) and $V' = lU' - UU'$ into eq. (20) yields

$$-l^2U' + 3lUU' - \frac{3}{2}U^2U' + \frac{1}{3}U''' = 0. \tag{15}$$

Integrating eq. (15) once yields the nonlinear ordinary differential equation as follows:

$$U'' - 3l^2U + \frac{9}{2}lU^2 - \frac{3}{2}U^3 = 0. \tag{16}$$

When homogeneous balance principle is used between U'' and U^3, we deduce a relationship as

$$M + 1 = N. \tag{17}$$

For suitable integer values of M and N, one can achieve different cases. We have chosen $M = 1$ and $N = 2$, then the following soliton solutions have been obtained.

Case 1.

$$a_0 = -\frac{(\lambda + \sqrt{\lambda^2-4\mu})b_0}{\sqrt{3}},$$

$$a_1 = \frac{-2b_0 - (\lambda + \sqrt{\lambda^2-4\mu})b_1}{\sqrt{3}},$$

$$a_2 = -\frac{2b_1}{\sqrt{3}}, \quad l = -\frac{\sqrt{\lambda^2-4\mu}}{\sqrt{3}}. \tag{18}$$

Using eq. (18) along with eqs (7) and (9) into eq. (5), we have dark soliton and singular soliton solutions as follows:

Family 1.

$$u_{1,1}(x,t) = -\frac{\left(\lambda(\lambda+\sqrt{\lambda^2-4\mu}) - 4\mu\right)\left(1 + \tanh\left[\frac{1}{2}\left(E + x - \frac{lt^\gamma}{\gamma}\right)\sqrt{\lambda^2-4\mu}\right]\right)}{\sqrt{3}\left(\lambda + \sqrt{\lambda^2-4\mu}\tanh\left[\frac{1}{2}\left(E + x - \frac{lt^\gamma}{\gamma}\right)\sqrt{\lambda^2-4\mu}\right]\right)}, \tag{19}$$

$$v_{1,1}(x,t) = lu_{1,1}(x,t) - \frac{u_{1,1}^2(x,t)}{2}, \tag{20}$$

where $\lambda^2 - 4\mu > 0$ (see figures 1a–1d and 2a, 2b).

Family 3.

$$u_{1,3}(x,t) = -\frac{\lambda}{\sqrt{3}} - \frac{\lambda}{\sqrt{3}} \times \coth\left[\frac{(-lt^{\gamma} + (EE + x)\gamma)\lambda}{2\gamma}\right], \tag{21}$$

$$v_{1,3}(x,t) = lu_{1,3}(x,t) - \frac{u_{1,3}^2(x,t)}{2}, \tag{22}$$

where $\mu = 0$ (see figures 3a–3d and 4a, 4b).

Case 2.

$$b_0 = \frac{3a_0}{3l+\sqrt{3}\lambda}, \quad b_1 = \frac{\sqrt{3}a_2}{2}, \quad a_1 = \frac{2a_0}{\lambda+\sqrt{3}l} + \frac{(\lambda+\sqrt{3}l)a_2}{2}, \quad \mu = \frac{-3l^2+\lambda^2}{4}. \tag{23}$$

Substituting eq. (23) with eq. (7) into eq. (5), will yield dark soliton solutions:

Family 1.

$$u_{2,1}(x,t) = \frac{(3l+\sqrt{3}\lambda)(\sqrt{3}l\lambda+\lambda^2-4\mu+(\sqrt{3}l+\lambda)f_1(x,t))}{3(\sqrt{3}l+\lambda)(\lambda+f_1(x,t))}, \tag{24}$$

$$v_{2,1}(x,t) = lu_{2,1}(x,t) - \frac{u_{2,1}^2(x,t)}{2}, \tag{25}$$

where

$$f_1(x,t) = \sqrt{\lambda^2-4\mu} \times \tanh\left[\frac{1}{2}\left(E + x - \frac{lt^{\gamma}}{\gamma}\right)\sqrt{\lambda^2-4\mu}\right]$$

and

$\lambda^2 - 4\mu > 0$

(see figures 5a–5d and 6a, 6b).

