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Application of mathematical methods on the system of dynamical equations for the ion sound and Langmuir waves

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Abstract. We investigated the new exact travelling wave solutions of the system of equations for the ion sound and Langmuir waves (SEISLWs). In this work, we use the extended form of two methods, auxiliary equation mapping and direct algebraic methods, to find the families of new exact travelling wave solutions of the SEISLWs. These new exact travelling solutions are derived in the form of trigonometric functions, hyperbolic functions, periodic solitary waves, bright and dark solitons, kink solutions of the SEISLWs. We used the Mathematica program to show these solutions in two and three dimensions graphically.

Keywords. System of equations for the ion sound and Langmuir waves; mathematical methods; new exact travelling wave solutions.

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1. Introduction

In this work, we consider the system of equations for the ion sound and Langmuir waves (SEISLWs) according to [1–4], given as

$$\mathrm{i}E_t + \frac{1}{2}E_{xx} - nE = 0,$$

$$n_{tt} - n_{xx} - 2\big(|E|^2\big)_{xx} = 0, \tag{1}$$

where $E\mathrm{e}^{-\mathrm{i}wpt}$ is the normalised electric field of the Langmuir oscillation and n is the normalised density perturbation. In the past few years, many researchers have found different types of solutions of the SEISLWs. Baskonus and Bulut [1] derived the exact solutions applied to the sine-Gordon expansion method. Manafian [2] investigated new families of exact travelling wave solutions with the help of the improved $\tan(\Phi(\xi)/2)$-expansion method (ITEM). Tuluce Demiray and Bulut [3] used the extended trial equation method and found the exact solutions and also the dark soliton solutions of the SEISLWs, with help of generalised Kudryashov method. Recently, new research has been done by Seadawy *et al* [4], in which they found new exact travelling solutions of the SEISLWs with the help of two integration schemes: modified Kudryashov method and hyperbolic function method.

The nonlinear form of the partial differential equations with the help of one or more independent variables, such as time t, has become very helpful to understand the physical phenomena of engineering and science. The nonlinear partial differential equations (NPDEs) have applications in various branches of physical sciences such as engineering, mathematical physics, mechanics, material science and mathematics. The nonlinear evolution equations (NLEEs) are very helpful for investigating the physical phenomena in different engineering and scientific fields, such as geophysics, plasma physics, optical fibres, chemistry, solid-state physics, biology, optics and so on. The exact travelling wave solutions of NPDEs play an important role in understanding the process and physical phenomena in many areas of physical sciences. In the last few decades, many new methods have been developed to investigate the exact solutions of NPDEs. Some important methods are: the modified simple equation method, exp-function method, Adomian decomposition method, extended auxiliary equation mapping method, modified extended tanh-expansion method, homotopy perturbation method, improved F-expansion method, extended modified direct algebraic method and so on [5–38].

The main purpose of this work is to find the families of new exact travelling wave solutions (SEISLWs) using the extended form of auxiliary equation mapping and direct algebraic mapping methods [39]. As a result, the new exact travelling wave solutions in the form of normalised electric field E and density perturbation n are obtained, which also represent two and three dimensional graphically.

This work is organised as follows: in §2, formulation of the SEISLWs is given. In §3, the method of finding the new exact travelling wave solutions is discussed in detail with the help of two families and the normalised electric field and density perturbation are formulated. Finally, the conclusion is given in §4.

2. Formulation of the SEISLWs

Let us consider the travelling wave solution of the SEISLWs as

$$E(x,t) = \mathrm{e}^{\mathrm{i}\theta}U(\xi), \quad n(x,t) = V(\xi),$$
$$\theta = (kx + \mu t), \quad \xi = (\omega x + \lambda t). \tag{2}$$

We obtain

$$\mathrm{i}(\lambda + \omega t)U' = 0, \tag{3}$$
$$kU'' - (2\mu + k^2)U - 2UV = 0, \tag{4}$$
$$(\lambda^2 - \omega^2)V'' - 2\omega^2(U^2)'' = 0. \tag{5}$$

We integrate eq. (5) twice with respect to ξ and keeping the integration constant as zero, we get

$$V(\xi) = \frac{2}{k^2 - 1}U^2(\xi), \quad \lambda = -\omega k. \tag{6}$$

Substituting eq. (6) into eq. (4), we finally obtain

$$\omega^2(k^2 - 1)U'' - (k^2 - 1)(2\mu^2 + k^2)U - 4U^3 = 0. \tag{7}$$

3. Families of solitary wave solutions

In this section we investigate the families of new exact travelling wave solutions in the form of normalised electric field E and density perturbation n for the SEISLWs. We apply the extended auxiliary equation mapping and extended direct algebraic mapping methods. As a result, different analytic solutions of eq. (1) for different values of the normalised electric field E and density perturbation n are obtained and the new families are given below.

3.1 *Families I*

3.1.1 *Extended auxiliary equation mapping method.*

Here we apply the extended auxiliary equation mapping method and the general solution of the SEISLWs is in series as

$$u(\xi) = \sum_{i=0}^{m} a_i F^i(\xi) + \sum_{i=-1}^{-m} b_{-i} F^i(\xi) + \sum_{i=2}^{m} c_i F^{i-2}(\xi)F'(\xi) + \sum_{i=-1}^{-m} d_{-i} F^i(\xi)F'(\xi), \tag{8}$$

where $a_0, a_1, \ldots, a_n, b_1, b_2, \ldots, b_n, c_2, c_3, \ldots, c_n, d_1, d_2, \ldots, d_n$ are constants, which are determined later, and the values of $F(\xi)$ and $F'(\xi)$ satisfy the following auxiliary equation:

$$F'(\xi) = \sqrt{pF^2(\xi) + qF^3(\xi) + rF^4(\xi)},$$
$$F''(\xi) = pF(\xi) + \frac{3}{2}qF^2(\xi) + 2rF^3(\xi),$$
$$F'''(\xi) = (p + 3qF(\xi) + 6rF^2(\xi))F'(\xi),$$
$$F''''(\xi) = \frac{1}{2}F(\xi)(2p^2 + 15pqF(\xi) + 5(3q^2 + 8pr)F^2(\xi) + 60qrF^3(\xi) + 48r^2F^4(\xi)). \tag{9}$$

Balancing the highest-order derivatives and highest-order nonlinear terms in eq. (7), we obtain $m = 1$. The general solution of eq. (1) takes the following form:

$$u(\xi) = a_0 + a_1F(\xi) + \frac{b_1}{F(\xi)} + \frac{d_1F'(\xi)}{F(\xi)}. \tag{10}$$

Substituting eq. (10) into eq. (8) and collecting all the coefficients of $F'^j(\xi)F^i(\xi)(j = 0, 1; i = 1, 2, 3, \ldots, n)$, we set every coefficient equal to zero to obtain a set of algebraic equations. By solving this system with the help of Mathematica, the parameters a_0, a_1, b_1, d_1 can be determined as follows.

