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Analytical behaviour of lump solution and interaction phenomenon to the Kadomtsev–Petviashvili-like equation

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Abstract. In this paper, we use the generalised Hirota bilinear method (GHBM). With the help of symbolic calculations and applying the used method, we solve the Kadomtsev–Petviashvili (KP)-like equation with $p = 3$ to obtain some new lump, periodic kink-wave and solitary wave solutions. All solutions have been verified with their corresponding equations with the aid of the Maple package program.

Keywords. Kadomtsev–Petviashvili-like equation; generalised Hirota bilinear method; lump solution; periodic kink-wave solution; solitary wave solution.

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1. Introduction

Consider the following $(2+1)$-dimensional dynamical model, which is called the nonlinear Kadomtsev–Petviashviii (KP) equation, first introduced by Kadomtsev and Petviashvili [1], i.e.

$$(u_t + 6uu_x + u_{xxx})_x - u_{yy} = 0. \tag{1}$$

The KP equation is completely integrable. Ma [2] obtained the lump solutions by using the following transformation:

$$u = 2(\ln f)_{xx} \tag{2}$$

and Manakov *et al* [3] obtained the subclass of the lump solutions which involve two free parameters. Besides, Zhao and Ma [4] used the Hirota bilinear method of the KP equation and obtained 12 classes of lump–kink solutions. Also, the B-KP (BKP) equation

$$(u_t + 15uu_{xxx} + 15u_x^2 - 15u_xu_y + u_{xxxxx})_x - 5u_{xxxy} -5u_{yy} = 0 \tag{3}$$

through the transformation

$$u = 2(\ln f)_x \tag{4}$$

obtains the following $(2+1)$-dimensional Hirota bilinear equation:

$$(D_x^6 - 5D_x^3D_y + D_xD_t - 5D_y^2)f \cdot f = 0, \tag{5}$$

which has been solved and the lump solutions were obtained by Yang and Ma [5].

The $(2+1)$-dimensional KP equation has been well studied by many scholars. For example, the exact solutions and other solutions, which contain the lump solutions [2], the multiple exp-function algorithm [6], the (G'/G)-expansion method [7,8], the Galerkin method [9], the improved $\tan(\phi/2)$-expansion method [10,11], the simplified homogeneous balance method [12], the similarity transformations method [13], the lump–kink solutions [14], the breather, lump and X soliton solutions [15], the line solitons [16], the hyperbolic function solutions, trigonometric function solutions, exponential solutions, rational solutions [17], the discrete spectral analysis [18], the rational solutions [19], the second-order rogue waves [20], the bell-type soliton solutions [21], the dust-acoustic solitary waves [22], the source generalisation method [23,24], the N-soliton solutions [25], the variational formulation of surface water waves [26], three-wave solution, periodic two-solitary-wave solutions [27], the Bäcklund transformation in bilinear form [28], the tanh–coth method and the Hirota bilinear method [29] were obtained. Also, researchers obtained further results including the exact solitary wave and periodic solutions, the non-travelling wave solutions, the new periodic soliton solutions, the conservation laws, the multiple soliton solutions, soliton-type solutions, rational solutions, the periodic cross-kink wave solutions, breather-type solutions, the lump and its interaction solutions, the multiple wave solutions, which have been presented in the literature. Meanwhile, there are many other related works on the (2+1)-dimensional KP-like equation [30], the reduced p-generalised Kadomtsev–Petviashvili (p-gKP) and p-generalised B-type Kadomtsev–Petviashvili (p-gbKP) equations [31], the (2 + 1)-dimensional KdV equation [32] and the related topics of lump solitons (see [33–47]).

Moreover, a variety of analytical methods on some physical models have been reported, such as using the Hirota bilinear method and Bäcklund transformation to investigate the KP-based system [48], utilising homoclinic test approach and Hirota bilinear method for the (2 + 1)-dimensional generalised KP equation [49], employing Hirota bilinear method to study phase shifts and trajectories during the overtaking collision of multi-solitons [50], studying two coupled KP equations by using the simplified Hereman form of Hirota bilinear method [50], applying the Hirota bilinear method to solve the (2 + 1)-dimensional bidirectional Sawada–Kotera equation [51], the multiresolution analysis [52], the multiscale analysis [53], and finally investigating the (4+1)-dimensional Fokas equation with the help of Hirota bilinear method in which single-soliton solution, double-soliton solution and triple-soliton solutions are obtained [54].

In this paper, we use the generalised Hirota bilinear method. With the help of symbolic calculation and applying the Hirota method, we want to solve the (2+1)-dimensional KP-like equation. As a result, we obtained some new lump solutions, periodic kink-wave, periodic soliton and solitary wave solutions. To the best of our knowledge, these solutions have not been reported. At the same time, this paper can also be regarded as a supplement to other associated works.

2. A KP-like differential equation

Consider the Kadomtsev–Petviashvili I (KPI) equation

$$(u_t + 6uu_x + u_{xxx})_x - u_{yy} = 0. \tag{6}$$

Use the following link between the functions $f(x, y, t)$ and $u(x, y, t)$:

$$u(x, y, t) = 2(\ln f(x, y, t))_{xx}. \tag{7}$$

Then eq. (6) is transformed into the bilinear form

$$\begin{aligned} B_{\mathrm{KP}}(f) &:= (D_x D_t + D_x^4 - D_y^2) f \cdot f \\ &= 2(ff_{xt} - f_x f_t + ff_{xxxx} - 4 f_x f_{xxx} \\ &\quad + 3f_{xx}^2 - ff_{yy} + f_y^2). \end{aligned} \tag{8}$$