Case 3.

$$a_0 = \frac{\sqrt{3}(a_1^2+l^2b_1^2)}{8b_1}, \quad a_2 = \frac{2b_1}{\sqrt{3}}, \quad b_0 = \frac{\sqrt{3}(a_1-lb_1)}{4}, \quad \mu = \frac{3(a_1^2-2la_1b_1+3l^2b_1^2)}{16b_1^2}, \quad \lambda = \frac{\sqrt{3}(a_1-lb_1)}{2b_1}. \tag{26}$$

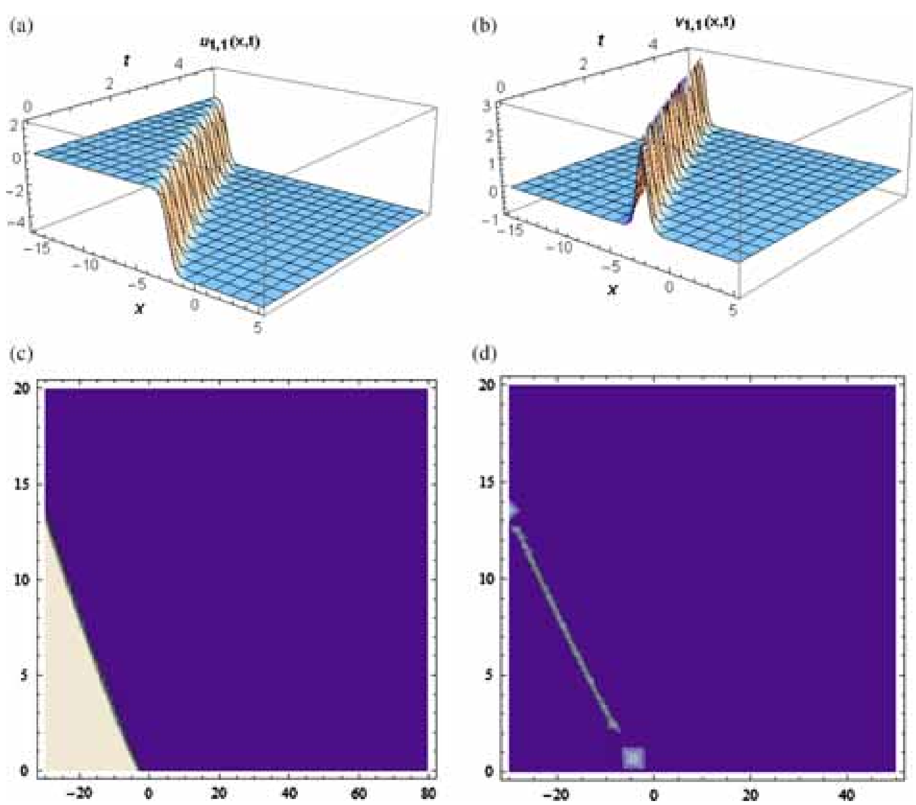

Figure 1. (**a**)–(**d**) The 3D and contour plots of eqs (19) and (20) for $\lambda = 5$, $\mu = 2$, $E = 2$ and $\gamma = 0.9$.