Case I

$$a_0 = 0, \quad a_1 = \pm\frac{\sqrt{k^2-1}\sqrt{r}\omega}{2\sqrt{2}}, \quad b_1 = 0,$$
$$d_1 = \pm\frac{\sqrt{k^2-1}\omega}{2\sqrt{2}}, \quad \mu = \frac{1}{4}(-2k^2 - p\omega^2). \tag{11}$$

Substituting eq. (11), for only the positive values of a_1 and d_1, into eq. (10), we get the normalised electric field E and density perturbation n of eq. (1) in the simplified form of new exact travelling solutions as follows:

$$E_1(x,t)=\frac{\sqrt{k^2-1}\omega e^{i(kx+\mu t)}(\sqrt{p}qs\operatorname{sech}^2(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)-2p\sqrt{r}(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)^2)}{4\sqrt{2}(qs\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+q)}, \quad (12)$$

$$n_1(x,t)=\frac{\omega^2(\sqrt{p}qs\operatorname{sech}^2(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)-2p\sqrt{r}(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)^2)^2}{16(qs\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+q)^2}, \quad (13)$$

$$E_2(x,t)=\frac{\sqrt{k^2-1}\omega e^{i(kx+\mu t)}\left(\sqrt{r}\sqrt{\frac{p}{r}}\left(\frac{s\sinh(\sqrt{p}(kx+\mu t))}{\cosh(\sqrt{p}(kx+\mu t))+\rho}+1\right)^2+\frac{2\sqrt{p}s(\rho\cosh(\sqrt{p}(kx+\mu t))+1)}{(\cosh(\sqrt{p}(kx+\mu t))+\rho)^2}\right)}{4\sqrt{2}\left(\frac{s\sinh(\sqrt{p}(kx+\mu t))}{\cosh(\sqrt{p}(kx+\mu t))+\rho}+1\right)}, \quad (14)$$

$$n_2(x,t)=\frac{\omega^2\left(\sqrt{r}\sqrt{\frac{p}{r}}\left(\frac{s\sinh(\sqrt{p}(kx+\mu t))}{\cosh(\sqrt{p}(kx+\mu t))+\rho}+1\right)^2+\frac{2\sqrt{p}s(\rho\cosh(\sqrt{p}(kx+\mu t))+1)}{(\cosh(\sqrt{p}(kx+\mu t))+\rho)^2}\right)^2}{16\left(\frac{s\sinh(\sqrt{p}(kx+\mu t))}{\cosh(\sqrt{p}(kx+\mu t))+\rho}+1\right)^2}, \quad (15)$$

$$\begin{aligned}E_3(x,t)=&\Bigg(e^{i(kx+\mu t)}\sqrt{k^2-1}\sqrt{p}\omega\Bigg(-s\cosh(\sqrt{p}(kx+\mu t))(\cosh(\sqrt{p}(kx+\mu t))+\rho\sqrt{\sigma^2+1})\\&+s\sinh(\sqrt{p}(kx+\mu t))\big(\sinh(\sqrt{p}(kx+\mu t))+\sigma\big)\\&-\frac{\sqrt{p}\sqrt{r}\left(s\cosh(\sqrt{p}(kx+\mu t))+\sinh(\sqrt{p}(kx+\mu t))+\rho s\sqrt{\sigma^2+1}+\sigma\right)^2}{q}\Bigg)\Bigg)\Big/\\&2\sqrt{2}\big(\sinh(\sqrt{p}(kx+\mu t))+\sigma\big)\big(s\cosh(\sqrt{p}(kx+\mu t))+\sinh(\sqrt{p}(kx+\mu t))+\rho s\sqrt{\sigma^2+1}+\sigma\big),\end{aligned} \quad (16)$$

$$\begin{aligned}n_3(x,t)=&\Bigg(p\omega^2\Bigg(s\cosh(\sqrt{p}(kx+\mu t))(\cosh(\sqrt{p}(kx+\mu t))+\rho\sqrt{\sigma^2+1})\\&-s\sinh(\sqrt{p}(kx+\mu t))\big(\sinh(\sqrt{p}(kx+\mu t))+\sigma\big)\\&+\frac{\sqrt{p}\sqrt{r}\left(s\cosh(\sqrt{p}(kx+\mu t))+\sinh(\sqrt{p}(kx+\mu t))+\rho s\sqrt{\sigma^2+1}+\sigma\right)^2}{q}\Bigg)^2\Bigg)\Big/\\&4\big(\sinh(\sqrt{p}(kx+\mu t))+\sigma\big)^2(s\cosh(\sqrt{p}(kx+\mu t))+\sinh(\sqrt{p}(kx+\mu t))+\rho s\sqrt{\sigma^2+1}+\sigma)^2.\end{aligned} \quad (17)$$

Case II

$$a_0=\pm\frac{\sqrt{k^2-1}\sqrt{p}\omega}{2\sqrt{2}},\quad a_1=\pm\frac{\sqrt{k^2-1}\sqrt{r}\omega}{\sqrt{2}},$$
$$b_1=0,\quad d_1=0,$$
$$\mu=\frac{1}{4}(-2k^2-p\omega^2),\quad q=2\sqrt{p}\sqrt{r}. \quad (18)$$

Substituting eq. (18), for only the positive values of a_0 and a_1, into eq. (10), the exact travelling wave solutions in the form of normalised electric field E and density perturbation n of eq. (1) are obtained as follows (see figures 1–4):

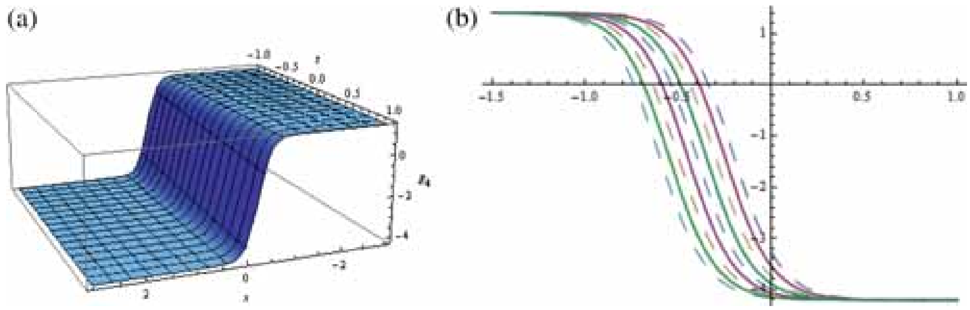

Figure 1. Exact solution for the real value of eq. (19) when $k = 8$, $p = 1$, $\omega = 0.5$, $r = 1$, $s = 1$, $\xi_0 = 0.8$, $q = 1$, $\mu = 0.4$, $\rho = 0.8$.