Suppose the Hirota derivatives are based on functions $f(x)$ and $g(x)$, given as

$$\prod_{i=1}^{3} D_{\xi_i}^{\alpha_i} f \cdot g = \prod_{i=1}^{3} \left(\frac{\partial}{\partial \xi_i} - \frac{\partial}{\partial \zeta_i} \right)^{\theta_i} f(\xi) g(\zeta) \bigg|_{\zeta=\xi}, \tag{9}$$

where the vectors $\xi = (\xi_1, \xi_2, \xi_3)$, $\zeta = (\zeta_1, \zeta_2, \zeta_3)$ and $\theta_1, \theta_2, \theta_3$ are arbitrary non-negative integers. Let us consider a generalised bilinear differential equation of the KP-like equation type as

$$\begin{aligned} \bar{B}_{\mathrm{KP}}(f) &:= (D_{3,x} D_{3,t} + D_{3,x}^4 - D_{3,y}^2) f \cdot f \\ &= 2(ff_{xt} - f_x f_t) + 6f_{xx}^2 + 2(f_y^2 - ff_{yy}). \end{aligned} \tag{10}$$

The generalised bilinear differential operator [31] is given as

$$\begin{aligned} & D_{p,x}^m D_{p,t}^n f \cdot f \\ &= \left(\frac{\partial}{\partial x} + \alpha_p \frac{\partial}{\partial x'} \right)^m \left(\frac{\partial}{\partial t} + \alpha_p \frac{\partial}{\partial t'} \right)^n f(x, t) \\ &\quad \times f(x', t') \big|_{x'=x, t'=t} \\ &= \sum_{i=0}^{m} \sum_{j=0}^{n} \binom{m}{i} \binom{n}{j} \frac{\partial^{m-i}}{\partial x^{m-i}} \frac{\partial^i}{\partial x'^{(i)}} \frac{\partial^{n-j}}{\partial t^{n-j}} \frac{\partial^j}{\partial t'^{(j)}} \\ &\quad \times f(x, t) f(x', t') \big|_{x'=x, t'=t'}, \quad m, n \geq 0, \end{aligned} \tag{11}$$

$$\alpha_p^s = (-1)^{r_p(s)}, \quad s = r_p(s) \bmod p. \tag{12}$$

By utilising a general Bell polynomial theory [55,56] and by using the transformation

$$u = 2(\ln f)_{xx}, \tag{13}$$

the generalised bilinear equation (10) related to the KP-like equation transforms to the following equation:

$$\begin{aligned} & 8\partial_x^{-1} u_t + 12u^2 + 12u(\partial_x^{-1} u)^2 + 3(\partial_x^{-1} u)^4 \\ & -8\partial_x^{-2} u_{yy} = 0. \end{aligned} \tag{14}$$

Theorem 2.1. *Function f solves* (10) *if and only if $u = 2(\ln f)_{xx}$ demonstrates a solution to the KP-like equation* (14):

$$\frac{8(D_{3,x}D_{3,t} + D_{3,x}^4 - D_{3,y}^2) f \cdot f}{f^2} = 8\partial_x^{-1}u_t + 12u^2 + 12u(\partial_x^{-1}u)^2 + 3(\partial_x^{-1}u)^4 - 8\partial_x^{-2}u_{yy}. \quad (15)$$

Proof. From expression (7), we obtain

$$u(x, y, t) = 2(\ln f(x, y, t))_{xx} \Longleftrightarrow f(x, y, t) = \exp\left(\frac{1}{2}\int\int u(x, y, t)\,\mathrm{d}x\,\mathrm{d}x\right). \quad (16)$$

Then by using expression (16), the expressions $\partial f/\partial x$, $\partial f/\partial y$, $\partial f/\partial t$, $\partial^2 f/\partial x^2$, $\partial^2 f/\partial y^2$ and $\partial^2 f/\partial x\partial t$, respectively, can be written as

$$\frac{\partial f}{\partial x} = \frac{1}{2}\left(\int u\,\mathrm{d}x\right)\exp\left(\frac{1}{2}\int\int u\,\mathrm{d}x\,\mathrm{d}x\right), \quad (17)$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}\left(\frac{1}{2}\int\int \frac{\partial u}{\partial y}\,\mathrm{d}x\,\mathrm{d}x\right)\exp\left(\frac{1}{2}\int\int u\,\mathrm{d}x\,\mathrm{d}x\right), \quad (18)$$

$$\frac{\partial f}{\partial t} = \frac{1}{2}\left(\frac{1}{2}\int\int \frac{\partial u}{\partial t}\,\mathrm{d}x\,\mathrm{d}x\right)\exp\left(\frac{1}{2}\int\int u\,\mathrm{d}x\,\mathrm{d}x\right), \quad (19)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{4}\exp\left(\frac{1}{2}\int\int u\,\mathrm{d}x\,\mathrm{d}x\right)\left[2u + \left(\int u\,\mathrm{d}x\right)^2\right], \quad (20)$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{1}{4}\exp\left(\frac{1}{2}\int\int u\,\mathrm{d}x\,\mathrm{d}x\right)\left[2\int\int \frac{\partial^2 u}{\partial y^2}\,\mathrm{d}x\,\mathrm{d}x + \left(\int\int \frac{\partial u}{\partial y}\,\mathrm{d}x\,\mathrm{d}x\right)^2\right], \quad (21)$$

$$\frac{\partial^2 f}{\partial x\partial t} = \frac{1}{4}\exp\left(\frac{1}{2}\int\int u\,\mathrm{d}x\,\mathrm{d}x\right)\left[2\int \frac{\partial u}{\partial t}\mathrm{d}x + \left(\int u\,\mathrm{d}x\right)\left(\int\int \frac{\partial u}{\partial t}\,\mathrm{d}x\,\mathrm{d}x\right)\right]. \quad (22)$$

