Pramana – J. Phys. (2019) 92:79
https://doi.org/10.1007/s12043-019-1741-3

Locally rotationally symmetric Bianchi type-I cosmological model with dynamical Λ and G in $f(R)$ gravity

RAKESH RAUSHAN[1], A K SHUKLA[1], R CHAUBEY[1,*] and T SINGH[2]

[1]Centre for Interdisciplinary Mathematical Sciences, Institute of Science, Banaras Hindu University, Varanasi 221 005, India
[2]Indian Institute of Advanced Study, Shimla 171 005, India
*Corresponding author. E-mail: yahoo_raghav@rediffmail.com; rchaubey@bhu.ac.in

MS received 7 February 2018; revised 10 October 2018; accepted 25 October 2018; published online 23 March 2019

Abstract. In this paper, we have studied the locally rotationally symmetric (LRS) Bianchi type-I cosmological model filled with a bulk viscous cosmological fluid in $f(R)$ gravity in the presence of time-varying gravitational and cosmological constant. We have used the power-law and intermediate scenario for scale factor to obtain the solution of the field equations. The evolution of temperature of a viscous Universe is also analysed.

Keywords. Viscous fluid; cosmological constant; gravitational constant; $f(R)$ gravity.

PACS Nos 98.80.Cq; 95.36.+x; 04.60.Pp

1. Introduction

The area of cosmology has undergone a paradigm shift since the idea of accelerated expansion of the Universe was proposed. The phenomenon is ascribed to the so-called 'Dark Energy', whose nature is still not well established and the scientific community is venturing into it. The analysis of large chunks of data collected from Type-Ia supernovae (SNe Ia) [1,2], cosmic microwave background radiation (CMBR) [3,4], constraints from Sloan digital sky survey (SDSS) galaxy clustering [4,5], baryonic acoustic oscillations (BAO) [6] and weak lensing [7] shed light upon this. Ample number of theories have been proposed to explain the late-time cosmic acceleration and dark energy. A mindful lot is exercising on it with the advent of newer results on the general relativity front. The simplest explanation is provided by a cosmological constant. However, this scenario is plagued by a severe fine-tuning problem associated with its energy scale [8]. The presence of the late-time cosmic acceleration of the Universe can indeed be explained by $f(R)$ gravity and $f(R)$ modified theories [9–11].

In Einstein's field equations (EFE), there are two important parameters: the cosmological constant Λ and the gravitational constant G. The Newtonian constant of gravitation G plays the role of coupling between geometry and matter in the Einstein's field equation. The idea of varying Λ and G was proposed in due course of time and some modified general theory of relativity with $\Lambda(t)$ and $G(t)$ for an evolving Universe in time were established. Variation of Newton's gravitational parameter G was originally suggested by Dirac [12] on the basis of his many hypotheses. Many extensions of general relativity with $G = G(t)$ have been made ever since Dirac first considered the possibility of a variable G. Abdusattar and Vishwakarma [13] have suggested the conservation of the energy–momentum tensor, which consequently renders G and Λ as coupled fields. This leaves Einstein's field equations formally unchanged. Bonanno and Reuter [14] have considered the scaling of $G(t)$ and $\Lambda(t)$ arising from an underlying renormalisation group flow near an infra-red attractive fixed point. The resulting cosmology [15] explains the high red-shift SNe Ia and observations of radio sources successfully. Various theories on variable G with induced gravity models were examined by Copi *et al* [16], Smolin [17] and Adler [18] in the context of induced gravity model, where G is generated by means of a non-vanishing vacuum expectation value of a scalar field. Recently, a constraint on the variation of G has been obtained by using Wilkinson microwave anisotropy probe (WMAP) and the Big Bang nucleosynthesis observations by Zee [19], which comes out to be $-3 \times 10^{-13} < (\dot{G}/G)_{\text{today}} < 4 \times 10^{-13}\ \text{yr}^{-1}$.