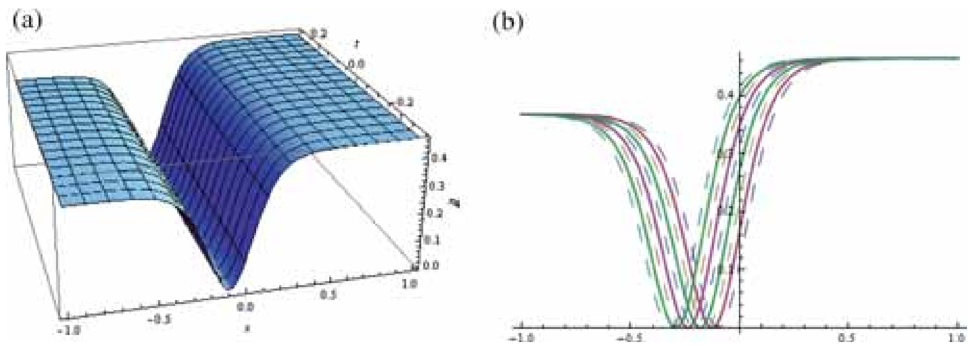

Figure 2. Exact solution given in eq. (20) when $k = 18$, $p = 0.4$, $\omega = 0.9$, $r = 0.8$, $s = 2$, $q = 1$, $\xi_0 = 0.5$, $\rho = 0.8$, $\mu = 0.5$.

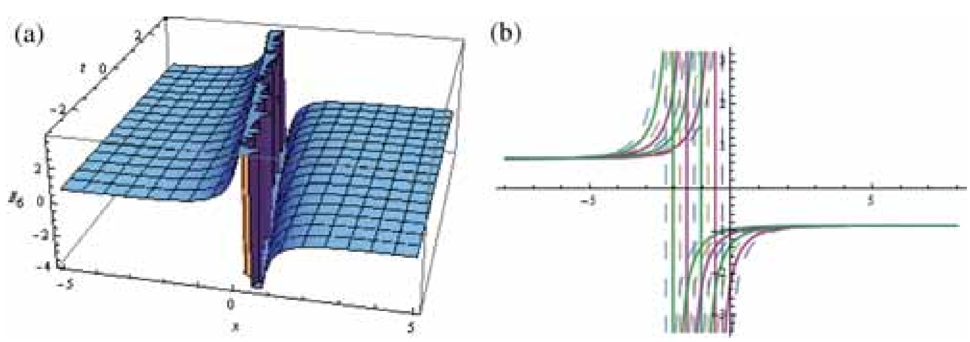

Figure 3. Exact solution for the real value of eq. (23) when $k = 2$, $p = 0.4$, $\omega = 2$, $r = 0.5$, $s = 2$, $q = 1$, $\sigma = 0.4$, $\rho = 0.8$, $\mu = 0.5$.

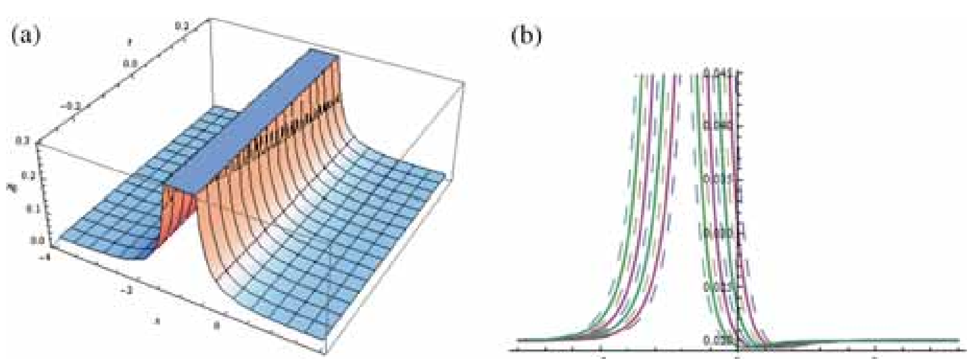

Figure 4. Exact solution given in eq. (24) when $k = 2$, $p = 0.5$, $\omega = 0.8$, $r = 0.5$, $s = 0.5$, $q = 1$, $\mu = 0.4$, $\rho = 0.8$, $\sigma = 2$.

$$E_4(x,t)=\frac{\sqrt{k^2-1}\sqrt{p}\omega e^{i(kx+\mu t)}(-2\sqrt{p}\sqrt{r}s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)-2\sqrt{p}\sqrt{r}+q)}{2\sqrt{2}q}, \quad (19)$$

$$n_4(x,t)=\frac{p\omega^2(-2\sqrt{p}\sqrt{r}s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)-2\sqrt{p}\sqrt{r}+q)^2}{4q^2}, \quad (20)$$

$$E_5(x,t)=\frac{\sqrt{k^2-1}\omega e^{i(kx+\mu t)}\left(\sqrt{r}\sqrt{\frac{p}{r}}\left(\frac{s\sinh(\sqrt{p}(kx+\mu t))}{\cosh(\sqrt{p}(kx+\mu t))+\rho}+1\right)+\sqrt{p}\right)}{2\sqrt{2}}, \quad (21)$$

$$n_5(x,t)=\frac{1}{4}\omega^2\left(\sqrt{r}\sqrt{\frac{p}{r}}\left(\frac{s\sinh(\sqrt{p}(kx+\mu t))}{\cosh(\sqrt{p}(kx+\mu t))+\rho}+1\right)+\sqrt{p}\right)^2, \quad (22)$$

$$E_6(x,t)=\frac{\sqrt{k^2-1}\sqrt{p}\omega e^{i(kx+\mu t)}\left(1-\frac{2\sqrt{p}\sqrt{r}\left(\frac{s(\cosh(\sqrt{p}(kx+\mu t))+\rho\sqrt{\sigma^2+1})}{\sinh(\sqrt{p}(kx+\mu t))+\sigma}+1\right)}{q}\right)}{2\sqrt{2}}, \quad (23)$$

$$n_6(x,t)=\frac{1}{4}p\omega^2\left(1-\frac{2\sqrt{p}\sqrt{r}\left(\frac{s(\cosh(\sqrt{p}(kx+\mu t))+\rho\sqrt{\sigma^2+1})}{\sinh(\sqrt{p}(kx+\mu t))+\sigma}+1\right)}{q}\right)^2. \quad (24)$$