Substituting (17)–(22) into (10) yields the bilinear form of eq. (14) as

$$2(ff_{xt} - f_x f_t) + 6f_{xx}^2 + 2(f_y^2 - ff_{yy}) = \frac{1}{8}\exp\left(\frac{1}{2}\int\int u\,\mathrm{d}x\,\mathrm{d}x\right) \times\left[3\left(\int\int u\,\mathrm{d}x\mathrm{d}x\right)^4 + 12u\left(\int u\,\mathrm{d}x\right)^2 + 12u^2 + 8\left(\int \frac{\partial u}{\partial t}\mathrm{d}x\right) - 8\left(\int\int \frac{\partial^2 u}{\partial y^2}\mathrm{d}x\,\mathrm{d}x\right)^2\right], \quad (23)$$

or can be rewritten as

$$\frac{16(ff_{xt} - f_x f_t) + 48f_{xx}^2 + 16(f_y^2 - ff_{yy})}{f^2} = \frac{8(D_{3,x}D_{3,t} + D_{3,x}^4 - D_{3,y}^2) f \cdot f}{f^2} = 8\partial_x^{-1}u_t + 12u^2 + 12u(\partial_x^{-1}u)^2 + 3(\partial_x^{-1}u)^4 - 8\partial_x^{-2}u_{yy}, \quad (24)$$

where $\partial_x^{-1}(\cdot) = \int(\cdot)\,\mathrm{d}x$. Therefore, eq. (24) is the KP-like equation. Hence, the theorem has been proved. □

3. Lump solution of KP-like equation

In this section, we search for the quadratic function solutions to eq. (10) by using the generalised Hirota bilinear method.

Assume that

$$f = g^2 + h^2 + a_9,$$
$$g(x, y, t) = a_1 x + a_2 y + a_3 t + a_4,$$
$$h(x, y, t) = a_5 x + a_6 y + a_7 t + a_8, \quad (25)$$

where a_i, $i = 1, \ldots, 9$, are free values which are to be determined later. Substituting (25) into eq. (10) and then collecting the coefficients in front of different polynomial functions, we get

Set I:

$$\begin{cases} a_1 = a_1, \quad a_2 = a_2, \quad a_3 = \dfrac{a_1(a_2^2 - a_6^2) + 2a_2 a_6 a_5}{a_1^2 + a_5^2}, \\ a_4 = a_4, \quad a_5 = a_5, \quad a_6 = a_6, \\ a_7 = \dfrac{a_5(a_6^2 - a_2^2) + 2a_1 a_2 a_6}{a_1^2 + a_5^2}, \quad a_8 = a_8, \\ a_9 = \dfrac{3(a_1^2 + a_5^2)^3}{(a_1 a_6 - a_2 a_5)^2} \end{cases} \quad (26)$$

and

$$a_1 \neq 0, \quad a_5 \neq 0, \quad a_6 a_1 - a_5 a_2 \neq 0. \quad (27)$$

Substituting (26) into (25), we get a lump solution of eq. (6) as follows:

$$u(x, y, t) = \frac{4(a_1^2 + a_5^2)}{f} - \frac{8(a_1 g + a_5 h)^2}{f^2}, \quad (28)$$

$$f=\left(a_1x+a_2y+\frac{a_1(a_2^2-a_6^2)+2a_2a_6a_5}{a_1^2+a_5^2}t+a_4\right)^2+\left(a_5x+a_6y+\frac{a_5(a_6^2-a_2^2)+2a_1a_2a_6}{a_1^2+a_5^2}t+a_8\right)^2+\frac{3(a_1^2+a_5^2)^3}{(a_1a_6-a_2a_5)^2}, \quad (29)$$

$$g=a_1x+a_2y+\frac{a_1(a_2^2-a_6^2)+2a_2a_6a_5}{a_1^2+a_5^2}t+a_4,$$
$$h=a_5x+a_6y+\frac{a_5(a_6^2-a_2^2)+2a_1a_2a_6}{a_1^2+a_5^2}t+a_8, \quad (30)$$

where a_1, a_2, a_4, a_5, a_6 and a_8 are arbitrary constants. Now, solution (28) is plotted in 3D, density, contour and 2D forms (see figures 1 and 2).