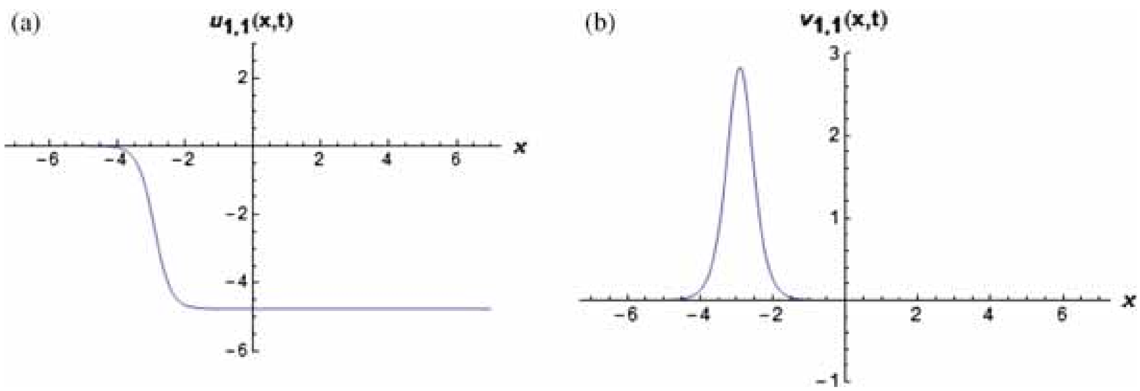

Figure 2. (**a**) and (**b**) The 2D plot of eqs (19) and (20) for $\lambda = 5$, $\mu = 2$, $E = 2$, $\gamma = 0.9$ and $t = 0.1$.

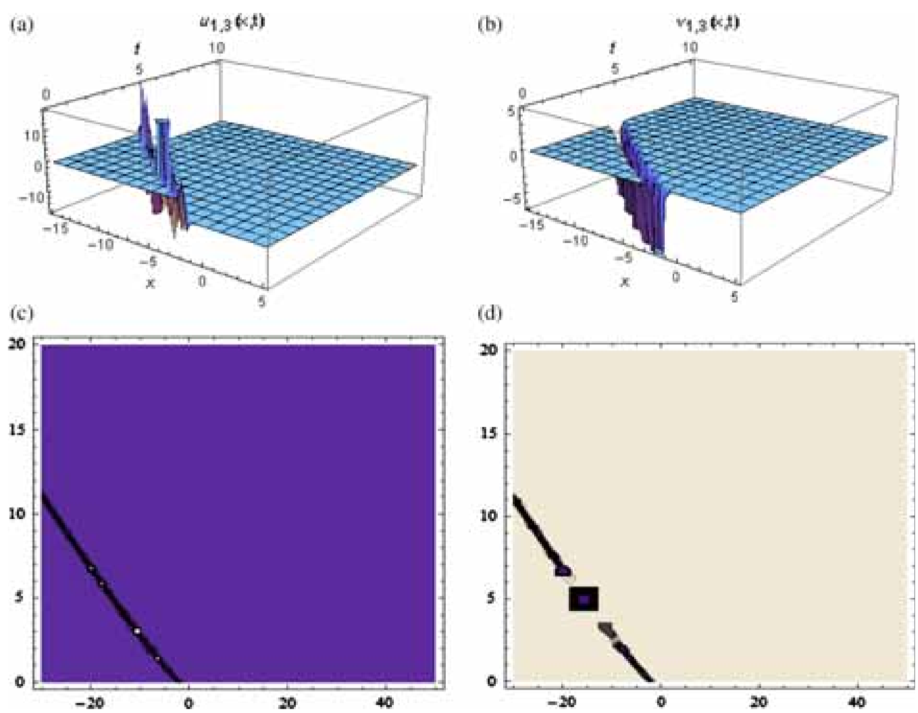

Figure 3. (**a–d**) The 3D and contour plots of eqs (21) and (22) for $\lambda = 5$, $\mu = 0$, $E = 2$ and $\gamma = 0.9$.