3.2 *Families II*

3.2.1 *Extended direct algebraic mapping method.* Here we apply the extended direct algebraic mapping method and the general solution of the SEISLWs is in series as

$$\phi(\xi)=\sum_{i=0}^{m}a_iF^i(\xi)+\sum_{i=-1}^{-m}b_{-i}F^i(\xi)+\sum_{i=2}^{m}c_iF^{i-2}(\xi)F'(\xi)+\sum_{i=-1}^{-m}d_{-i}F^i(\xi)F'(\xi), \quad (25)$$

where $a_0, a_1, \ldots, a_n, b_1, b_2, \ldots, b_n, c_2, c_3, \ldots, c_n, d_1, d_2, \ldots, d_n$ are arbitrary constants, which are determined later, and the values of $F(\xi)$ and $F'(\xi)$ satisfy the following auxiliary equation:

$$F'(\xi)=\sqrt{pF^2(\xi)+qF^4(\xi)+rF^6(\xi)},$$

$$F''(\xi)=pF(\xi)+2qF^3(\xi)+3rF^5(\xi),$$

$$F'''(\xi)=(p+6qF^2(\xi)+15rF^4(\xi))F'(\xi),$$

$$F''''(\xi)=F(\xi)(p^2+20pqF^2(\xi)+6(4q^2+13pr)F^4(\xi)+120qrF^6(\xi)+105r^2F^8(\xi)). \quad (26)$$

Balancing the highest-order partial derivatives and highest-order nonlinear terms in eq. (7), we get $m=1$. The solution of eq. (1) takes the following form:

$$u(\xi)=a_0+a_1F(\xi)+\frac{b_1}{F(\xi)}+\frac{d_1F'(\xi)}{F(\xi)}. \quad (27)$$

Substituting eq. (27) into eq. (25) and collecting the coefficients of $F'^j(\xi)F^i(\xi)(j=0,1;i=1,2,3,\ldots,n)$, we set every coefficient equal to zero to obtain a set of algebraic equations. By solving this system of equations with the aid of Mathematica, the parameters a_0, a_1, b_1, d_1 can be obtained as follows:

Case I

$$a_0=0,\quad a_1=\pm\frac{\sqrt{k^2-1}\sqrt{q}\omega}{2\sqrt{2}},\quad b_1=0,$$
$$d_1=\frac{\sqrt{k^2-1}\omega}{2\sqrt{2}},\quad \mu=\frac{1}{4}(-2k^2-p\omega^2),\quad r=0. \quad (28)$$

Substituting eq. (28), for only the positive values of a_1, into eq. (27), the new exact travelling wave solutions in the form of normalised electric field E and density perturbation n of eq. (1) are obtained in following simplified forms (see figures 5–8):

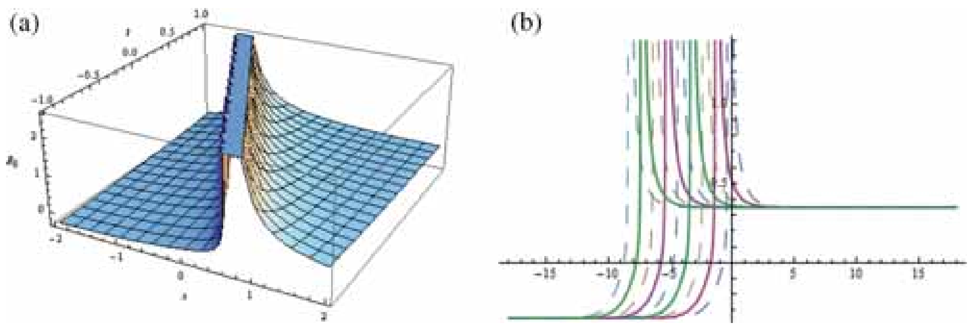

Figure 5. Exact solution for the real value of eq. (31) when $k = 2$, $p = 0.4$, $\omega = 0.9$, $r = 4$, $s = 4$, $\xi_0 = 0.5$, $q = 0.5$, $A = 1$, $B = 1$, $\mu = 2$.

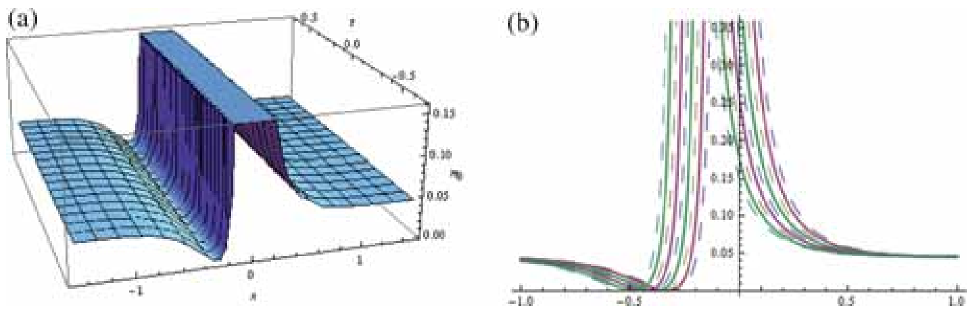

Figure 6. Exact solution given in eq. (32) when $k = 8$, $p = 0.5$, $\omega = 0.6$, $\xi_0 = 0.5$, $q = 1.7$, $A = 1$, $B = 1$, $\mu = 0.2$.

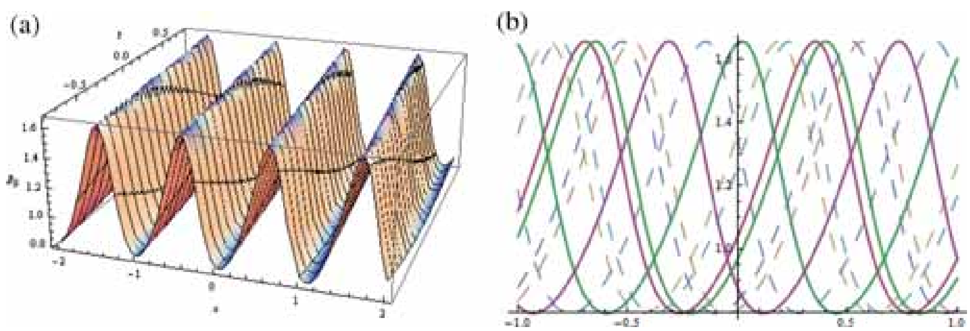

Figure 7. Exact solution for the real value of eq. (33) when $k = 3$, $p = -1$, $\omega = 0.8$, $\xi_0 = 0.9$, $q = 2$, $C = 0.8$, $\mu = 0.5$.