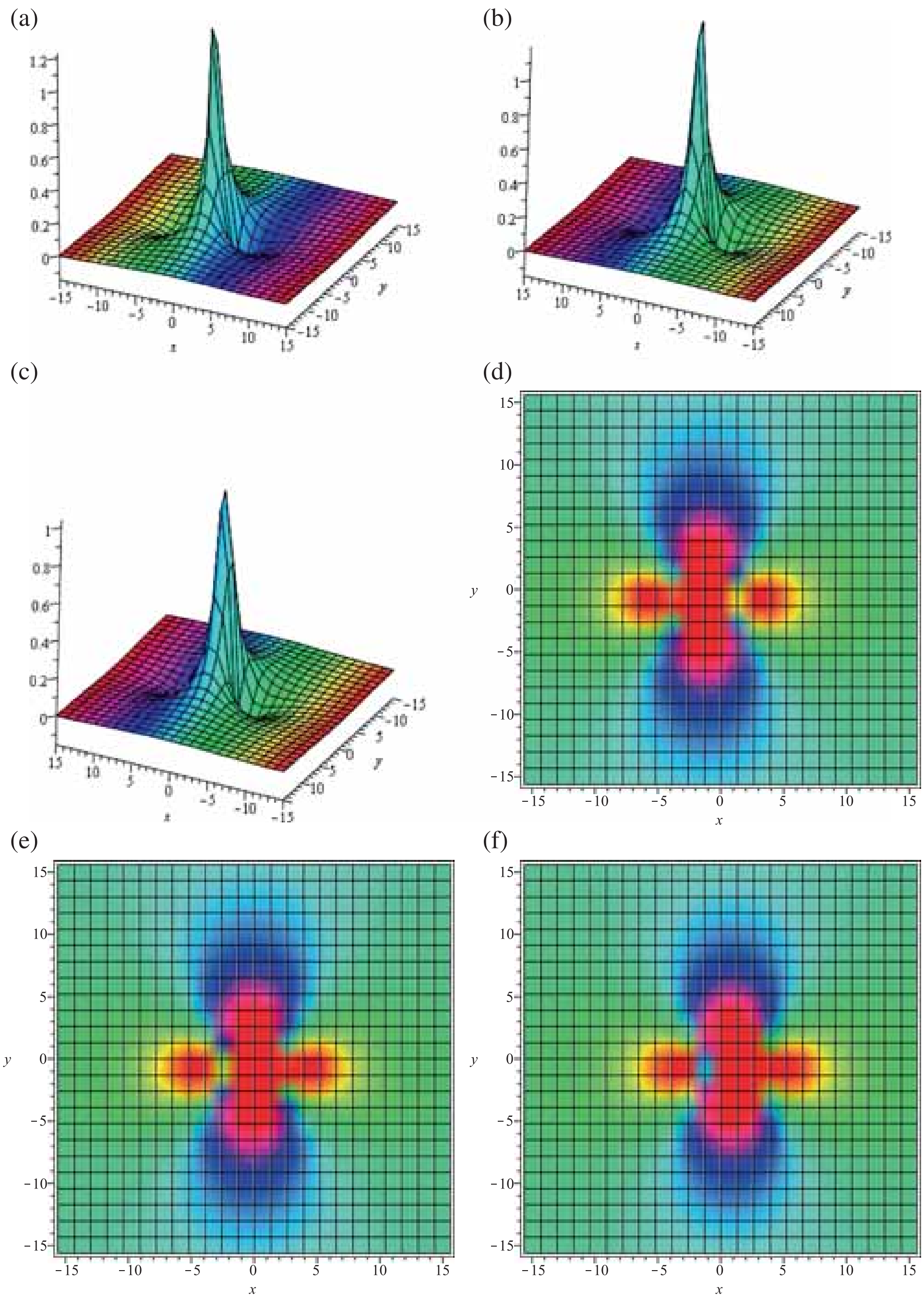

Figure 1. Characteristic form of the lump solution (28) with parameters $a_1 = a_6 = 0.8, a_2 = a_4 = 0.6, a_5 = -0.6$, $a_5 = -0.8, a_8 = 0.5$. (**a**) and (**d**) $t = -1$, (**b**) and (**e**) $t = 0$ and (**c**) and (**f**) $t = 1$. (**a**)–(**b**) 3D plot and (**d**)–(**f**) density plot.

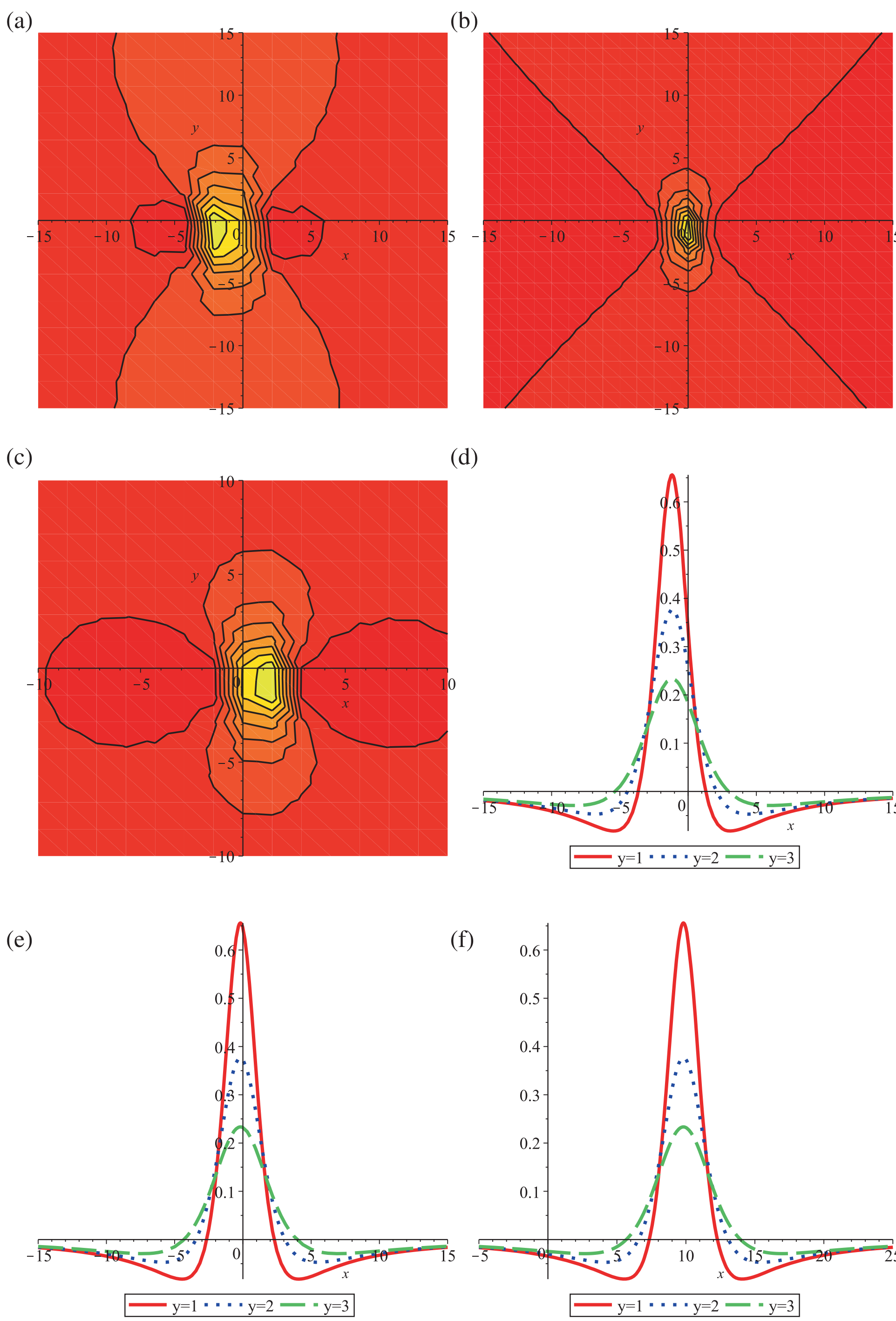

Figure 2. Characteristic form of the lump solution (28) with parameters $a_1 = a_6 = 0.8, a_2 = a_4 = 0.6, a_5 = -0.6$, $a_5 = -0.8, a_8 = 0.5$. (**a**) and (**e**) $t = -1$, (**b**) and (**f**) $t = 0$ and (**c**) and (**d**) $t = 1$. (**a**)–(**c**) Contour plot and (**d**)–(**f**) 2D plot.

