Pramana – J. Phys. (2019) 93:1
https://doi.org/10.1007/s12043-019-1748-9

Adaptive synchronisation of complex networks with non-dissipatively coupled and uncertain inner coupling matrix

SHUGUO WANG

Department of Mathematics and Physics, Changzhou Campus, Hohai University, Changzhou, Jiangsu 213022, People's Republic of China
E-mail: wsg97@163.com

MS received 19 June 2018; revised 9 September 2018; accepted 13 November 2018; published online 16 April 2019

Abstract. In this paper, the adaptive synchronisation of time-varying perturbed complex networks with non-dissipatively coupled and uncertain inner coupling matrix is studied. In order to describe the actual network better, the out-coupling configuration matrix is not limited by the dissipatively coupled conditions. It is also worth pointing out that the drive system and the response system described in this paper are uncertain, and uncertainty arises in linear inner coupling matrix and unavoidable uncertain external disturbances, which is different from the past. On the basis of Lyapunov stability theory, adaptive law can be obtained and at the same time unknown bounded disturbances can be overcome.

Keywords. Synchronisation; time-varying delays; non-dissipatively coupled; uncertain complex.

PACS Nos **05.45.Xt; 05.45.Gg; 05.45.Pq**

1. Introduction

In the past several years, a lot of control schemes including pinning techniques, adaptive techniques, backstepping methods, impulsive control, sliding mode techniques, etc. are widely adopted to realise the synchronisation of complex dynamic networks [1–10]. Various types of synchronisation have been put forward in the existing literatures [11–18], which can be summarised as cluster synchronisation, exponential synchronisation, lag synchronisation, projective synchronisation, complete synchronisation, outer synchronisation, generalised synchronisation, etc.

Although many synchronisation criteria have been proposed in the previous studies, one of the basic assumptions adopted by them is that the nodes in the complex dynamical network are dissipative coupled [11–24]. Sivaranjani *et al* [11] proposed the synchronisation problem of nonlinear singularly perturbed complex networks with dissipative coupling and time-varying coupling delays. They studied the synchronisation of fractional-order neural networks with multiple time delays based on the inequality of fractional-order and comparison principles of linear fractional equation with multiple time delays, and some sufficient conditions for synchronisation of master–slave systems were obtained in [12]. A novel finite-time analysis was given to investigate the global projective synchronisation on coloured networks with dissipative coupling by Cai *et al* [13], and some less conservative conditions were derived in [13]. Jiang and Lu [14] studied the problem concerning the synchronisation of coloured delayed networks with dissipative coupling under decentralised pinning intermittent control. Lei *et al* [15] investigated the problem of aperiodically intermittent pinning control synchronisation of two dissipatively coupled complex dynamical networks. Zheng [16] investigated the pinning and impulsive synchronisation issue of two derivative coupled complex networks with non-derivative systems, and some new standards of synchronisation were derived. Due to the problem of function projective synchronisation of asymmetric-coupled complex networks, a new function projection synchronisation criterion was proposed in [17]. Wang *et al* [18] studied the problem concerning projective lag synchronisation of dynamical complex networks, and the dynamical network was a delay coupled complex network with uncertainty and time-varying coupling strength. Li *et al* [19] obtained outer synchronisation condition of uncertain networks with different node number, and some new synchronisation criteria were proposed. In [20], the problem concerning synchronisation of asymmetric coupled complex networks with time delay was studied, and sufficient conditions were obtained for synchronisation by decomposing the asymmetric matrix method. The finite-time lag synchronisation problem of the master–slave complex networks with unknown signal propagation delays through the linear and adaptive error state feedback methods was studied in [21]. Wu and Leng [22] studied a generalisation of hybrid projective synchronisation in dynamical networks, and several sufficient conditions were derived to achieve synchronisation. The impulsive pinning synchronisation problem concerning a class of time-varying delay complex dynamical networks with dissipative coupling was considered in [23]. Ma and Wang [24] studied the problem concerning the outer synchronisation of nonlinear coupling networks, both inner delay and coupling delay were included in the coupled network model, and some pinning synchronisation standards were deduced. However, the dissipative coupled complex networks are not always consistent with the real network. Therefore, it is very important to study the synchronisation of complex networks with non-dissipative coupling. Lei *et al* [25] investigated the matrix projective outer synchronisation of two complex networks with non-dissipative coupling by using open-plus-closed-loop controllers. The problem of generalised outer synchronisation of two complex dynamical networks with time-varying delays and non-dissipative coupling was proposed in [26]. However, uncertain inner coupling matrix of the complex networks and uncertain external disturbances are not considered in [25,26].

On the basis of the above discussion, synchronisation schemes for uncertain complex networks with time-varying delay dynamical nodes and non-dissipatively coupled configuration matrix were explored in this study. Compared with the existing literatures, the main contributions of this paper are as follows: (i) a general adaptive synchronisation scheme for uncertain complex networks with non-dissipative coupling and time-varying delay dynamical nodes was proposed. The outer coupling matrices of the drive network and the response network are not constrained by the dissipatively coupled condition. (ii) In this paper, the uncertain complex networks that can simulate more realistic networks, the coupling delay and external disturbances, and also the uncertainty that arises in inner coupling matrix were studied. Therefore, it is of great significance to study uncertain complex networks. (iii) The Lyapunov–Krasovskii functional of the network was constructed and some new synchronisation criteria are obtained, so that there was no need to solve the Jacobian matrix of the state equations on the network nodes. In addition, unknown bounded disturbances can also be conquered at the same time by using updated laws in the synchronisation process. All these unique features show that this solution is practical and effective. This study undoubtedly provides new insights for the actual engineering design compared to the previous literatures [25,26].