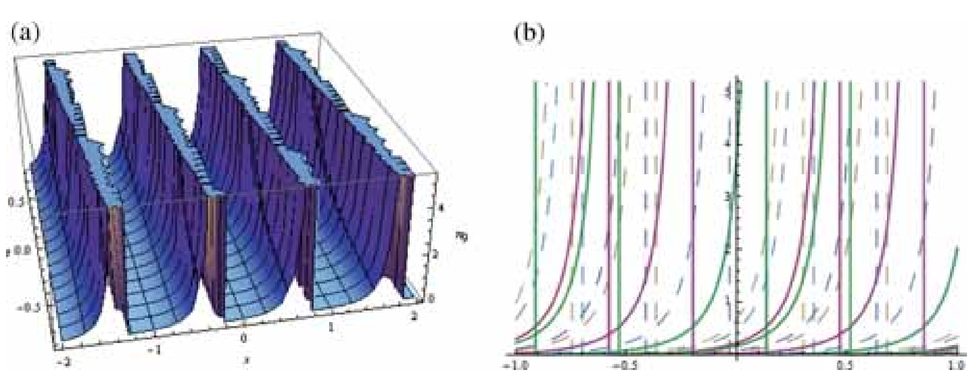

Figure 8. Exact solution given in eq. (34) when $k = 3$, $p = -1$, $\omega = 0.8$, $\xi_0 = 0.9$, $q = 2$, $C = 2$, $\mu = 0.5$.

$$E_7(x,t)=\frac{\sqrt{k^2-1}\omega e^{i(kx+\mu t)}(\sqrt{p}\sqrt{q}s\,\mathrm{sech}^2(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)-2p(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)^2)}{4\sqrt{2}\sqrt{q}(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)}, \tag{29}$$

$$n_7(x,t)=\frac{\omega^2(\sqrt{p}\sqrt{q}s\,\mathrm{sech}^2(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)-2p(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)^2)^2}{16q(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)^2}, \tag{30}$$

$$E_8(x,t)=\frac{1}{4}\sqrt{k^2-1}\omega \times e^{i(kx+\mu t)}\left(2\sqrt{q}\sqrt{\frac{A}{\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B}} + \frac{\sqrt{2}\sqrt{p}\sinh(2(\sqrt{p}(kx+\mu t)+\xi_0))}{\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B}\right), \tag{31}$$

$$n_8(x,t) =\frac{1}{8}\omega^2\left(2\sqrt{q}\sqrt{\frac{A}{\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B}} + \frac{\sqrt{2}\sqrt{p}\sinh(2(\sqrt{p}(kx+\mu t)+\xi_0))}{\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B}\right)^2, \tag{32}$$

$$E_9(x,t)=\frac{1}{4}\sqrt{k^2-1}\omega e^{i(kx+\mu t)} \times\left(2\sqrt{q}\sqrt{\frac{p}{Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0)-q}} + \frac{\sqrt{2}C\sqrt{-p}p\cos(2\sqrt{-p}(kx+\mu t)+\xi_0)}{q-Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0)}\right), \tag{33}$$

Case II

$$a_0=0,\quad a_1=0,\quad b_1=0,\quad d_1=\pm\frac{\sqrt{k^2-1}\omega}{\sqrt{2}},$$
$$\mu=\frac{1}{2}(-k^2-2p\omega^2),\quad r=0. \tag{35}$$

Substituting eq. (35), for only the positive value of d_1, into eq. (27), the normalised electric field E and density perturbation n of eq. (1) can be obtained as follows (see figures 9–12):

$$E_{10}(x,t) =\frac{\sqrt{k^2-1}\sqrt{p}s\omega e^{i(kx+\mu t)}\mathrm{sech}^2(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)}{2\sqrt{2}(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)}, \tag{36}$$

$$n_{10}(x,t)=\frac{ps^2\omega^2\mathrm{sech}^4(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)}{4(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)^2}, \tag{37}$$

$$E_{11}(x,t) =\frac{\sqrt{k^2-1}\sqrt{p}\omega e^{i(kx+\mu t)}\sinh(2(\sqrt{p}(kx+\mu t)+\xi_0))}{\sqrt{2}\big(\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B\big)}, \tag{38}$$

$$n_{11}(x,t)=\frac{p\omega^2\sinh^2(2(\sqrt{p}(kx+\mu t)+\xi_0))}{(B-\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0)))^2}, \tag{39}$$

$$E_{12}(x,t)=\frac{C\sqrt{k^2-1}\sqrt{-p}p\omega e^{i(kx+\mu t)}\cos(2\sqrt{-p}(kx+\mu t)+\xi_0)}{\sqrt{2}(q-Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0))}, \tag{40}$$

$$n_9(x,t) =\frac{1}{8}\omega^2\left(2\sqrt{q}\sqrt{\frac{p}{Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0)-q}} + \frac{\sqrt{2}C\sqrt{-p}p\cos(2\sqrt{-p}(kx+\mu t)+\xi_0)}{q-Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0)}\right)^2. \tag{34}$$

$$n_{12}(x,t)=-\frac{C^2p^3\omega^2\cos^2(2\sqrt{-p}(kx+\mu t)+\xi_0)}{(q-Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0))^2}. \tag{41}$$

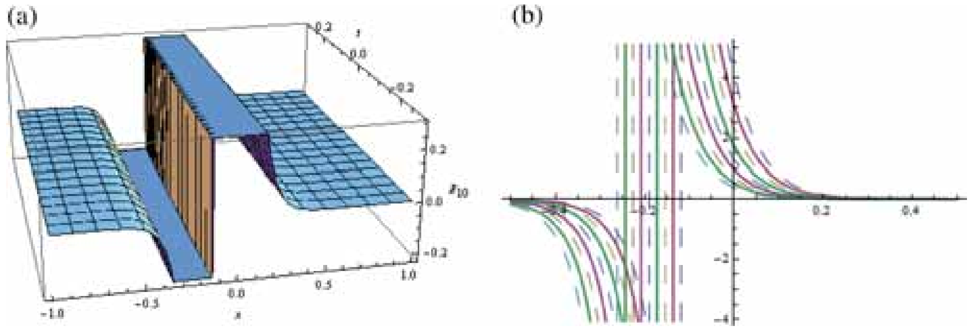

Figure 9. Exact solution for the real value of eq. (36) when $k = 28$, $p = 0.4$, $\omega = 0.8$, $s = 2$, $\xi_0 = 0.5$, $\mu = 0.5$.

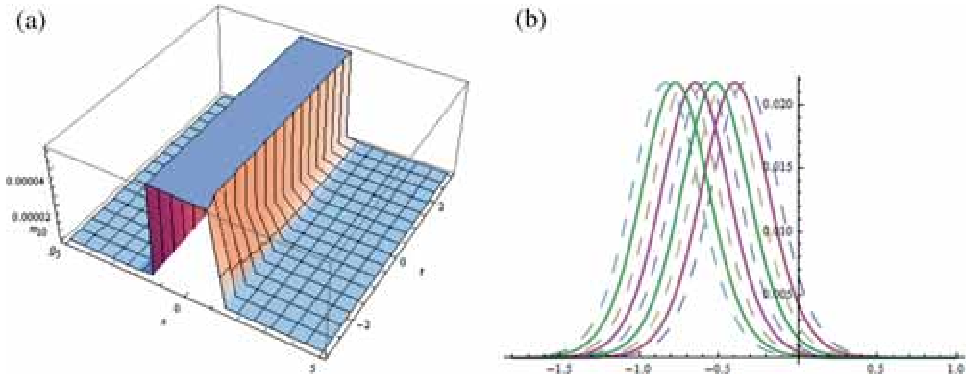

Figure 10. Exact solution given in eq. (37) when $k = 8$, $p = 0.5$, $\omega = 0.7$, $s = 0.6$, $\xi_0 = 0.5$, $\mu = 0.5$.