4. Kinky breather-soliton solution

In this section, we search for the periodic kink-wave solution to eq. (6) by using the generalised Hirota bilinear method. We suppose that

$$f = \mathrm{e}^{-\lambda g} + b_1 \mathrm{e}^{\lambda g} + b_0 \cos(\gamma h) + a_9,$$

$$g = a_1 x + a_2 y + a_3 t + a_4,$$

$$h = a_5 x + a_6 y + a_7 t + a_8, \quad (31)$$

where $a_i, i = 1, \ldots, 8$ and b_1 are free values to be determined later. Substituting (31) into eq. (10) and then collecting the coefficients in front of different exponential and trigonometric functions, we get

Set I:

$$\begin{cases} a_1 = a_9 = 0, \quad a_2 = a_2, \quad a_3 = \dfrac{2a_2a_6}{a_5}, \quad a_4 = a_4, \\ a_5 = a_5, \quad a_6 = a_6, \quad a_7 = \dfrac{\gamma^2 a_6^2 - \lambda^2 a_2^2}{\gamma^2 a_5}, \\ a_8 = a_8, \quad b_0 = b_0, \quad b_1 = \dfrac{1}{4}b_0^2. \end{cases} \tag{32}$$

Submitting (32) into (31), we get a periodic kink-wave solution of eq. (6) as follows:

$$u = \frac{-2b_0\gamma^2 a_5^2 \cos(\gamma h)}{f} - \frac{8b_0^2\gamma^2 a_5^2 \sin^2(\gamma h)}{f^2}, \tag{33}$$

$$g = a_2 y + \frac{2a_2a_6}{a_5}t + a_4,$$

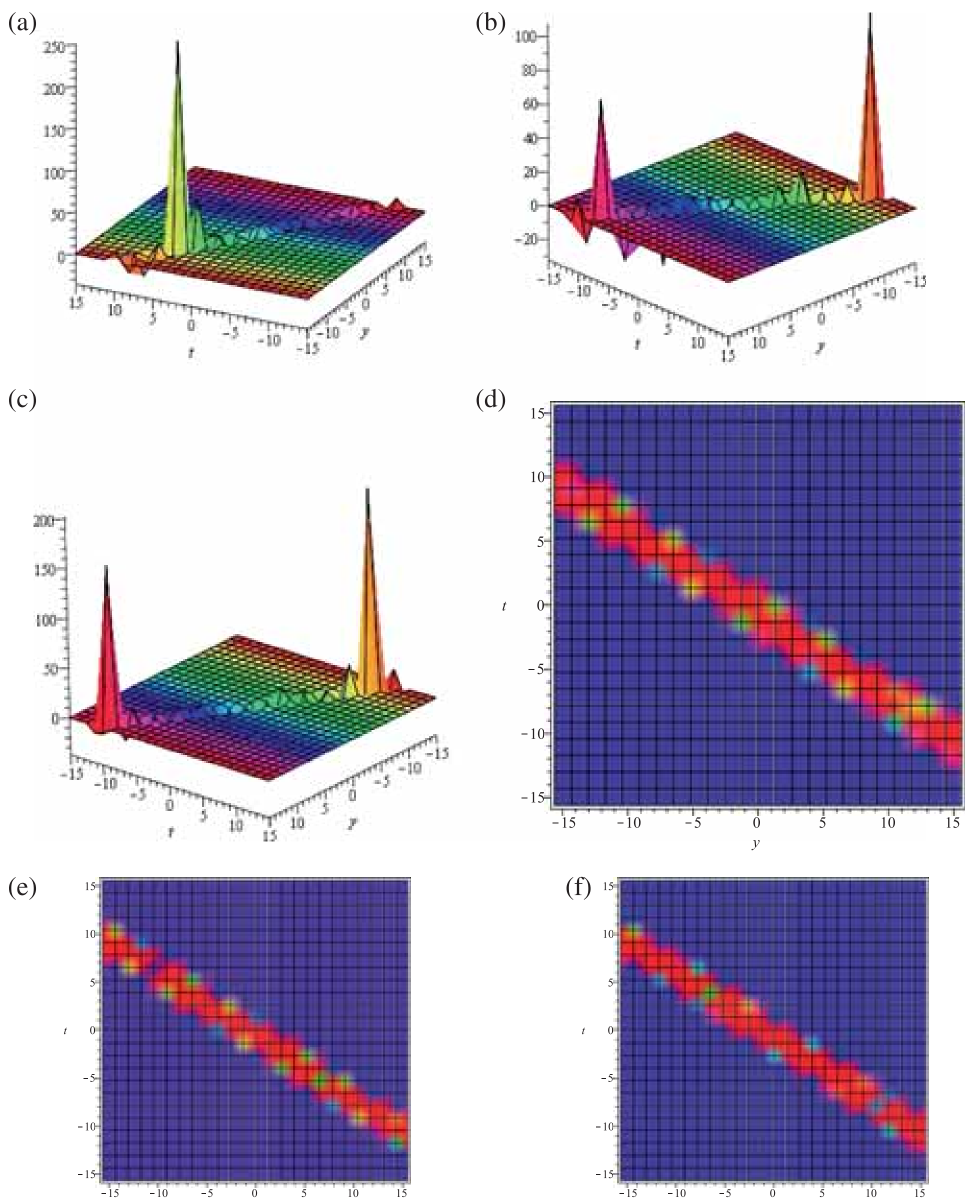

Figure 3. Characteristic form of the periodic kink-wave solution (33) with parameters $a_2 = a_4 = a_8 = 1$, $\lambda = \gamma = b_0 = a_5 = 2, a_6 = 1.5$. (**a**) and (**b**) $x = -1$, (**b**) and (**e**) $x = 0$ and (**c**) and (**f**) $x = 1$. (**a**)–(**c**) 3D plot and (**d**)–(**f**) density plot.

$$h = a_5 x + a_6 y + \frac{\gamma^2 a_6^2 - \lambda^2 a_2^2}{\gamma^2 a_5} t + a_8, \tag{34}$$

where $a_2, a_4, a_5\ (\neq 0), a_6, a_8$ are arbitrary constants. The exact solution (33) is a periodic kink-wave solution.