2. Preliminaries

Consider a non-dissipatively coupled complex network consisting of N identical nodes:

$$\dot{x}_i(t) = f_i(t, x_i(t), x_i(t-\tau_1(t))) + \sum_{j=1, j\neq i}^{N} b_{ij}(\Gamma + \Delta\Gamma(t))(x_j(t-\tau_2) - x_i(t-\tau_3)), \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\mathrm{T}} \in R^n$ is their state vector and $f : R^+ \times R^n \times R^n \to R^n$ are continuous nonlinear vector-valued functions. $\tau_1(t) > 0$ is the node time-varying delay. $\tau_2, \tau_3 > 0$ are unknown coupling delays. $\tau_1(t)$, τ_2 and τ_3 are different and independent of each other. $\Gamma = \mathrm{diag}(1, 1, \ldots, 1)$ is the inner coupling matrix. $\Delta\Gamma(t)$ is the uncertainty of the inner coupling matrix. Denote $B = (b_{ij})_{N\times N} \in R^{N\times N}$ as the outer coupling configuration matrix representing the topological structure and the coupling strength of the network. b_{ij} is defined as follows: if there is a connection from node i to node j $(j \neq i)$, then $b_{ij} \neq 0$. Otherwise, $b_{ij} = 0\,(j \neq i)$. The outer coupling matrix $B = (b_{ij})_{N\times N} \in R^{N\times N}$ may not be zero-row sum, i.e. the diagonal entries b_{ii} may be random scalar generated, and all coupling coefficients $b_{ij}\,(i, j = 1, 2, \ldots, N)$ are bounded. The initial values of network (1) are given by $x_i(t) = \varphi_i(t) \in C([-\tau, 0], R^n)$.

To achieve synchronisation of two complex networks, model (1) is called the drive network and the response network is written as

$$\dot{y}_i(t) = f_i(t, y_i(t), y_i(t-\tau_1(t))) + \sum_{j=1, j\neq i}^{N} b_{ij}(\Gamma+\Delta\Gamma(t))(y_j(t-\tau_2)-y_i(t-\tau_3)) + \Theta_i(t) + u_i(t), \tag{2}$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \ldots, y_{in}(t))^{\mathrm{T}} \in R^n$ is the state vector of the ith node, $u_i(t)$ is its control input. $\Theta_i(t)$ is the disturbance. The initial values of network (2) are given by $y_i(t) = \phi_i(t) \in C([-\tau, 0], R^n)$ and other notations are the same as above.

Remark 1. In this paper, a general adaptive synchronisation scheme for uncertain complex networks with non-dissipative coupling and time-varying delay dynamical nodes is proposed. The outer coupling matrices of the drive network and the response network are not constrained by the dissipatively coupled condition, i.e. the diagonal entries b_{ii} may be random scalar generated.

DEFINITION 1

The drive networks (1) and response networks (2) are said to achieve synchronisation if $\lim_{t\to\infty}\|y_i(t)-x_i(t)\|=0,\ i=1,2,\ldots,N$.

Let the error term be

$$e_i(t)=y_i(t)-x_i(t),\quad i=1,2,\ldots,N. \tag{3}$$

Assumption 1. The disturbances $\Theta_i(t)$ is bounded, i.e., $\|\Theta_i(t)\|\le\varepsilon_i$, where ε_i are any positive constants.

Assumption 2. The uncertain function matrix $\Delta\Gamma(t)$ can be given by $\Delta\Gamma(t)=F\Phi(t)E$, where F and E are known constant matrices with an appropriate dimension, and the uncertain matrix $\Phi(t)$ satisfies $(\Phi(t))^{\mathrm{T}}\Phi(t)\le I$.

Assumption 3. There exists a constant $\delta\in R^{+}$ such that $|b_{ij}|<\delta_1,\ (i,j=1,2,\ldots,N)$.

Assumption 4. $\tau(t)$ is a differential function which satisfies $0\le\dot{\tau}(t)\le\varepsilon\le1$, where ε is constant.

Lemma 1. *$Q\in R^{n\times n}$ is a positive definite matrix, and the following matrix inequality holds*: $2x^{\mathrm{T}}y\le x^{\mathrm{T}}Qx+y^{\mathrm{T}}Q^{-1}y$, *for any vectors* $x,y\in R^n$.

Lemma 2 [21]. *For the vector-valued function* $f(t,x_i(t),x_i(t-\tau(t)))$, *assuming that there exist positive constants* $\gamma>0,\theta>0$ *such that f satisfies the semi-Lipschitz condition*:

$$\begin{aligned}&(x_i(t)-y_i(t))^{\mathrm{T}}(f(t,x_i(t),x_i(t-\tau(t)))\\&\quad-f(t,y_i(t),y_i(t-\tau(t))))\\&\quad\le\gamma(x_i(t)-y_i(t))^{\mathrm{T}}(x_i(t)-y_i(t))+\theta(x_i(t-\tau(t))\\&\quad-y_i(t-\tau(t)))^{\mathrm{T}}(x_i(t-\tau(t))-y_i(t-\tau(t))),\end{aligned}$$

for all $x,y\in R^n$ *and* $\tau(t)\ge0,i=1,2,\ldots,N$.