The Friedmann–Robertson–Walker (FRW) models assume the present day Universe to be homogeneous and isotropic. However, the latest observational data of the cosmic microwave background (CMB) by WMAP satellite show hints of anomalies that the isotropy seems to be broken in cosmological data [20]. Large-angle anomalies in the CMB can be a very important means to understand the very early Universe and effects this early Universe had on the present day large-scale structure. According to the theories proposed by Misner [21] and Gibbons and Hawking [22], anisotropy at the early stage of the Universe turns into an isotropic present Universe with initial anisotropies dying away.

Several authors [23,24] have suggested that the anisotropic Bianchi Universes can play important roles in observational cosmology (see also [21,25–29]). The WMAP data [30–32] seem to suggest that the standard cosmological model requires the addition of a positive cosmological constant that bears resemblance to the Bianchi morphology [33–35]. According to this, the Universe should have a slightly anisotropic spatial geometry in spite of the inflation, contrary to generic inflationary models [36–40].

In many cosmological and astrophysical situations, an idealised fluid model of the matter is inappropriate. Dissipative effects, including both the bulk and shear viscosities, are supposed to play important roles in the early evolution of the Universe. From a physical point of view, the inclusion of dissipative terms in the energy–momentum tensor of the cosmological fluid seems to be the best motivated generalisation of the matter term of the gravitational field equations. Fayaz *et al* [41] have studied the dark energy and viscous fluid cosmology with variables G and Λ in an anisotropic space–time by considering the constant deceleration parameter (Berman law). The $f(R)$ modified gravity including higher-order terms based on different equations of state parameter in variables G and Λ has been studied by Khurshudyan *et al* [42]. Recently, Chaubey *et al* [43] have studied several anisotropic cosmological models with variables Λ and G in viscous cosmology and $f(R,T)$ gravity with $\Lambda(T)$. The study of varying Λ and G with bulk viscous fluid in R^2 gravity was carried out by Paul and Debnath [44]. The variations of G and Λ lead to modification of Einstein's field equations and the conservation laws [42,44,45]. This is because, if we allow G and Λ to be variables in Einstein equations, the energy conservation law is violated. Therefore, the study of varying G and Λ can be carried out using the modified field equations and modified conservation law [46].

The present paper is organised as follows. In §1, a brief introduction is given. Section 2 deals with the basic equations of cosmological model. Gibbs equation is also defined in this section. In §3, we have obtained the cosmological solutions of our model for power law. We have analysed the variation of temperature in the presence and absence of viscosity. The cosmological solutions and variation of temperature for intermediate scenario are also given in §3. The paper ends with a conclusion given in §4.

2. Model and basic equations

A gravitational action associated with higher-order term in the scalar curvature R containing a varying G is given by

$$I = -\int \left[\frac{1}{16\pi G(t)} f(R) + L_{\text{m}}\right]\sqrt{-g}\,\text{d}^4x, \tag{1}$$

where $f(R)$ is a function of R and its higher power including a variable cosmological constant $\Lambda(t)$, g is the determinant of the four-dimensional metric and L_{m} represents the matter Lagrangian.

Variation of the action equation (1) with respect to g_{ij} is given by [44]

$$\begin{aligned} & f_R(R)R_{ij} - \frac{1}{2}f(R)g_{ij} \\ & + f_{RR}(R)(\nabla_i\nabla_j R - g_{ij}\nabla^i\nabla^j g_{ij}) \\ & + f_{RRR}(R)(\nabla_i R\nabla_j R - \nabla^k R\nabla_k R g_{ij}) \\ & = -8\pi G(t)T_{ij}, \end{aligned} \tag{2}$$

where ∇_i is the covariant differential operator. T_{ij} is the energy–momentum tensor for the matter determined by L_{m} which is defined as $T_{ij} = \rho u_i u_j + p g_{ij}$, where ρ and p are respectively the energy density and pressure of the cosmic fluid. Here $f_R(R)$ denotes the derivative of $f(R)$ with respect to R.