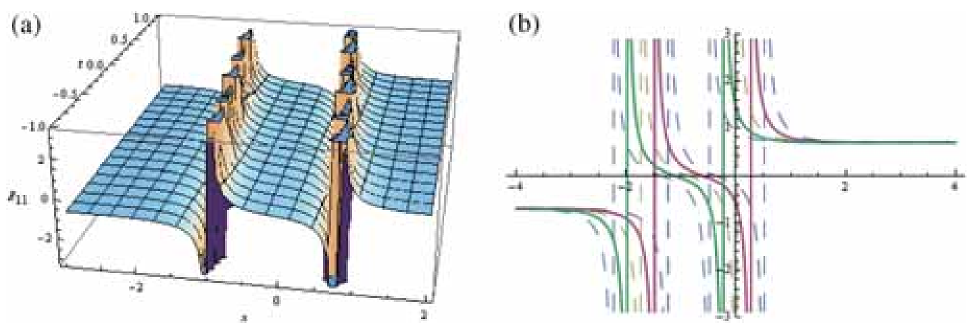

Figure 11. Exact solution for the real value of eq. (38) when $k = 2$, $p = 0.5$, $\omega = 0.8$, $\xi_0 = 0.5$, $q = 1$, $B = 6$, $\mu = 0.5$.

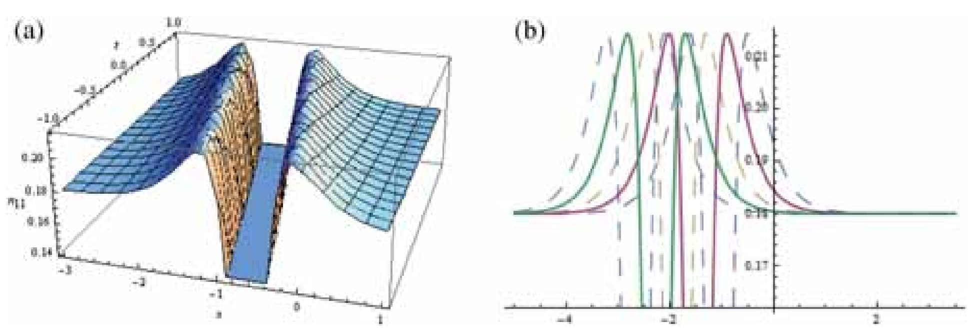

Figure 12. Exact solution given in eq. (39) when $k = 2$, $p = 0.7$, $\omega = 0.6$, $\xi_0 = 1.5$, $B = 0.4$, $\mu = 0.8$.

$$E_{13}(x,t)=\mathrm{e}^{\mathrm{i}(kx+\mu t)}\left(\frac{\sqrt{k^2-1}\sqrt{p}s\omega\,\mathrm{sech}^2(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)}{4\sqrt{2}(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)}-\frac{a_1(ps\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+p)}{q}\right), \tag{43}$$

$$n_{13}(x,t)=\frac{(\sqrt{2}\sqrt{k^2-1}\sqrt{p}qs\omega\,\mathrm{sech}^2(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)-8a_1p(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)^2)^2}{32(k^2-1)(qs\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+q)^2}, \tag{44}$$

$$E_{14}(x,t)=\frac{\mathrm{e}^{\mathrm{i}(kx+\mu t)}\left(4a_1\sqrt{\frac{A}{\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B}}+\frac{\sqrt{k^2-1}\sqrt{p}\omega\sinh(2(\sqrt{p}(kx+\mu t)+\xi_0))}{\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B}\right)}{2\sqrt{2}}, \tag{45}$$

$$n_{14}(x,t)=\frac{\left(4a_1\sqrt{\frac{A}{\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B}}+\frac{\sqrt{k^2-1}\sqrt{p}\omega\sinh(2(\sqrt{p}(kx+\mu t)+\xi_0))}{\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B}\right)^2}{4(k^2-1)}, \tag{46}$$

$$E_{15}(x,t)=\frac{\mathrm{e}^{\mathrm{i}(kx+\mu t)}\left(4a_1\sqrt{\frac{p}{Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0)-q}}+\frac{C\sqrt{k^2-1}\sqrt{-p}p\omega\cos(2\sqrt{-p}(kx+\mu t)+\xi_0)}{q-Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0)}\right)}{2\sqrt{2}}, \tag{47}$$

$$n_{15}(x,t)=\frac{\left(4a_1\sqrt{\frac{p}{Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0)-q}}+\frac{C\sqrt{k^2-1}\sqrt{-p}p\omega\cos(2\sqrt{-p}(kx+\mu t)+\xi_0)}{q-Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0)}\right)^2}{4(k^2-1)}. \tag{48}$$

Case III

$$a_0=0,\quad a_1=a_1,\quad b_1=0,\quad d_1=\frac{\sqrt{k^2-1}\omega}{2\sqrt{2}},$$
$$\mu=\frac{1}{4}(-2k^2-p\omega^2),$$
$$q=\frac{8a_1^2}{(k^2-1)\omega^2},\quad r=0. \tag{42}$$

Substituting eq. (42) into eq. (27), the normalised electric field E and density perturbation n of eq. (1) can be obtained as the new exact travelling wave solutions in the simplified form as (see figures 13–18)

Case IV

$$a_0=a_1=b_1=0,\quad d_1=\pm\sqrt{2}\sqrt{k^2-1}\omega,$$
$$\mu\to\frac{1}{2}(-k^2-8p\omega^2),\quad q=0. \tag{49}$$

Substituting eq. (49), for only the positive values of d_1, into eq. (27), the normalised electric field E and density perturbation n of eq. (1) can be obtained as a new exact travelling wave solutions in the simplified form as

$$E_{16}(x,t)=\frac{\sqrt{k^2-1}\sqrt{p}s\omega\mathrm{e}^{\mathrm{i}(kx+\mu t)}\mathrm{sech}^2(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)}{\sqrt{2}(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)}, \tag{50}$$

$$n_{16}(x,t)=\frac{ps^2\omega^2\mathrm{sech}^4(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)}{(s\tanh(\frac{1}{2}\sqrt{p}(kx+\mu t)+\xi_0)+1)^2}, \tag{51}$$

$$E_{17}(x,t)=\frac{\sqrt{2}\sqrt{k^2-1}\sqrt{p}\omega\mathrm{e}^{\mathrm{i}(kx+\mu t)}\sinh(2(\sqrt{p}(kx+\mu t)+\xi_0))}{\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0))-B}, \tag{52}$$

$$n_{17}(x,t)=\frac{4p\omega^2\sinh^2(2(\sqrt{p}(kx+\mu t)+\xi_0))}{(B-\cosh(2(\sqrt{p}(kx+\mu t)+\xi_0)))^2}, \tag{53}$$

$$E_{18}(x,t)=\frac{\sqrt{2}C\sqrt{k^2-1}(-p)^{3/2}\omega\mathrm{e}^{\mathrm{i}(kx+\mu t)}\cos(2\sqrt{-p}(kx+\mu t)+\xi_0)}{Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0)-q}, \tag{54}$$

$$n_{18}(x,t)=-\frac{4C^2p^3\omega^2\cos^2(2\sqrt{-p}(kx+\mu t)+\xi_0)}{(q-Cp\sin(2\sqrt{-p}(kx+\mu t)+\xi_0))^2}. \tag{55}$$