3. Synchronisation schemes

In this section, a method for the synchronisation of the drive dynamical network (1) and the response dynamical network (2) is proposed. The error dynamical system can be derived as

$$\begin{aligned}\dot{e}_i(t)&=\tilde{f}(t,e_i(t),e_i(t-\tau_1(t)))\\&\quad+\sum_{j=1,j\ne i}^{N}b_{ij}(\Gamma+\Delta\Gamma(t))(e_j(t-\tau_2)\\&\quad-e_i(t-\tau_3))+\Theta_i(t)+u_i(t),\end{aligned}\tag{4}$$

where

$$\begin{aligned}&\tilde{f}(t,e_i(t),e_i(t-\tau_1(t)))\\&\quad=f(t,y_i(t),y_i(t-\tau_1(t)))\\&\quad-f(t,x_i(t),x_i(t-\tau_1(t))).\end{aligned}$$

Theorem 1. *If Assumptions* 1–4 *hold*, *the drive networks* (1) *and the response networks* (2) *can realise synchronisation via the control law as shown below*:

$$\begin{aligned}u_i(t)&=-q_ie_i(t)-\beta_i(t)\mathrm{sgn}(e_i(t))\\&\quad+\sum_{j=1}^{N}b_{ij}(\Delta\Gamma(t))e_i(t-\tau_3),\end{aligned}\tag{5}$$

$$\dot{q}_i=k_1e_i^{\mathrm{T}}(t)e_i(t),\tag{6}$$

$$\dot{\beta}_i(t)=k_2e_i^{\mathrm{T}}(t)\mathrm{sgn}(e_i(t)),\tag{7}$$

where k_1 and k_2 are positive constants, $\tilde{\beta}_i(t)=\beta_i(t)-\beta_i^*$, β_i^* *are the positive constants designed later. By taking appropriate q^* and β_i^*, such that the following inequalities hold*

$$\begin{aligned}&\gamma+\frac{\theta}{(1-\varepsilon)}+2\delta N+\frac{1}{2}\lambda_{\max}^{(P)}\\&\quad+\frac{1}{2}\lambda_{\max}^{(B,E)}-q^*<0,\end{aligned}\tag{8}$$

$$\varepsilon_i-\beta_i^*<0,\quad\text{for } i=1,2,\ldots,N,\tag{9}$$

where $P=(I_N\otimes FF^{\mathrm{T}})$, $\lambda_{\max}^{(B,E)}=\lambda_{\max}(B\otimes E)^{\mathrm{T}}(B\otimes E)$, *then the drive networks* (1) *and the response networks* (2) *can achieve synchronisation.*

Proof. Choose the following Lyapunov function candidate:

$$\begin{aligned}V(t)&=\frac{1}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)\\&\quad+\frac{\theta}{(1-\varepsilon)}\sum_{i=1}^{N}\int_{t-\tau_1(t)}^{t}e_i^{\mathrm{T}}(\xi)e_i(\xi)\,\mathrm{d}\xi\\&\quad+\frac{\delta N}{2}\sum_{i=1}^{N}\int_{t-\tau_3}^{t}e_i^{\mathrm{T}}(\mu)e_i(\mu)\,\mathrm{d}\mu\\&\quad+\frac{1}{2}\sum_{i=1}^{N}\frac{1}{k_1}(q_i-q^*)^2\end{aligned}$$

$$+\left(\frac{\delta N}{2}+\frac{1}{2}\lambda_{\max}^{(B,E)}\right)\sum_{i=1}^{N}\int_{t-\tau_2}^{t} e_i^{\mathrm{T}}(\zeta)e_i(\zeta)\,\mathrm{d}\zeta$$
$$+\frac{1}{2k_2}\sum_{i=1}^{N}\tilde{\beta}_i^2(t). \tag{10}$$

By differentiating the function $V(t)$ along the trajectories of the error the dynamics (3), the following equation is obtained:

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)(\tilde{f}(t,e_i(t),e_i(t-\tau_1(t)))\\
&\quad+\sum_{j=1}^{N} b_{ij}(\Gamma+\Delta\Gamma(t))(e_j(t-\tau_2)-e_i(t-\tau_3))\\
&\quad+\Theta_i(t)+u_i(t))+\frac{\theta}{(1-\varepsilon)}\sum_{i=1}^{N}(e_i^{\mathrm{T}}(t)e_i(t)\\
&\quad-(1-\dot{\tau}_1(t))e_i^{\mathrm{T}}(t-\tau_1(t))e_i(t-\tau_1(t)))\\
&\quad+\sum_{i=1}^{N}\frac{1}{k_1}(q_i-q^*)\dot{q}_i+\frac{1}{k_2}\sum_{i=1}^{N}\dot{\beta}_i(t)\tilde{\beta}_i(t)\\
&\quad+\frac{\delta N}{2}\sum_{i=1}^{N}(e_i^{\mathrm{T}}(t)e_i(t)-e_i^{\mathrm{T}}(t-\tau_3)e_i(t-\tau_3))\\
&\quad+\left(\frac{\delta N}{2}+\frac{1}{2}\lambda_{\max}^{(B,E)}\right)\sum_{i=1}^{N}(e_i^{\mathrm{T}}(t)e_i(t)\\
&\quad-e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2))\\
&\le \sum_{i=1}^{N}\left[\gamma e_i^{\mathrm{T}}(t)e_i(t)+\theta e_i^{\mathrm{T}}(t-\tau_1(t))e_i(t-\tau_1(t))\right]\\
&\quad+\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\Theta_i(t)+\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)u_i(t)\\
&\quad+\frac{\theta}{(1-\varepsilon)}\sum_{i=1}^{N}(e_i^{\mathrm{T}}(t)e_i(t)\\
&\quad-(1-\dot{\tau}_1(t))e_i^{\mathrm{T}}(t-\tau_1(t))e_i(t-\tau_1(t)))\\
&\quad+\frac{1}{k_2}\sum_{i=1}^{N}\dot{\beta}_i(t)\tilde{\beta}_i(t)+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}(\Gamma\\
&\quad+\Delta\Gamma(t))(e_j(t-\tau_2)-e_i(t-\tau_3))\\
&\quad+\sum_{i=1}^{N}(q_i-q^*)e_i^{\mathrm{T}}(t)e_i(t)\\
&\quad+\left(\frac{\delta N}{2}+\frac{1}{2}\lambda_{\max}^{(B,E)}\right)\sum_{i=1}^{N}(e_i^{\mathrm{T}}(t)e_i(t)\\
&\quad-e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2))\\
&\quad+\frac{\delta N}{2}\sum_{i=1}^{N}(e_i^{\mathrm{T}}(t)e_i(t)-e_i^{\mathrm{T}}(t-\tau_3)e_i(t-\tau_3))\\
&\le \sum_{i=1}^{N}\gamma e_i^{\mathrm{T}}(t)e_i(t)\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}(\Gamma+\Delta\Gamma(t))(e_j(t-\tau_2)\\
&\quad-e_i(t-\tau_3))+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\Theta_i(t)\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)(-q_ie_i(t)-\beta_i(t)\mathrm{sgn}(e_i(t)))\\
&\quad+\frac{\theta}{(1-\varepsilon)}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)+\sum_{i=1}^{N}q_ie_i^{\mathrm{T}}(t)e_i(t)\\
&\quad-\sum_{i=1}^{N}q^*e_i^{\mathrm{T}}(t)e_i(t)+\frac{1}{k_2}\sum_{i=1}^{N}\dot{\beta}_i(t)\tilde{\beta}_i(t)\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}(\Delta\Gamma(t))e_i(t-\tau_3)\\
&\quad+\left(\frac{\delta N}{2}+\frac{1}{2}\lambda_{\max}^{(B,E)}\right)\sum_{i=1}^{N}(e_i^{\mathrm{T}}(t)e_i(t)\\
&\quad-e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2))\\
&\quad+\frac{\delta N}{2}\sum_{i=1}^{N}(e_i^{\mathrm{T}}(t)e_i(t)-e_i^{\mathrm{T}}(t-\tau_3)e_i(t-\tau_3))\\
&\le \sum_{i=1}^{N}\left(\gamma+\frac{\theta}{(1-\varepsilon)}+\delta N+\frac{1}{2}\lambda_{\max}^{(B,E)}-q^*\right)\\
&\quad\times e_i^{\mathrm{T}}(t)e_i(t)-\sum_{i=1}^{N}\beta_i^*e_i^{\mathrm{T}}(t)\mathrm{sgn}(e_i(t))\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\Theta_i(t)+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}(\Gamma+\Delta\Gamma(t))\\
&\quad\times(e_j(t-\tau_2)-e_i(t-\tau_3))\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}(\Delta\Gamma(t))e_i(t-\tau_3)\\
&\quad-\frac{\delta N}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau_3)e_i(t-\tau_3)\\
&\quad-\left(\frac{\delta N}{2}+\frac{1}{2}\lambda_{\max}^{(B,E)}\right)\frac{\delta N}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2)
\end{aligned}$$

$$
\begin{aligned}
&\le \sum_{i=1}^{N}\left(\gamma+\frac{\theta}{(1-\varepsilon)}+\delta N+\frac{1}{2}\lambda_{\max}^{(B,E)}-q^{*}\right)e_i^{\mathrm{T}}(t)e_i(t)\\
&+\sum_{i=1}^{N}[\varepsilon_i-\beta_i^{*}]\,\|e_i(t)\|\\
&+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}(\Gamma+\Delta\Gamma(t))(e_j(t-\tau_2)\\
&-e_i(t-\tau_3))-\frac{\delta N}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau_3)e_i(t-\tau_3)\\
&-\left(\frac{\delta N}{2}+\frac{1}{2}\lambda_{\max}^{(B,E)}\right)\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2)\\
&+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}(\Delta\Gamma(t))e_i(t-\tau_3). \qquad (11)
\end{aligned}
$$

According to Lemma 1 and Assumption 3,

$$
\begin{aligned}
&\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}(\Gamma+\Delta\Gamma(t))(e_j(t-\tau_2)-e_i(t-\tau_3))\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}(\Delta\Gamma(t))e_i(t-\tau_3)\\
&=\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Gamma(e_j(t-\tau_2)-e_i(t-\tau_3))\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Delta\Gamma(t)e_j(t-\tau_2)\\
&\le\sum_{i=1}^{N}\sum_{j=1}^{N}|b_{ij}|\,\|e_i^{\mathrm{T}}(t)\|\,\|\Gamma\|\,\|e_j(t-\tau_2)-e_i(t-\tau_3)\|\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Delta\Gamma(t)e_j(t-\tau_2)\\
&\le\delta\sum_{i=1}^{N}\sum_{j=1}^{N}\|e_i^{\mathrm{T}}(t)\|\,\|e_j(t-\tau_2)-e_i(t-\tau_3)\|\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Delta\Gamma(t)e_j(t-\tau_2)\\
&\le\delta\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_i^{\mathrm{T}}(t)e_j(t-\tau_2)\right|+\left|e_i^{\mathrm{T}}(t)e_i(t-\tau_3)\right|\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Delta\Gamma(t)e_j(t-\tau_2)\\
&\le\delta\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{e_i^{\mathrm{T}}(t)e_i(t)+e_j^{\mathrm{T}}(t-\tau_2)e_j(t-\tau_2)}{2}\right.\\
&\quad\left.+\frac{e_i^{\mathrm{T}}(t)e_i(t)+e_j^{\mathrm{T}}(t-\tau_3)e_j(t-\tau_3)}{2}\right)\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Delta\Gamma(t)e_j(t-\tau_2)\\
&=\delta N\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)\\
&\quad+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Delta\Gamma(t)e_j(t-\tau_2)\\
&\quad+\frac{\delta N}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2)\\
&\quad+\frac{\delta N}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau_3)e_i(t-\tau_3). \qquad (12)
\end{aligned}
$$