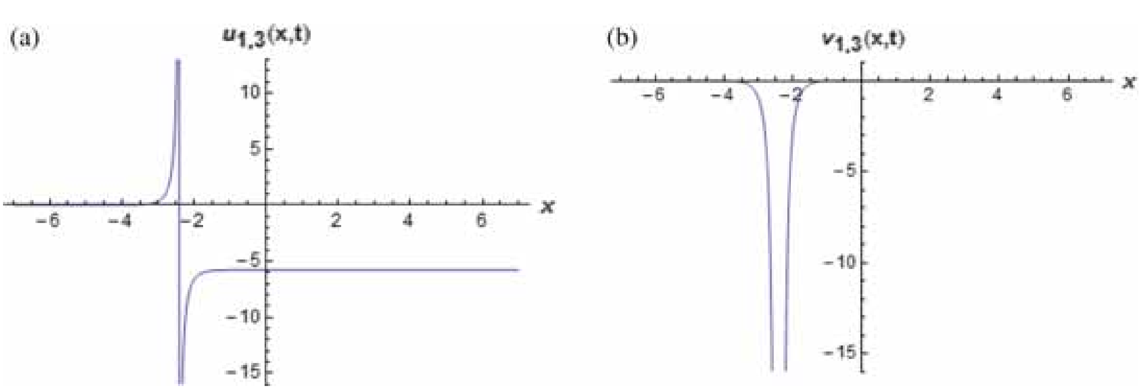

Figure 4. (**a**) and (**b**) The 2D plot of eqs (21) and (22) for $\lambda = 5$, $\mu = 0$, $E = 2$, $\gamma = 0.9$ and $t = 0.1$.

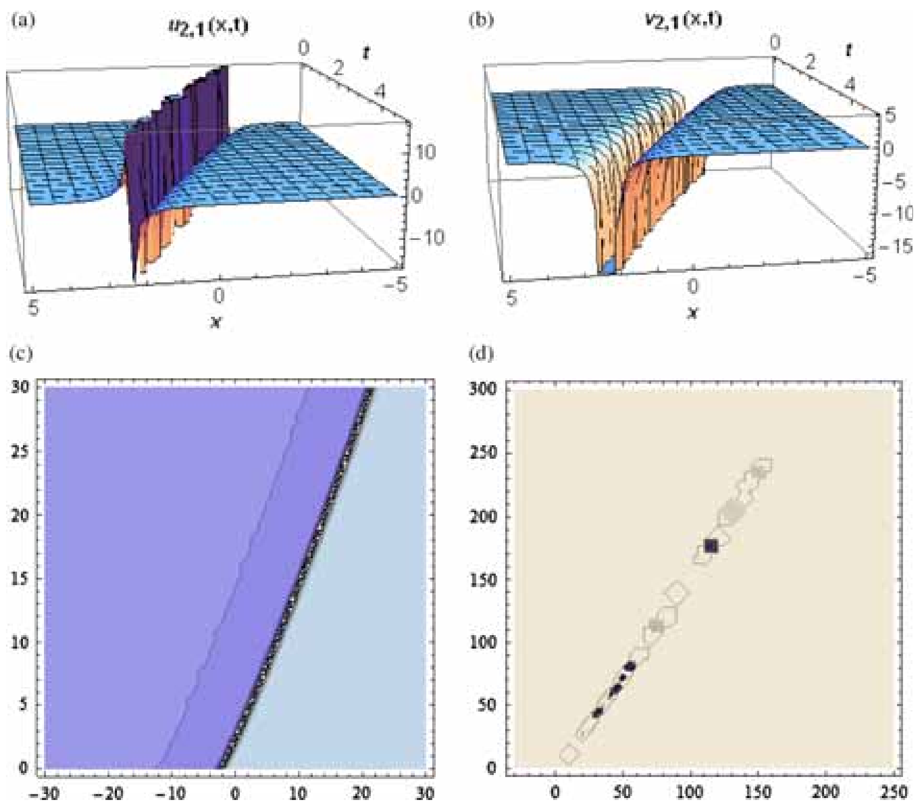

Figure 5. **(a)**–**(d)** The 3D and contour plots of eqs (24) and (25) for $\lambda = 0.5$, $l = 1$, $E = 2$ and $\gamma = 0.9$.