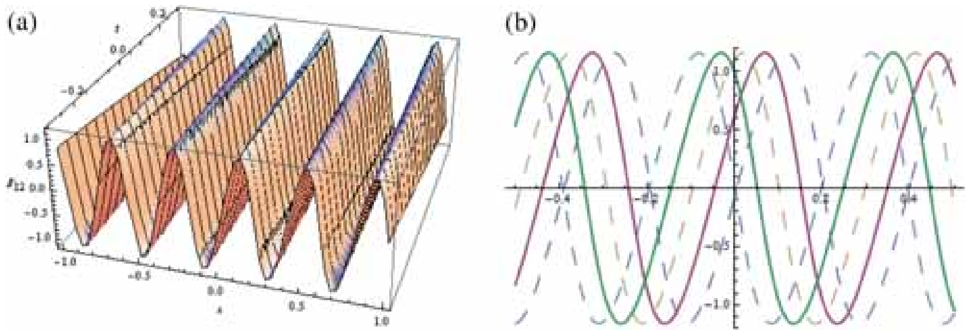

Figure 13. Exact solution for the real value of eq. (40) when $k = 8$, $p = -1$, $\omega = 0.8$, $\xi_0 = 1.5$, $q = 2$, $C = 0.5$, $\mu = 0.4$.

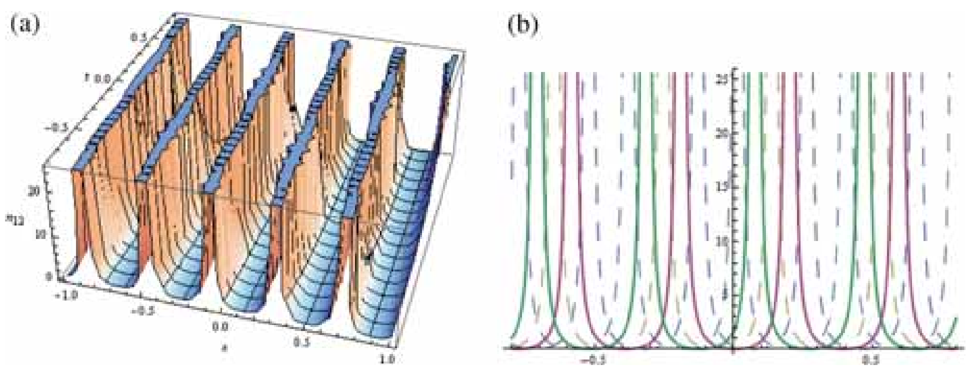

Figure 14. Exact solution given in eq. (41) when $k = 8$, $p = -1$, $\omega = 0.9$, $\xi_0 = 0.5$, $q = 2$, $C = 2$, $\mu = 0.5$.

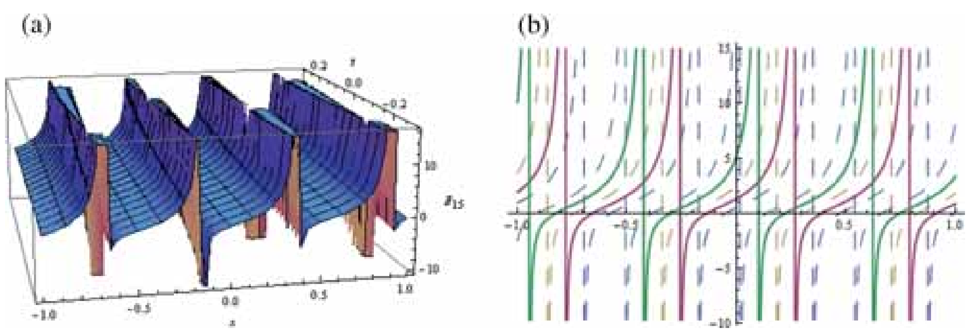

Figure 15. Exact solution for the real value of eq. (47) when $k = 6$, $p = -1$, $\omega = 0.9$, $\xi_0 = 0.5$, $q = 2$, $a_1 = 2$, $C = 2$, $\mu = 0.5$.

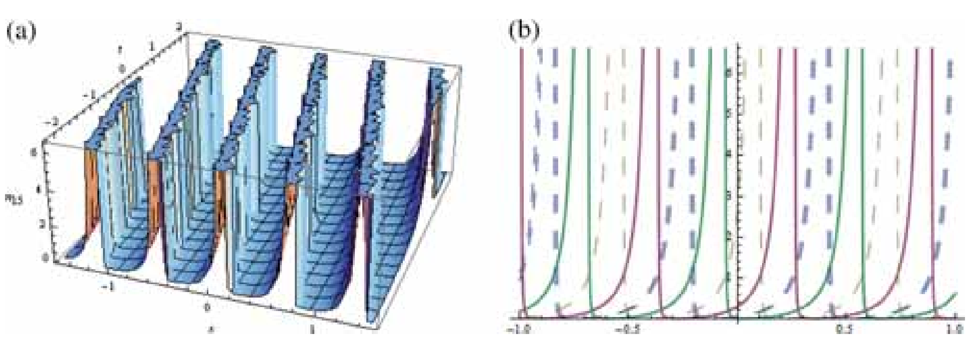

Figure 16. Exact solution given in eq. (48) when $k = 5$, $p = -1$, $\omega = 0.9$, $\xi_0 = 0.5$, $q = 2$, $a_1 = 2$, $C = 2$, $\mu = 0.8$.

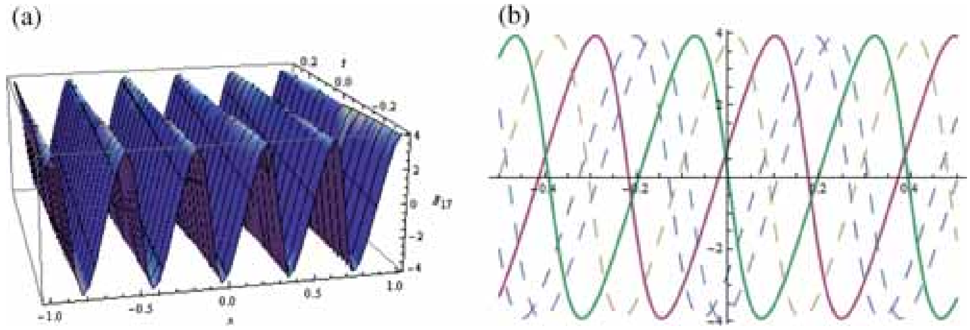

Figure 17. Exact solution for the real value of eq. (54) when $k = 8$, $p = -1$, $\omega = 0.8$, $\xi_0 = 0.5$, $q = 0.5$, $C = 0.2$, $\mu = 0.7$.