According to Lemma 1 and Assumption 2,

$$
\begin{aligned}
&\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Delta\Gamma e_j(t-\tau_2)\\
&=\sum_{i=1}^{N}\sum_{j=1}^{N}e_i^{\mathrm{T}}(t)b_{ij}\Delta\Gamma e_j(t-\tau_2)\\
&=e^{\mathrm{T}}(t)(B\otimes\Delta\Gamma)e(t-\tau_2)\\
&=e^{\mathrm{T}}(t)(B\otimes F\Phi(t)E)e(t-\tau_2)\\
&=e^{\mathrm{T}}(t)(I_N\otimes F)(B\otimes\Phi(t))(I_N\otimes E)e(t-\tau_2)\\
&\le\frac{1}{2}e^{\mathrm{T}}(t)(I_N\otimes F)(I_N\otimes F^{\mathrm{T}})e(t)\\
&\quad+\frac{1}{2}e^{\mathrm{T}}(t-\tau_2)(I_N\otimes E^{\mathrm{T}})(B\otimes\Phi(t))^{\mathrm{T}}\\
&\quad\times(B\otimes\Phi(t))(I_N\otimes E)e(t-\tau_2)\\
&=\frac{1}{2}e^{\mathrm{T}}(t)(I_N\otimes FF^{\mathrm{T}})e(t)\\
&\quad+\frac{1}{2}e^{\mathrm{T}}(t-\tau_2)(I_N\otimes E^{\mathrm{T}})\\
&\quad\times(B^{\mathrm{T}}B\otimes\Phi^{\mathrm{T}}(t)\Phi(t))(I_N\otimes E)e(t-\tau_2)\\
&\le\frac{1}{2}e^{\mathrm{T}}(t)Pe(t)+\frac{1}{2}e^{\mathrm{T}}(t-\tau_2)(B\otimes E)^{\mathrm{T}}\\
&\quad\times(B\otimes E)e(t-\tau_2)\\
&\le\frac{1}{2}\lambda_{\max}^{(P)}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)\\
&\quad+\frac{1}{2}\lambda_{\max}^{(B,E)}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2). \qquad (13)
\end{aligned}
$$

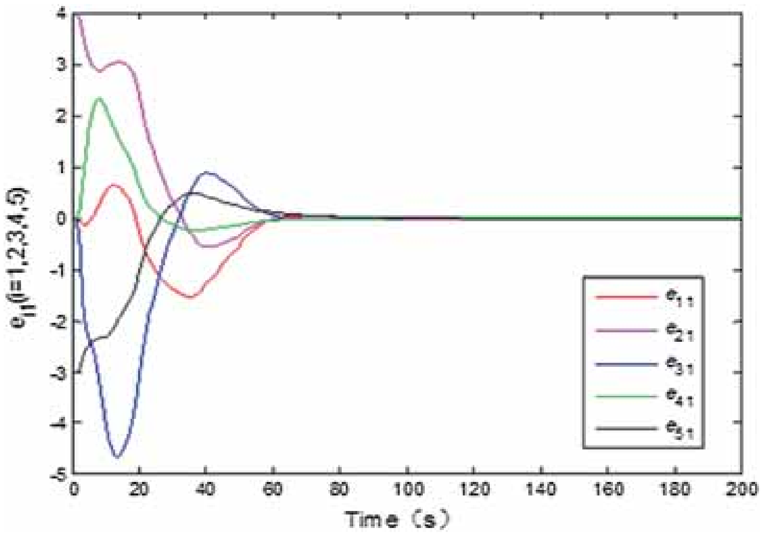

Figure 1. The synchronisation of errors e_{i1} ($i = 1, 2, \ldots, 5$).

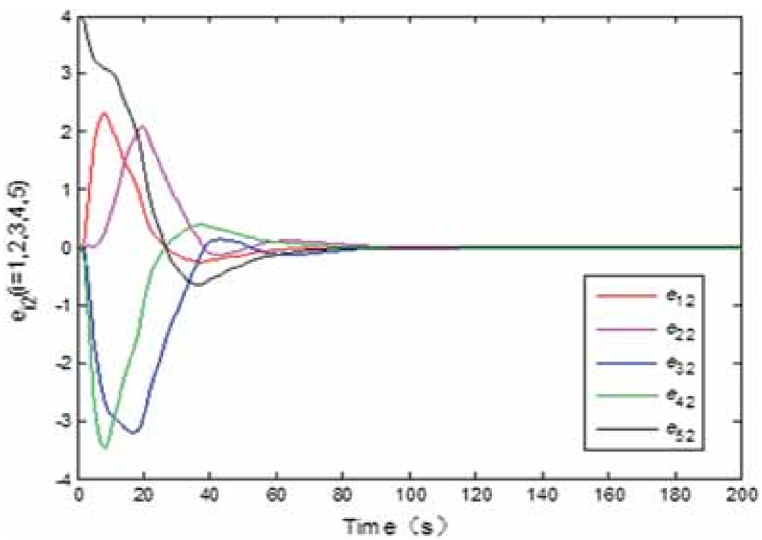

Figure 2. The synchronisation of errors e_{i2} ($i = 1, 2, \ldots, 5$).