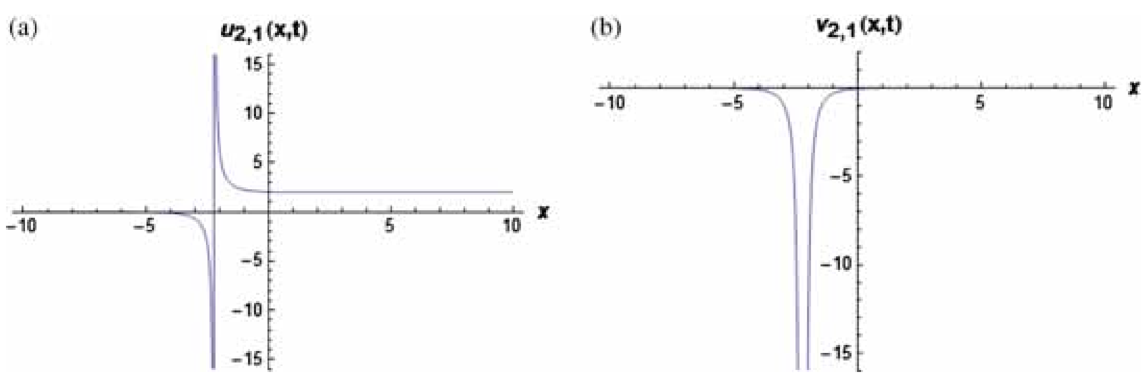

Figure 6. **(a)** and **(b)** The 2D plot of eqs (24) and (25) for $\lambda = 0.5$, $l = 1$, $E = 2$, $\gamma = 0.9$ and $t = 0.1$.

Substituting eq. (26) with eq. (8) into eq. (5), will yield dark soliton solutions:

Family 2.

$$u_{3,2}(x,t) = \left(\frac{\sqrt{3}(a_1^2 + l^2 b_1^2)}{8b_1} + \frac{8\mu^2 b_1}{\sqrt{3} f_2^2(x,t)} - \frac{2\mu a_1}{f_2(x,t)}\right) \times \left(\frac{4 f_2(x,t)}{\sqrt{3}(a_1 - l b_1) f_2(x,t) - 8\mu b_1}\right), \quad (27)$$

$$v_{3,2}(x,t) = l u_{3,2}(x,t) - \frac{u_{3,2}^2(x,t)}{2}, \quad (28)$$

where

$$f_2(x,t) = \lambda - \sqrt{-\lambda^2 + 4\mu} \times \tan\left[\frac{1}{2}\left(E + x - \frac{l t^{\gamma}}{\gamma}\right)\sqrt{-\lambda^2 + 4\mu}\right]$$

and

$$\lambda^2 - 4\mu < 0$$

(see figures 7a–7d and 8a, 8b).