Now, solution (33) is plotted in 3D, density, contour and 2D forms (see figures 3 and 4).

5. Solitary wave solution

Finally, in this section, we search for the solitary wave solutions to eq. (6) by using the generalised Hirota

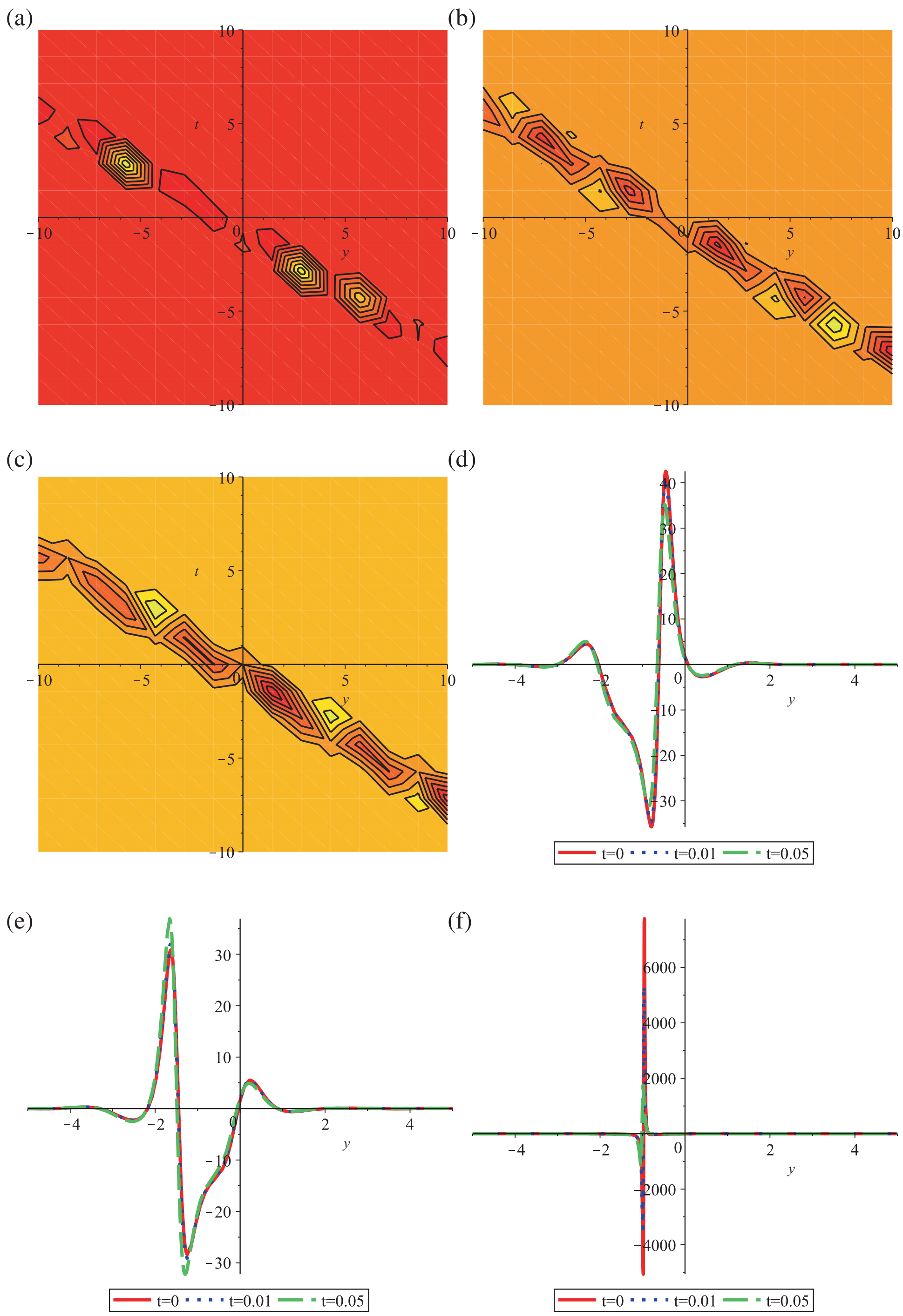

Figure 4. Characteristic form of the periodic kink-wave solution (33) with parameters $a_2 = a_4 = a_8 = 1$, $\lambda = \gamma = b_0 = a_5 = 2$, $a_6 = 1.5$. (**a**) and (**d**) $x = -1$, (**b**) and (**e**) $x = 0$ and (**c**) and (**f**) $x = 1$. (**a**)–(**c**) Contour plot and (**d**)–(**f**) 2D plot.