By substituting (13) into (12), we get

$$
\begin{aligned}
&\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) \sum_{j=1}^{N} b_{ij}(\Gamma + \Delta\Gamma(t))(e_j(t-\tau_2) - e_i(t-\tau_3)) \\
&\quad + \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) \sum_{j=1}^{N} b_{ij}(\Delta\Gamma(t)) e_i(t-\tau_3) \\
&\leq \delta N \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) + \frac{\delta N}{2} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_2) e_i(t-\tau_2) \\
&\quad + \frac{\delta N}{2} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_3) e_i(t-\tau_3) \\
&\quad + \frac{1}{2}\lambda_{\max}^{(P)} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \\
&\quad + \frac{1}{2}\lambda_{\max}^{(B,E)} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_2) e_i(t-\tau_2) \\
&= \left(\delta N + \frac{1}{2}\lambda_{\max}^{(P)}\right) \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \\
&\quad + \left(\frac{\delta N}{2} + \frac{1}{2}\lambda_{\max}^{(B,E)}\right) \sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_2) e_i(t-\tau_2) \\
&\quad + \frac{\delta N}{2} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_3) e_i(t-\tau_3). \qquad (14)
\end{aligned}
$$

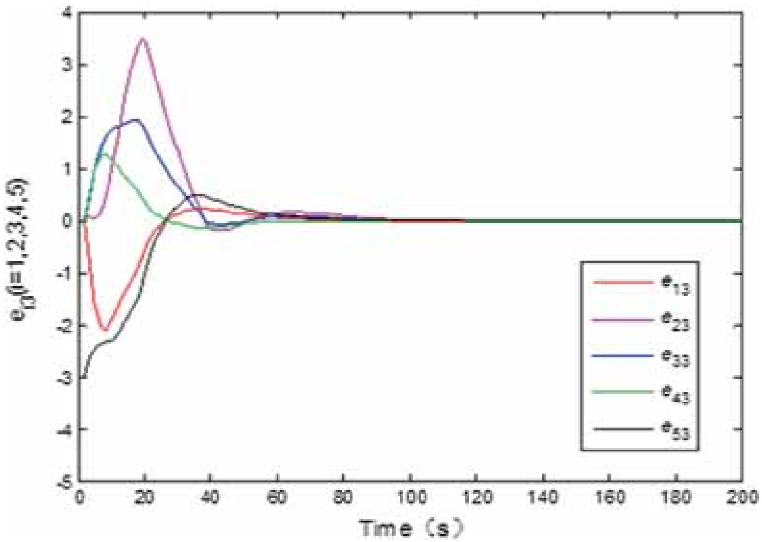

Figure 3. The synchronisation of errors e_{i3} $(i = 1, 2, \ldots, 5)$.

By substituting (14) into (11), we get

$$\begin{aligned}
\dot{V}(t) \leq & \sum_{i=1}^{N}\left(\gamma + \frac{\theta}{(1-\varepsilon)} + \delta N + \frac{1}{2}\lambda_{\max}^{(B,E)} - q^*\right) e_i^{\mathrm{T}}(t)e_i(t) \\
& + \sum_{i=1}^{N}[\varepsilon_i - \beta_i^*]\,\|e_i(t)\| \\
& - \frac{\delta N}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_3)e_i(t-\tau_3) \\
& - \left(\frac{\delta N}{2} + \frac{1}{2}\lambda_{\max}^{(B,E)}\right)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2) \\
& + \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\sum_{j=1}^{N} b_{ij}(\Gamma + \Delta\Gamma(t))(e_j(t-\tau_2) \\
& - e_i(t-\tau_3)) + \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) \\
& \times \sum_{j=1}^{N} b_{ij}(\Delta\Gamma(t))e_i(t-\tau_3) \\
= & \sum_{i=1}^{N}\left(\gamma + \frac{\theta}{(1-\varepsilon)} + \delta N + \frac{1}{2}\lambda_{\max}^{(B,E)} - q^*\right) \\
& \times e_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N}[\varepsilon_i - \beta_i^*]\,\|e_i(t)\| \\
& - \frac{\delta N}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_3)e_i(t-\tau_3) \\
& - \left(\frac{\delta N}{2} + \frac{1}{2}\lambda_{\max}^{(B,E)}\right)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2) \\
& + \left(\delta N + \frac{1}{2}\lambda_{\max}^{(P)}\right)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) \\
& + \left(\frac{\delta N}{2} + \frac{1}{2}\lambda_{\max}^{(B,E)}\right)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_2)e_i(t-\tau_2) \\
& + \frac{\delta N}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t-\tau_3)e_i(t-\tau_3) \\
= & \sum_{i=1}^{N}\left(\gamma + \frac{\theta}{(1-\varepsilon)} + 2\delta N + \frac{1}{2}\lambda_{\max}^{(P)} + \frac{1}{2}\lambda_{\max}^{(B,E)} - q^*\right) e_i^{\mathrm{T}}(t)e_i(t) \\
& + \sum_{i=1}^{N}[\varepsilon_i - \beta_i^*]\,\|e_i(t)\|.
\end{aligned}$$

Therefore, by taking appropriate q^* and β_i^* such that $\gamma + (\theta/(1-\varepsilon)) + 2\delta N + (1/2)\lambda_{\max}^{(P)} + (1/2)\lambda_{\max}^{(B,E)} - q^* < 0$ and $\varepsilon_i - \beta_i^* < 0$, for $i = 1, 2, \ldots, N$, we get $\dot{V}(t) \leq 0$, and hence, the proof is completed. □

