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Comment on "New optical soliton solutions for nonlinear complex fractional Schrödinger equation via new auxiliary equation method and novel (G′/G)-expansion method"

SHOUKRY EL-GANAINI[1],* and ELSAYED M E ZAYED[2]

[1]Department of Mathematics, Faculty of Science, Damanhour University, Bahira 22514, Egypt
[2]Mathematics Department, Faculty of Science, Zagazig University, Zagazig, Egypt
*Corresponding author. E-mail: shoukryelganaini@yahoo.com

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Abstract. This comment deals with the new auxiliary equation method (Khater method) introduced by Mostafa M A Khater, Aly R Seadawy and Dianchen Lu in *Pramana – J. Phys.* **90**, 59 (2018). By simple calculation, it is shown that this method is wrong. The exact solutions obtained in this paper are also wrong. In this comment, the errors in the above paper are pointed out and modified. The right method (amended Khater method) has been introduced.

Keywords. Doubtful Khater method; amended Khater method; doubtful exact solutions.

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1. New auxiliary equation method (Khater method) is wrong

In §2.1 of [1], Khater *et al* have applied the new auxiliary equation method (Khater method) obtained in [2–7] and claimed that the solution of eq. (2.4) of [1] has the form

$$u(\xi) = a_0 + a_1 a^{f(\xi)}, \tag{1}$$

where $f(\xi)$ is a function of ξ, while a_0, a_1 and a are constants, such that $a_1 \neq 0,\ a \neq 1$ and $a \succ 0$.

Khater *et al* [2] claimed that $a^{f(\xi)}$ has many formulas (2.10)–(2.39) (see also [3]) where $f(\xi)$ is the solution of the equation:

$$f'(\xi) = \frac{1}{\ln(a)}\left[\alpha a^{-f(\xi)} + \beta + \sigma a^{f(\xi)}\right], \tag{2}$$

where α, β and σ are constants and $' = \mathrm{d}/\mathrm{d}\xi$.

The first four formulas (2.10)–(2.13) of the paper [2] are given as follows:

If $\beta^2 - \alpha\sigma \prec 0$ and $\sigma \neq 0$, then

$$a^{f(\xi)} = -\frac{\beta}{\sigma} + \frac{\sqrt{\alpha\sigma - \beta^2}}{\sigma}\tan\left(\frac{\xi}{2}\sqrt{\alpha\sigma - \beta^2}\right) \tag{3}$$

or

$$a^{f(\xi)} = -\frac{\beta}{\sigma} + \frac{\sqrt{\alpha\sigma - \beta^2}}{\sigma}\cot\left(\frac{\xi}{2}\sqrt{\alpha\sigma - \beta^2}\right). \tag{4}$$

If $\beta^2 - \alpha\sigma \succ 0$ and $\sigma \neq 0$, then

$$a^{f(\xi)} = -\frac{\beta}{\sigma} - \frac{\sqrt{\alpha\sigma - \beta^2}}{\sigma}\tanh\left(\frac{\xi}{2}\sqrt{\beta^2 - \alpha\sigma}\right) \tag{5}$$

or

$$a^{f(\xi)} = -\frac{\beta}{\sigma} - \frac{\sqrt{\alpha\sigma - \beta^2}}{\sigma}\coth\left(\frac{\xi}{2}\sqrt{\beta^2 - \alpha\sigma}\right). \tag{6}$$

Recently, Zayed *et al* [8–10] have shown that eqs (3)–(6) obtained in [2] are not solutions of eq. (2). Therefore, the Khater method is absolutely wrong and all the exact solutions of the nonlinear evolution equations obtained in [2–7] are also incorrect. Recently, Seadawy *et al* [11] have written a false reply on the paper [8], while Zayed *et al* [12] have written a note on this false reply showing that Seadawy *et al* [11] were not honest in writing their responses.

Based on the recent papers [8–10], all the exact solutions (2.7)–(2.83) of eq. (2.4) obtained in [1] are absolutely wrong for two reasons.

Firstly, the method used is wrong and secondly all those solutions do not satisfy eq. (2.4) of [1].