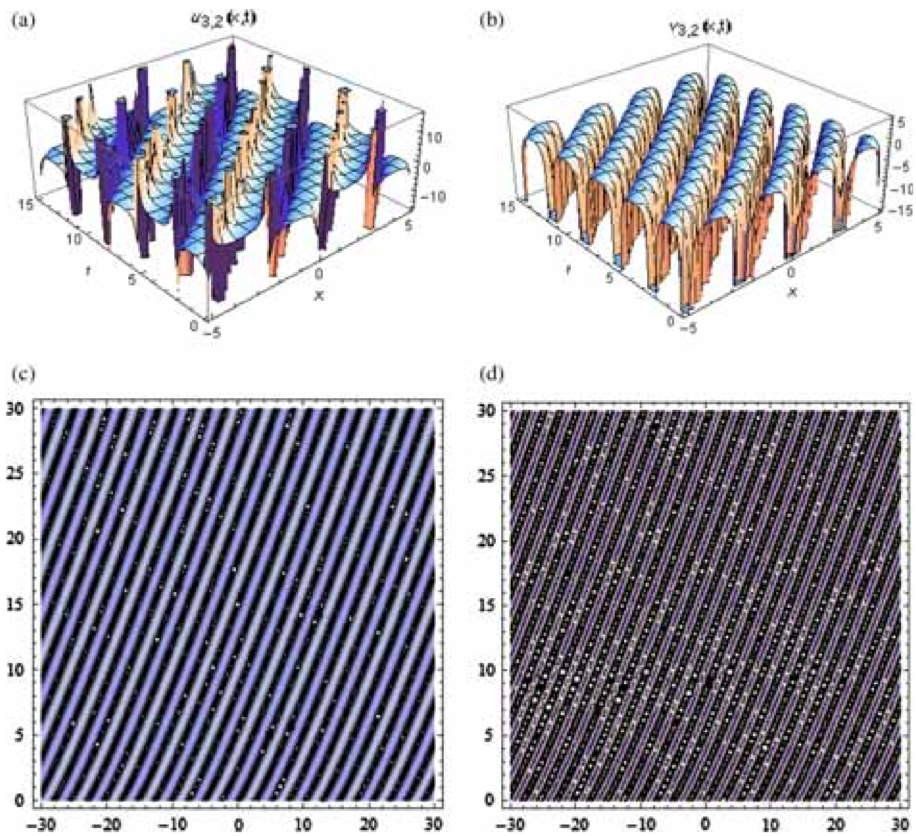

Figure 7. **(a)**–**(d)** The 3D and contour plots of eqs (27) and (28) for $a_1 = 3$, $b_1 = 0.5$, $l = 1$, $E = 2$ and $\gamma = 0.9$.

Case 4.

$$b_0 = -\frac{\sqrt{3}\lambda a_2}{4}, \quad b_1 = -\frac{\sqrt{3}a_2}{2},$$

$$a_0 = \frac{(3l^2 - 4\sqrt{3}l\lambda + 4\lambda^2)a_2}{16},$$

$$a_1 = -\frac{(\sqrt{3}l - 2\lambda)a_2}{2}, \quad \mu = \frac{-3l^2 + 4\lambda^2}{16}. \tag{29}$$

When we use eq. (29) with eq. (7) into eq. (5), we have combined soliton solutions:

Family 1.

$$u_{4,1}(x,t) = \frac{-f_3^2(x,t)[2\mu(3l^2-4\lambda^2+16\mu)+(-4\sqrt{3}l\lambda(\lambda^2-4\mu)+3l^2(\lambda^2-2\mu)+4(\lambda^4-6\lambda^2\mu+8\mu^2))f_4(x,t)}{4\sqrt{3}(\lambda+f_1(x,t))(\lambda^2-4\mu+\lambda f_1(x,t))}$$

$$+\frac{\sqrt{\lambda^2-4\mu}(3l^2\lambda - 4\sqrt{3}l(\lambda^2-2\mu)+4(\lambda^3-4\lambda\mu))f_5(x,t)]}{4\sqrt{3}(\lambda+f_1(x,t))(\lambda^2-4\mu+\lambda f_1(x,t))}, \tag{30}$$

where

$$f_3(x,t) = \operatorname{sech}\left[\frac{1}{2}\left(E+x-\frac{lt^\gamma}{\gamma}\right)\sqrt{\lambda^2-4\mu}\right]^2,$$

$$f_4(x,t) = \cosh\left[\left(E+x-\frac{lt^\gamma}{\gamma}\right)\sqrt{\lambda^2-4\mu}\right],$$

$$f_5(x,t) = \sinh\left[\left(E+x-\frac{lt^\gamma}{\gamma}\right)\sqrt{\lambda^2-4\mu}\right]$$

and

$\lambda^2 - 4\mu > 0$.

(see figures 9a–9d and 10a, 10b).

$$v_{4,1}(x,t) = lu_{4,1}(x,t) - \frac{u_{4,1}^2(x,t)}{2}, \tag{31}$$

Remark. All solutions were substituted in the corresponding system and the system was verified using computational programs.