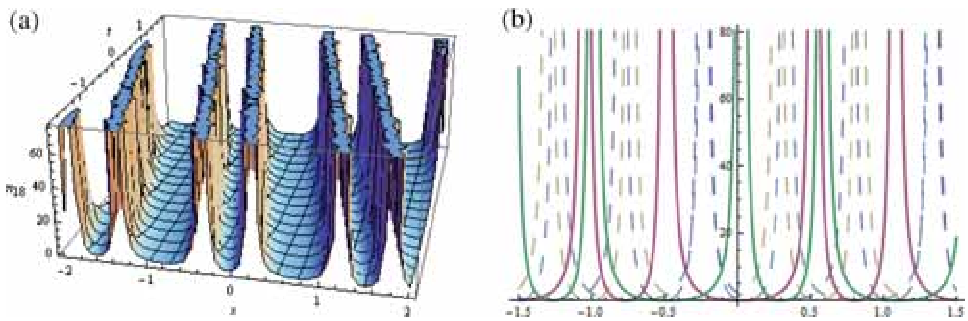

Figure 18. Exact solution given in eq. (55) when $k = 2$, $p = -1$, $\omega = 0.8$, $\xi_0 = 0.5$, $q = 2$, $C = 4.5$, $\mu = 0.8$.

4. Conclusion

In this research paper, we studied two new techniques, extended auxiliary equation mapping and extended direct algebraic methods, to determine different kinds of new exact travelling wave solutions of the SEISLWs. Many NPDEs in science and engineering and other branches of physical sciences can also be solved by these fruitful and powerful methods. These methods are applied successfully on SEISLWs. As a result, we obtained different kinds of new exact travelling wave solutions in the form of normalised electric field and normalised density perturbation, which show the effectiveness and reliability of these methods. As a result, new exact travelling wave solutions are obtained in the form of trigonometric functions, hyperbolic functions, periodic solitary waves, bright and dark solitons, kink solutions which are shown graphically in two and three dimensions graphically. These new solutions play important roles in different areas of engineering, physics and other branches of physical sciences and are also helpful to other researchers in understanding and investigating the physical meaning of the system.

References

[1] H M Baskonus and H Bulut, *Waves Random Complex Media* **27**, 163 (2017)
[2] J Manafian, *Opt. Quant. Electron.* **49(1)**, 17 (2017)
[3] S Tuluce Demiray and H Bulut, *Math. Comput. Appl.* **21(2)**, 21 (2016)
[4] A Seadawy, D Kumar, K Hosseini and F Samadani, *Results Phys.* **9**, 1631 (2018)
[5] A R Seadawy and K El-Rashidy, *Pramana – J. Phys.* **87**: 20 (2016)
[6] Abdullah, Aly Seadawy and Jun Wang, *Pramana – J. Phys.* **91**: 26 (2018)
[7] M N Ali, A R Seadawy and S M Husnine, *Pramana – J. Phys.* **91**: 48 (2018)
[8] K U-H Tariq and A R Seadawy, *Pramana – J. Phys.* **91**: 68 (2018)
[9] A R Seadawy, *Pramana – J. Phys.* **89**: 49 (2017)
[10] M Arshad, A R Seadawy, D Lu and J Wang, *Chin. J. Phys.* **55**, 780 (2017)
[11] A Asghar, A R Seadawya and D Lu, *Optik* **145**, 79 (2017)
[12] A Seadawy, *Optik – Int. J. Light. Electron. Opt.* **139**, 31 (2017)
[13] M Iqbal, A R Seadawy and D Lu, *Mod. Phys. Lett. A* **33(32)**, 1850183 (2018)
[14] Asghar Ali, Aly R Seadawy and Dianchen Lu, *Open Phys.* **16**, 219 (2018)
[15] A R Seadawy, *Appl. Math. Sci.* **6(82)**, 4081 (2012)
[16] A R Seadawy, *Physica A* **455**, 44 (2016)
[17] D Lu, A R Seadawy, M Arshad and W Jun, *Results Phys.* **7**, 899 (2017)
[18] K Tariq and A R Seadawy, *Results Phys.* **7**, 1143 (2017)
[19] A R Seadawy and J Manafian, *Results Phys.* **8**, 1158 (2018)

[20] K Khan, H Koppelaar and A Akbar, *Comput. Math. Eng.* **2**, 5 (2016)
[21] M Arshad, A R Seadawy and D Lu, *Optik* **138**, 40 (2017)
[22] I Kourakis, W M Moslem, U M Abdelsalam, R Sabry and P K Shukla, *Plasma Fusion Res.* **4**, 1 (2009)
[23] A Seadawy, *J. Electromagn. Waves Appl.* **31(14)**, 1353 (2017)
[24] A R Seadawy and K El-Rashidy, *Math. Comput. Model* **57**, 13 (2013)
[25] S Zhang, J L Tong and W Wang, *Phys. Lett. A* **372**, 2254 (2008)
[26] A R Seadawy, *Appl. Math. Lett.* **25**, 687 (2012)
[27] A H Khater, D K Callebaut and A R Seadawy, *Phys. Scr.* **62**, 353 (2000)
[28] A H Khater, D K Callebaut, M A Helal and A R Seadawy, *Eur. Phys. J. D* **39**, 237 (2006)
[29] M Iqbal, A R Seadawy and D Lu, *Mod. Phys. Lett. A* **33**, 1850217 (2018)
[30] M A Helal and A R Seadawy, *Phys. Scr.* **80**, 350 (2009)
[31] J Manafian, *Comput. Math. Appl.* **76(5)**, 1246 4 (2018)
[32] J Manafian, *Eur. Phys. J. Plus* **130**, 1 (2015)
[33] M Foroutan, I Zamanpour and J Manafian, *Eur. Phys. J. Plus* **132**, 421 (2017)
[34] S H Seyedi, B N Saray and M R H Nobari, *Appl. Math. Comput.* **269**, 488 (2015)
[35] M Dehghan, J M Heris and A Saadatmandi, *Math. Methods Appl. Sci.* **33**, 1384 (2010)
[36] S Hadi Seyedi, B N Saray and A Ramazani, *Powder Technol.* **340**, 264 (2018)
[37] M Dehghan, J Manafian and A Saadatmandi, *Numer. Methods Partial Differ. Equ. J.* **26**, 448 (2010)
[38] M Dehghan, J Manafian and A Saadatmandi, *Z. Naturforsch.* **65a**, 935 (2010)
[39] A R Seadawy and D Lu, *Results Phys.* **6**, 590 (2016)