4. Numerical results

In this section, the numerical simulation example is given. The node dynamic system of the networks is given by

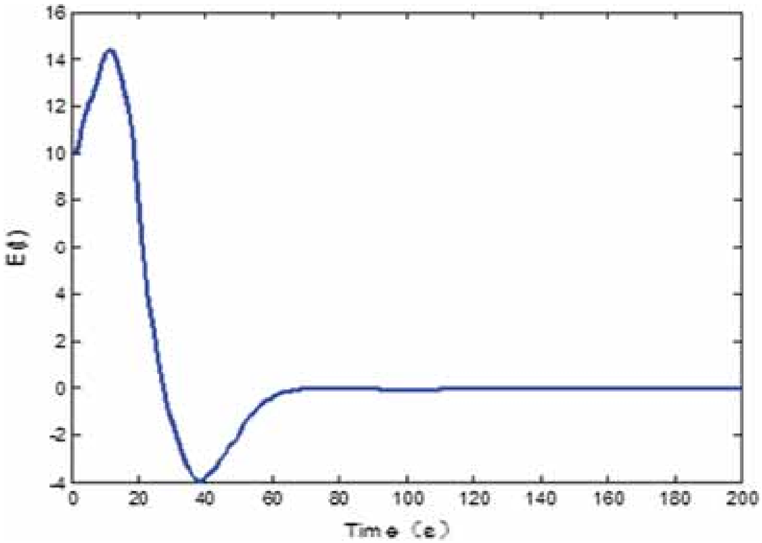

Figure 4. The synchronisation of errors $E(t)$.

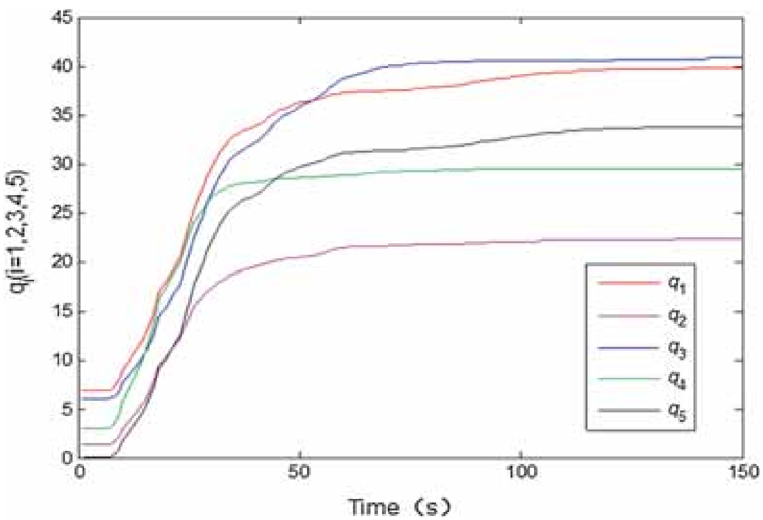

Figure 5. The state trajectories of feedback gains.

$$\begin{cases} \dot{x}_1(t) = p(x_2(t) - x_1(t) - g(x_1(t))), \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t), \\ \dot{x}_3(t) = -qx_2(t) - \mu x_3(t) - q\kappa \sin(\upsilon - x_1(t-\tau)), \end{cases}$$

where

$$g(x_1(t)) = m_1 x_1(t) + (1/2)(m_2 - m_1)\,(|x_1(t)+1| - |x_1(t)-1|)$$

and

$$p = 10, \quad q = 19.53, \quad \mu = 0.1636,$$
$$m_1 = -0.7831, \quad m_2 = -1.4325, \quad \kappa = 0.2,$$
$$\upsilon = 0.5, \quad \tau = 0.02.$$

The delayed Chua chaotic system [16] satisfies Lemma 2 with $\gamma = 12.0948$ and $\theta = 0.3225$,

$$k_1 = 2, \quad k_2 = 6\Gamma = I_{3\times 3}, \; N = 5,$$

$$\Delta\Gamma(t) = \begin{bmatrix} 2\cos t & 2\sin t - \cos t & 3\cos t + 2\sin t \\ \sin t & 3\sin t - 4\cos t & -2\cos t + 3\sin t \\ -2\cos t & 3\sin t - 2\cos t & \cos t + 3\sin t \end{bmatrix},$$

$$\tau_2 = 0.5, \; \tau_3 = 1.$$

The coupling matrix $B = \left[b_{ij}\right]_{5\times 5}$ is a scalar matrix randomly generated by Matlab. The disturbance

$$\Theta_i(t) = \begin{bmatrix} 0.3\sin(t) \\ 0.2\cos(t)\sin(t) \\ 0.5\cos(t) \end{bmatrix}$$

was chosen. The controller $u_i(t)$ can be given by (5) in Theorem 1. $\lambda_{\max}^{(P)} = 1.4832$, $\lambda_{\max}^{(B,E)} = 4.5366$, and taking $q^* = 40$, then $\dot{V}(t) \leq 0$ can be obtained. Networks (1) and (2) can achieve synchronisation. The simulation