For example, we show that eq. (2.12) of [1], which is given by

$$u(\xi) = a_1\left[-\frac{\beta}{\sigma} + \frac{\sqrt{-\beta^2}}{\sigma}\tan\left(\frac{\xi}{2}\sqrt{-\beta^2}\right)\right], \tag{7}$$

is not a solution of eq. (2.4) obtained in [1], which is given by

$$4\omega u^3 + 4(\varepsilon - \omega^2)u^2 + 6ku^2u' + k^2u'^2 - 2kuu'' = 0. \tag{8}$$

With this aim, eq. (7) is differentiated twice with respect to ξ, and the following equations are obtained:

$$u'(\xi) = -a_1 \frac{\beta^2}{2\sigma} \sec^2\left(\frac{\xi}{2}\sqrt{-\beta^2}\right) \tag{9}$$

and

$$u''(\xi) = -\frac{a_1\beta^2\sqrt{-\beta^2}}{2\sigma}\left[\tan\left(\frac{\xi}{2}\sqrt{-\beta^2}\right) + \tan^3\left(\frac{\xi}{2}\sqrt{-\beta^2}\right)\right]. \tag{10}$$

Substituting (7), (9) and (10) into the LHS of eq. (8), a non-zero value is obtained. This proves that eq. (7) is not a solution of eq. (8). Similarly, the remaining wrong eqs (2.7)–(2.83) obtained in [1] are not solutions of eq. (8).

In the next section, the right method (amended Khater method) is derived, which Khater *et al* [2] hoped to find, but failed.

2. Derivation of the right method (amended Khater method)

Zhu [13] assumed that the solution of the nonlinear evolution equation has the form

$$u(\xi) = \sum_{i=0}^{N} \alpha_i Q^i(\xi), \tag{11}$$

where α_i are constants and N is the balance number to be determined, while $Q(\xi)$ is the solution of the Riccati equation

$$Q'(\xi) = r + pQ(\xi) + qQ^2(\xi), \tag{12}$$

where $\xi = x - ct$, and $r,\ p,\ q,\ c$ are constants, such that $q \neq 0$ and $' = \mathrm{d}/\mathrm{d}\xi$.

It is well known [13] that eq. (12) has 27 special solutions. The first four solutions are as follows:

(i) If $p^2 - 4rq \prec 0,\ \ pq \neq 0$, or $qr \neq 0$, then

$$Q(\xi) = -\frac{1}{2q}\left[p - \sqrt{4rq - p^2}\tan\left(\frac{\xi}{2}\sqrt{4rq - p^2}\right)\right] \tag{13}$$

or

$$Q(\xi) = -\frac{1}{2q}\left[p + \sqrt{4rq - p^2}\cot\left(\frac{\xi}{2}\sqrt{4rq - p^2}\right)\right]. \tag{14}$$

(ii) If $p^2 - 4rq \succ 0,\ \ pq \neq 0$, or $qr \neq 0$, then

$$Q(\xi) = -\frac{1}{2q}\left[p + \sqrt{p^2 - 4rq}\tanh\left(\frac{\xi}{2}\sqrt{p^2 - 4rq}\right)\right] \tag{15}$$

or

$$Q(\xi) = -\frac{1}{2q}\left[p + \sqrt{p^2 - v4rq}\coth\left(\frac{\xi}{2}\sqrt{p^2 - 4rq}\right)\right]. \tag{16}$$

In order to derive the right method (amended Khater method), the following transformation is used:

$$Q(\xi) = a^{f(\xi)}, \tag{17}$$

where $f(\xi)$ is a new function of $\xi,\ \ a \succ 0$ and $a \neq 1$. Substituting (17) into (11) and (12) we get

$$u(\xi) = \sum_{i=0}^{N} \alpha_i \left[a^{f(\xi)}\right]^i, \tag{18}$$

where $f(\xi)$ is the solution of the equation

$$f'(\xi) = \frac{1}{\mathrm{In}(a)}\left[ra^{-f(\xi)} + p + qa^{f(\xi)}\right]. \tag{19}$$

From (13)–(16) we deduce that $a^{f(\xi)}$ has the following corrected forms:

(iii) If $p^2 - 4rq \prec 0,\ \ pq \neq 0$ or $qr \neq 0$, then

$$a^{f(\xi)} = -\frac{1}{2q}\left[p - \sqrt{4rq - p^2}\tan\left(\frac{\xi}{2}\sqrt{4rq - p^2}\right)\right] \tag{20}$$

or

$$a^{f(\xi)} = -\frac{1}{2q}\left[p + \sqrt{4rq - p^2}\cot\left(\frac{\xi}{2}\sqrt{4rq - p^2}\right)\right]. \tag{21}$$

(iv) If $p^2 - 4rq \succ 0,\ \ pq \neq 0$ or $qr \neq 0$, then

$$a^{f(\xi)} = -\frac{1}{2q}\left[p + \sqrt{p^2 - 4rq}\tanh\left(\frac{\xi}{2}\sqrt{p^2 - 4rq}\right)\right] \tag{22}$$

or

$$a^{f(\xi)} = -\frac{1}{2q}\left[p + \sqrt{p^2 - 4rq}\coth\left(\frac{\xi}{2}\sqrt{p^2 - 4rq}\right)\right]. \tag{23}$$

Similarly, the other 23 formulas can be found for $a^{f(\xi)}$ by using the other 23 solutions of eq. (12) listed in [13] which are omitted here for simplicity. Thus, the right method (amended Khater method) has been derived.