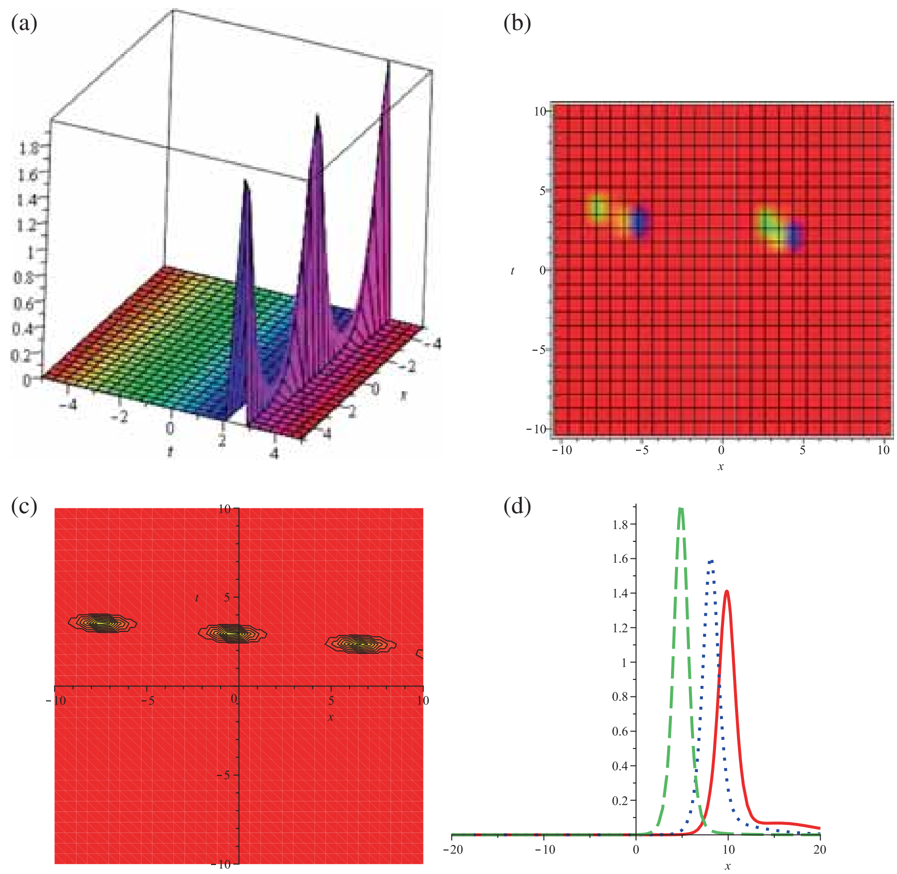

Figure 5. Characteristic form of the solitary wave solution (37) with parameters $k_2 = 2$, $k_3 = 3$, $a_2 = 2\sqrt{6}$, $a_4 = 1$, $b_4 = 2$, $c_4 = 5$, $y = 10$. (**a**) 3D plot, (**b**) density plot, (**c**) contour plot and (**d**) 2D plot.

bilinear method. We assume that

$$\begin{aligned} f &= k_1 \exp(\theta_1) + \exp(-\theta_1) + k_2 \sinh(\theta_2) + k_3 \cosh(\theta_3), \\ \theta_1 &= a_1 x + a_2 y + a_3 t + a_4, \\ \theta_2 &= b_1 x + b_2 y + b_3 t + b_4, \\ \theta_3 &= c_1 x + c_2 y + c_3 t + c_4, \end{aligned} \tag{35}$$

where $a_i, b_i, c_i, i = 1, \ldots, 4$, are free values to be determined later. Substituting (35) into eq. (10) and then collecting the coefficients in front of different hyperbolic functions, we get

Set I:

$$\begin{cases} a_1 = -c_1, \quad a_2 = a_2, \quad a_3 = -\dfrac{a_2^2 - 3c_1^4}{c_1}, \quad a_4 = a_4, \\ b_1 = \sqrt{\dfrac{2}{5}} c_1, \quad b_2 = -\dfrac{\sqrt{6}a_2^2 - 3c_1^2}{\sqrt{15}}, \\ b_3 = \dfrac{(\sqrt{6}a_2^2 - 6c_1^2 a_2 + 3\sqrt{6}c_1^4}{\sqrt{15}c_1)}, \quad c_2 = -a_2 + \sqrt{6}c_1^2, \\ c_3 = \dfrac{a_2^2 - 2\sqrt{6}a_2 c_1^2 + 3c_1^4}{c_1}, \quad c_4 = c_4, \\ k_1 = -\dfrac{1}{300} \dfrac{75k_3^2c_1^4 + 9c_1^4k_2^2 - 10a_2^2k_2^2 + 10\sqrt{6}a_2k_2^2c_1^2}{c_1^4}, \\ k_2 = k_2, \quad k_3 = k_3. \end{cases} \tag{36}$$

Submitting (36) into (35), we get a solitary wave solution of eq. (6) as follows:

$$\begin{aligned} u = {} & \frac{2(a_1^2 k_1 \exp(\theta_1) + a_1^2 \exp(-\theta_1) + k_2 b_1^2 \sinh(\theta_2) + k_3 c_1^2 \cosh(\theta_3))}{f} \\ & - \frac{2(a_1 k_1 \exp(\theta_1) - a_1 \exp(-\theta_1) + k_2 b_1 \sinh(\theta_2) + k_3 c_1 \cosh(\theta_3))^2}{f^2}, \end{aligned}$$