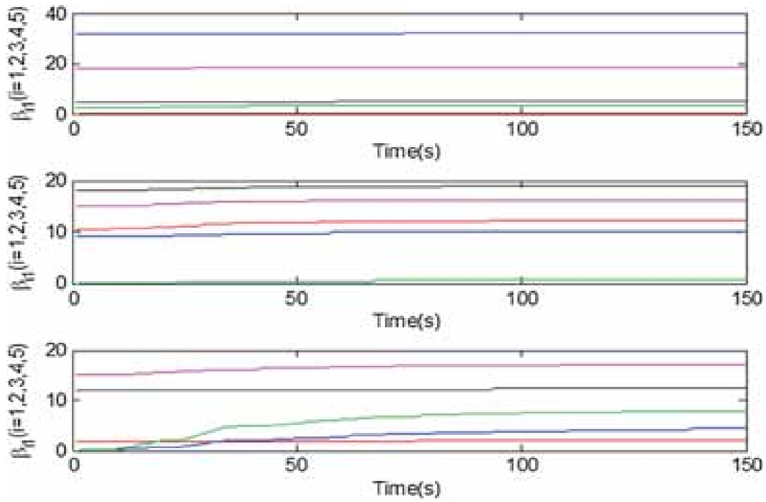

Figure 6. The state trajectories of parameters $\hat{\beta}_i$.

results are given in figures 1–6. Figures 1–3 show the time evolution of errors e_{i1}, e_{i2}, e_{i3} $(i = 1, 2, \ldots, 5)$, figure 4 shows the time evolution of the total synchronisation errors $E(t) = \sum_{i=1}^{5} \sum_{j=1}^{3} e_{ij}$, figure 5 shows the state trajectories of feedback gains and figure 6 shows the state trajectories of parameters $\hat{\beta}_i$.

5. Conclusions

In this paper, the adaptive synchronisation problem of uncertain complex dynamic networks with non-dissipative coupling and time-varying delay dynamic nodes is studied. It is assumed that the dynamical complex networks are uncertain, and uncertainty occurs in linear inner coupling matrix. First, an adaptive controller was designed. Secondly, based on the Lyapunov stability theory, the criteria for adaptive synchronisation were derived. Thirdly, the adaptive synchronisation of uncertain complex networks was proved by theoretical analysis and numerical simulations. In the future work, the time-varying coupling delays $\tau_2(t)$ and $\tau_3(t)$ and nonlinear inner coupling for the networks will be considered.

Acknowledgements

This research was partially supported by the Fundamental Research Funds for the Central Universities, China (No. 2017B17914) and the National Nature Science Foundation of China (Grant No. 11402226).

References

[1] L M Pecora and T L Carroll, *Phys. Rev. Lett.* **80**, 2109 (1998)
[2] A Selivanov, A Fradkov and E Fridman, *J. Franklin Inst.* **352**, 52 (2015)
[3] R Rakkiyappan and N Sakthivel, *Neurocomputing* **162**, 26 (2015)
[4] G Y Zhou, C R Li, T T Li, Y Yang, C Wang, F J He and J C Sun, *Physica A* **457**, 506 (2016)
[5] L M Wang, *Pramana – J. Phys.* **89**: 38 (2017)
[6] S Rajavel, R Samidurai, J D Cao, A Alsaedi and B Ahmad, *Appl. Math. Comput.* **297**, 145 (2017)
[7] F Z Nian and W L Liu, *Pramana – J. Phys.* **86**, 1209 (2016)
[8] C D Huang, J D Cao, M Xiao, A Alsaedi and T Hayat, *Appl. Math. Comput.* **292**, 210 (2017)
[9] H Tirandza, *Pramana – J. Phys.* **89**: 85 (2017)
[10] X Y Chen, J D Cao, J L Qiu, A Alsaedi and F E Alsaadi, *Adv. Differ. Equ.* **2016**, 231 (2016)
[11] K Sivaranjani, R Rakkiyappan, J D Cao and A Alsaedi, *Appl. Math. Comput.* **311**, 283 (2017)
[12] W W Zhang, R C Wu, J D Cao, A Alsaedi and T Hayat, *Nonlinear Anal.: Model. Control* **22**, 636 (2017)
[13] G L Cai, S Q Jiang, S M Cai and L X Tian, *Pramana – J. Phys.* **86**, 545 (2016)
[14] S Q Jiang and X B Lu, *Pramana – J. Phys.* **86**, 1243 (2016)
[15] X Q Lei, S M Cai, S Q Jiang and Z R Liu, *Neurocomputing* **222**, 26 (2017)
[16] S Zheng, *J. Franklin Inst.* **354**, 6341 (2017)
[17] L Shi, H Zhu, S M Zhong, K B Shi and J Cheng, *ISA Trans.* **65**, 81 (2016)
[18] S G Wang, S Zheng, B W Zhang and H T Cao, *Optik* **127**, 4716 (2016)
[19] C R Li, L Lü, Y M Yang, S Zhou and Y X Hong, *Physica A* **492**, 2301 (2018)

[20] L Shi, H L Yang, X Wang, S M Zhong and W Q Wang, *Chaos Solitons Fractals* **111**, 180 (2018)
[21] X Wang, X Z Liu, K She and S M Zhong, *J. Franklin Inst.* **354**, 4913 (2017)
[22] Z Y Wu and H Leng, *J. Franklin Inst.* **354**, 689 (2017)
[23] X Wang, X Z Liu, K She and S M Zhong, *Nonlinear Anal.: Hybrid Syst.* **26**, 307 (2017)
[24] X H Ma and J A Wang, *Neurocomputing* **199**, 197 (2016)
[25] Y F Lei, L L Zhang, Y H Wang and Y Q Fan, *Neurocomputing* **230**, 390 (2017)
[26] L L Zhang, Y F Lei, Y H Wang and H G Chen, *Appl. Math. Model.* **55**, 248 (2018)