3. Important attention to the authors of [1]

The wrong formulas (3)–(6) are found in the paper of Khater *et al* [1] and these are not solutions of eq. (2). The corrected formulas (20)–(23) are not found in the paper of Khater *et al* [1] and these represent solutions of eq. (19). Nobody can show that formulas (2)–(6) are equivalent to formulas (19)–(23). This fact has been pointed out in [8,12]. Seadawy *et al* [11] have written a false reply in [8], while Zayed *et al* [12] have written a note on this false reply showing that Seadawy *et al* [11] were not honest in their response. This shows that the mathematical contents of the paper of Khater *et al* [1] have serious errors. So, Khater *et al* [1] should recognise these serious errors and we have advised them to be careful and honest while solving the nonlinear evolution equations. In this paper, these errors are removed and amended the studies on the exact solutions of the nonlinear evolution equations obtained in [1]. We hope our present work will be interesting to our readers.

4. Conclusions and other studies on exact solutions

In this comment, it is shown that solutions (2.7)–(2.83) of eq. (2.4) obtained in §2.1 of the paper of Khater *et al* [1] are wrong because the authors have applied the wrong Khater method presented in [2–7]. In §2 of our present comment, we have amended the Khater method presented in [2–7]. This amendment follows directly from the well-known Riccati equation mapping method presented in [13].

It is worth mentioning here that the general solutions to the Riccati equation and the Bernoulli equation are even presented in [14]. All those methods are special cases of the transformed rational function method [15]. There are also other studies on an interesting kind of exact solutions called lump solutions [16,17] and interaction solutions between lump solutions and soliton solutions for integrable equations in $(2+1)$ dimensions [18,19]. For lump–kink interaction solutions, see [20], while for lump–soliton interaction solutions, see [21]. Finally, please see [22,23] which are related to the subject of the present paper. Matinfar *et al* [22] have applied the first integral method for the $(2+1)$ dimensional Jaulent–Miodek equation, while Mishra *et al* [23] have applied an auxiliary equation method for the chemotaxis diffusion reaction equation and found their exact solutions.

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References

[1] M M A Khater, A R Seadawy and D Lu, *Pramana – J. Phys.* **90**: 59 (2018)
[2] M M A Khater, A R Seadawy and D Lu, *Results Phys.* **7**, 2325 (2017)
[3] S Bibi, S T Mohyud-Din, U Khan and N Ahmed, *Results Phys.* **7**, 4440 (2017)
[4] M M A Khater, A R Seadawy and D Lu, *Superlattices Microstruct.* **113**, 346 (2018)
[5] M M A Khater, A R Seadawy and D Lu, *Optik* **158**, 434 (2018)
[6] D Lu, A R Seadawy and M M A Khater, *Optik* **164**, 54 (2018)
[7] M M A Khater, A R Seadawy and D Lu, *Opt. Quantum Electron.* **50**, 155 (2018)
[8] E M E Zayed and A-G Al-Nowehy, *Superlattices Microstruct.* **123**, 94 (2018)
[9] E M E Zayed, A-G Al-Nowehy, S El-Ganaini and R M A Shohib, *Results Phys.* **9**, 1436 (2018)
[10] E M E Zayed, S El-Ganaini, A-G Al-Nowehy and M E M Alngar, *Optik* **170**, 339 (2018)
[11] A R Seadawy, D Lu and M M A Khater, *Superlattices Microstruct.* **123**, 460 (2018)
[12] E M E Zayed, A-G Al-Nowehy, M E M Alngar and R M A Shohib, *Superlattices Microstruct.* **120**, 262 (2018)
[13] S D Zhu, *Chaos Solitons Fractals* **37**, 1335 (2008)
[14] W X Ma and B Fuchssteiner, *Int. J. Non-Linear Mech.* **31**, 329 (1996)
[15] W X Ma and J H Lee, *Chaos Solitons Fractals* **42**, 1356 (2009)
[16] W X Ma and Y Zhou, *J. Differ. Equ.* **264**, 2633 (2018)
[17] S T Chen and W X Ma, *Front. Math. China* **13**, 525 (2018)
[18] J Zhang and W X Ma, *Comput. Math. Appl.* **74**, 591 (2017)
[19] H Zhao and W X Ma, *Comput. Math. Appl.* **74**, 1399 (2017)
[20] W X Ma, X Yong and H Q Zhang, *Comput. Math. Appl.* **75**, 289 (2018)
[21] J Y Yang, W X Ma and Z Qin, *Anal. Math. Phys.* **8**, 427 (2018)
[22] M Matinfar, M Eslami and M Kordy, *Pramana – J. Phys.* **85**, 593 (2015)
[23] A Mishra, R S Kaushal and A Prasad, *Pramana – J. Phys.* **86**, 1043 (2016)