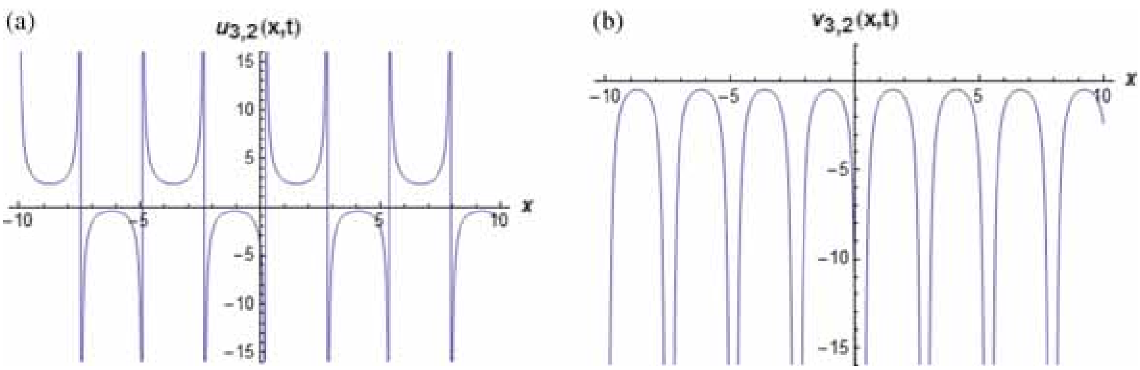

Figure 8. (**a**) and (**b**) The 2D plot of eqs (27) and (28) for $a_1 = 3$, $b_1 = 0.5$, $l = 1$, $E = 2$, $\gamma = 0.9$ and $t = 0.1$.

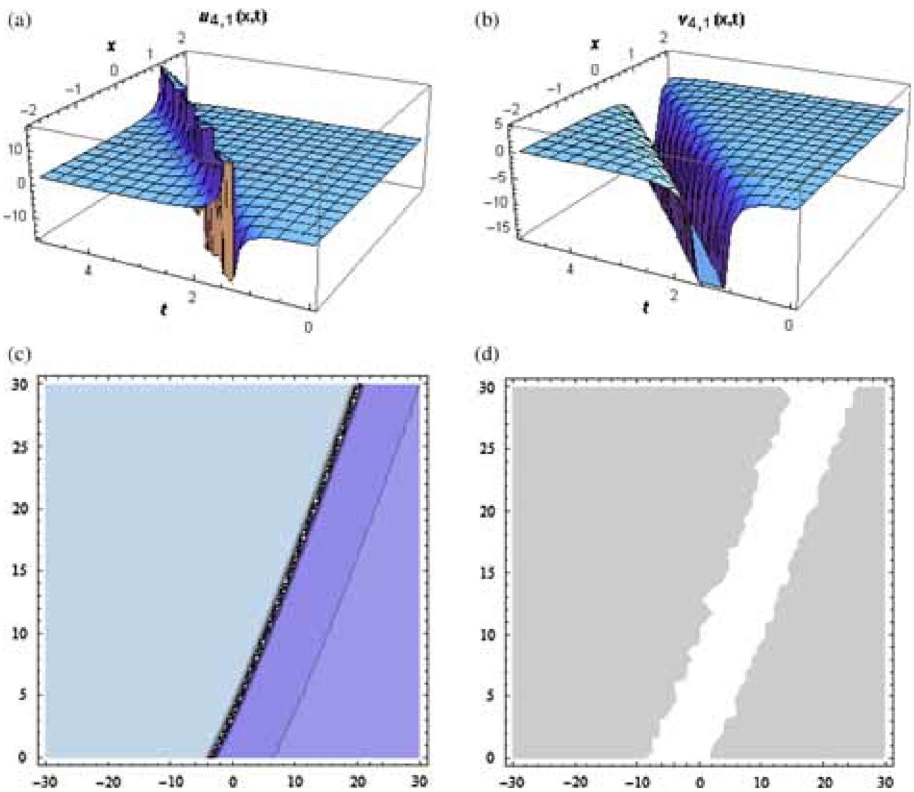

Figure 9. (**a**)–(**d**) The 3D and contour plots of eqs (30) and (31) for $\lambda = 0.5$, $l = 1$, $E = 2$ and $\gamma = 0.9$.