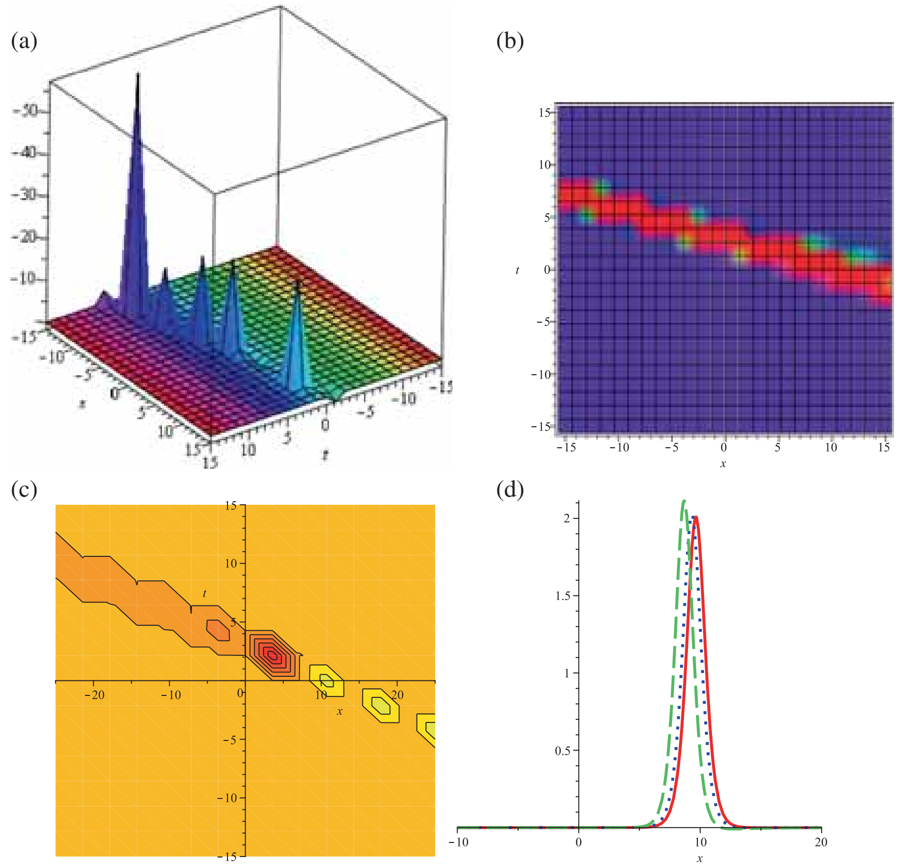

Figure 6. Characteristic form of the solitary wave solution (39) with parameters $k_2 = 2$, $k_3 = -2$, $a_2 = 1$, $a_4 = 1$, $b_4 = 1$, $c_1 = 1$, $c_4 = 1$, $y = -5$. (**a**) 3D plot, (**b**) density plot, (**c**) contour plot and (**d**) 2D plot.

$$f = -\frac{1}{300}\frac{75k_3^2c_1^4 + 9c_1^4k_2^2 - 10a_2^2k_2^2 + 10\sqrt{6}a_2k_2^2c_1^2}{c_1^4} \times \exp(\theta_1)+\exp(-\theta_1)+k_2\sinh(\theta_2)+k_3\cosh(\theta_3),$$

$$\theta_1 = -c_1x + a_2y - \frac{a_2^2 - 3c_1^4}{c_1}t + a_4,$$

$$\theta_2 = \sqrt{\frac{2}{5}}c_1x - \frac{\sqrt{6}a_2^2 - 6c_1^2a_2 - 3\sqrt{6}c_1^4}{\sqrt{15}c_1}y + b_3t + b_4,$$

$$\theta_3 = c_1x + (-a_2 + \sqrt{6}c_1^2)y + \frac{a_2^2 - 2\sqrt{6}a_2c_1^2 + 3c_1^4}{c_1}t + c_4, \tag{37}$$

where $c_1(\neq 0)$, a_2, a_4, c_4, k_2, k_3 are arbitrary constants.

Set II:

$$\left\{\begin{aligned} &a_1 = c_1,\quad a_2 = a_2,\quad a_3 = \frac{a_2^2 - 3c_1^4}{c_1},\quad a_4 = a_4,\\ &b_1 = \sqrt{\frac{2}{5}}c_1,\quad b_2 = -\frac{-2a_2^2 + \sqrt{6}c_1^2a_2 + 3c_1^4}{\sqrt{15}c_1^2},\\ &b_3 = \frac{(\sqrt{6}a_2^2 + 6c_1^2a_2 + 3\sqrt{6}c_1^4}{\sqrt{15}c_1)},\quad c_2 = a_2 + \sqrt{6}c_1^2,\\ &c_3 = \frac{a_2^2 + 2\sqrt{6}a_2c_1^2 + 3c_1^4}{c_1},\quad c_4 = c_4,\\ &k_1 = -\frac{1}{300}\frac{75k_3^2c_1^4+9c_1^4k_2^2-10a_2^2k_2^2-10\sqrt{6}a_2k_2^2c_1^2}{c_1^4},\\ &k_2 = k_2,\quad k_3 = k_3. \end{aligned}\right. \tag{38}$$

Submitting (38) into (35), we get a solitary wave solution of eq. (6) as follows:

$$u = \frac{2(a_1^2k_1\exp(\theta_1) + a_1^2\exp(-\theta_1) + k_2b_1^2\sinh(\theta_2) + k_3c_1^2\cosh(\theta_3))}{f} - \frac{2(a_1k_1\exp(\theta_1) - a_1\exp(-\theta_1) + k_2b_1\sinh(\theta_2) + k_3c_1\cosh(\theta_3))^2}{f^2},$$

$$f=-\frac{1}{300}\frac{75k_3^2c_1^4+9c_1^4k_2^2-10a_2^2k_2^2-10\sqrt{6}a_2k_2^2c_1^2}{c_1^4}$$
$$\times\exp(\theta_1)+\exp(-\theta_1)+k_2\sinh(\theta_2)+k_3\cosh(\theta_3),$$
$$\theta_1=c_1x+a_2y+\frac{a_2^2-3c_1^4}{c_1}t+a_4,$$
$$\theta_2=\sqrt{\frac{2}{5}}c_1x-\frac{-2a_2^2+\sqrt{6}c_1^2a_2+3c_1^4}{\sqrt{15}c_1^2}y+b_3t+b_4,$$
$$\theta_3=c_1x+(a_2+\sqrt{6}c_1^2)y+\frac{a_2^2+2\sqrt{6}a_2c_1^2+3c_1^4}{c_1}t$$
$$+c_4, \tag{39}$$

where c_1 ($\neq 0$), a_2, a_4, c_4, k_2, k_3 are arbitrary constants. Assuming that the relations including ($k_2, k_3 \neq 0$) are satisfied, then the solutions of (37) and (39) will be like the solitary wave solutions. Now, solutions (37) and (39) are plotted in 3D, density, contour and 2D forms (see figures 5 and 6).

6. Conclusion

With the help of symbolic calculation and applying the generalised Hirota bilinear method, we have found some new exact solutions for the KP-like equation with $p = 3$. As a result, some new solutions, which include the lump, periodic kink-wave and solitary wave solutions, were obtained. We thus believe that these results will help in conducting future research in various areas of physics such as mathematical physics, nonlinear mechanics and other applied fields.

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