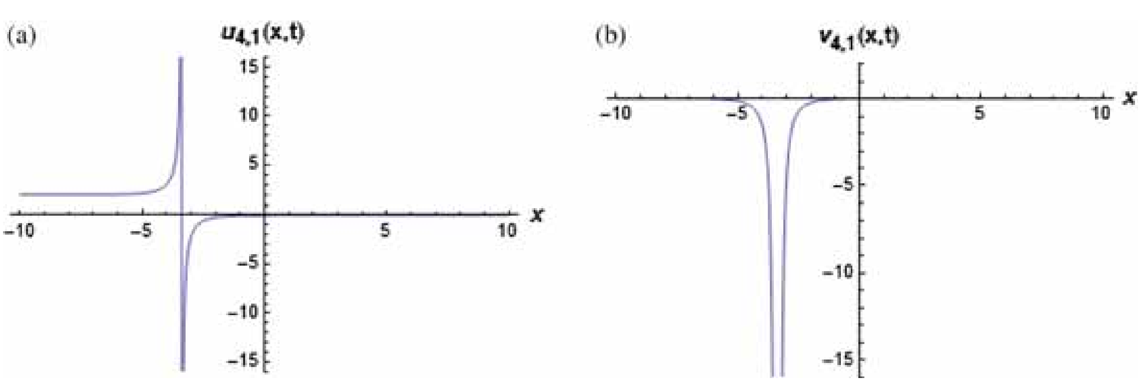

Figure 10. (**a**) and (**b**) The 2D plot of eqs (30) and (31) for $\lambda = 0.5$, $l = 1$, $E = 2$, $\gamma = 0.9$ and $t = 0.1$.

5. Conclusions and discussions

In this paper, we used the modified $\exp(-\Omega(\zeta))$-expansion function method on nonlinear conformable time-fractional Wu–Zhang system to present some travelling solutions. Using conformable fractional derivative definition in travelling transformation, eq. (1) was converted to its nonlinear ordinary differential equation form. For four different cases, we obtained five soliton-type solutions which are dark, singular and combined soliton solutions. We have plotted the 2D–3D and contour surfaces under suitable values of constants. When we compare the obtained solutions with [15], we saw that eqs (21) and (22) are the same solutions and all others are completely different. According to our knowledge, there is not enough research on the mentioned problem in the literature. We think that, because of the convenience provided by the numerical calculation of the submitted soliton solutions, they may be very helpful, especially in the engineering field.

Furthermore, the hyperbolic functions are circular functions which have arisen in both mathematics and physics. For example, the hyperbolic cosine functions have catenary shape, the hyperbolic tangent functions arise during calculations of magnetic moment and rapidity of special relativity, the hyperbolic secant functions arise in the profile of a laminar jet and the hyperbolic cotangent functions arise in the Langevin function for magnetic polarisation [35].

During this study, we encountered some peculiar situations. In Case 1, when the given coefficients were used for Family 2, the obtained solutions satisfied eq. (1). However, there are no values to λ and μ such that both $l = -((\sqrt{\lambda^2 - 4\mu})/\sqrt{3})$ and $\lambda^2 - 4\mu < 0$ are provided by them. In Case 2, when the coefficient $\mu = (-3l^2 + \lambda^2)/4$ was substituted into $-\lambda^2 + 4\mu$, it gave $-3l^2$. However, there is no real value for l such that $-\lambda^2 + 4\mu > 0$ for the condition of Family 2. A similar situation arose when the coefficient $\mu = (-3l^2 + 4\lambda^2)/16$ was substituted into $-\lambda^2 + 4\mu$, and it gave $-3l^2/4$ in Case 4 for Family 2. In Case 3, the values of coefficients λ and μ were put into the condition of Family 1 that is $\lambda^2 - 4\mu > 0$ and it was found to be $-3l^2/2$ although we cannot find any real number l such that $(-3l^2/2) > 0$. For this reason, even though the solutions corresponding to the obtained coefficients are verified to the mentioned equation system, they have not been assumed solution families.

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