We consider the locally rotationally symmetric (LRS) Bianchi type-I (BI) metric given by

$$\text{d}s^2 = \text{d}t^2 - a_1^2\text{d}x^2 - a_2^2(\text{d}y^2 + \text{d}z^2), \tag{3}$$

where the metric functions $a_1(t)$ and $a_2(t)$ are functions of cosmic time t only. The scale factor $a_1(t)$ is the expansion in the x-direction and $a_2(t)$ is the expansion in y- and z-directions of three-dimensional space. The directional Hubble parameters in the respective directions are defined as

$$H_i = \frac{\dot{a}_i}{a_i}, \quad i = 1, 2. \tag{4}$$

The overhead dot denotes the differential with respect to cosmic time t. We define the generalised Hubble parameter H as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right) = \frac{1}{3}(H_1 + 2H_2). \tag{5}$$

Here we assume $a_1 \propto a_2{}^n$, where n is a positive constant. For $n = 1$, LRS BI model reduces to FRW cosmological model whereas for other values of n, the model becomes anisotropic.

From eq. (5), one can obtain the relation between directional Hubble parameters and generalised Hubble parameter as

$$H_1 = nH_2 = \left(\frac{3n}{n+2}\right)H. \tag{6}$$

The scalar curvature, R, is given by

$$R = 2\left[\frac{\ddot{a}_1}{a_1} + 2\frac{\ddot{a}_2}{a_2} + 2\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2\right]. \tag{7}$$

Now from eqs (6) and (7), the scalar curvature in terms of H and $\dot{H}$ is given by

$$R = 6\left[\dot{H} + \frac{3(n^2+2n+3)}{(n+2)^2}H^2\right]. \tag{8}$$

From eqs (5) and (8), the (0, 0) and trace components of eq. (2) yield

$$\begin{aligned}&\dot{H}_1 + H_1^2 + 2(\dot{H}_2 + H_2^2)\\ &\quad = \frac{1}{f_R(R)}\left[\frac{f(R)}{2} + f_{RR}(R)\dot{R}(H_1 + 2H_2)\right]\\ &\qquad - \frac{8\pi G(t) T_{00}}{f_R(R)},\end{aligned} \tag{9}$$

$$\begin{aligned}&6\left[\dot{H} + \frac{3(n^2+2n+3)}{(n+2)^2}H^2\right]\\ &\quad = \frac{1}{f_R(R)}[2f(R) + 3f_{RR}(R)\\ &\quad \times (\ddot{R} + (H_1 + 2H_2)\dot{R} + 3f_{RRR}(R)\dot{R}]\\ &\qquad - \frac{8\pi G(t) T}{f_R(R)}.\end{aligned} \tag{10}$$

Let us consider a higher-order gravity [44]

$$f(R) = R + \alpha R^2 - 2\Lambda(t). \tag{11}$$

It is a well-known fact in the cosmology that the model based on eq. (11) cannot provide late-time acceleration for the present cosmic acceleration but can be used for the inflation in the early Universe [45]. Kahya *et al* [47] have also considered eq. (11) for their study in a higher derivative theory of gravity in the presence of time-varying Λ and G.

From eqs (6), (8)–(11) we get

$$\begin{aligned}&\frac{3(2n+1)}{(n+2)^2}H + \frac{54\alpha(n^2+2n+3)(-n^2+2n-1)}{(n+2)^4}H^4\\ &\quad - 6\alpha\dot{H}^2 - 36\alpha\dot{H}H^2 - 12\alpha H\ddot{H}\\ &\quad = \frac{\Lambda(t) - 8\pi G(t)\rho}{3}.\end{aligned} \tag{12}$$

The conservation equation for matter ($T^{ij};\, j = 0$) becomes

$$\dot{\rho} + 3(\rho + p)H = 0. \tag{13}$$

By using eq. (11) into eq. (2), the divergence of eq. (2) leads to

$$\Lambda(t)_{,j}\, g^{ij} = -8\pi(G(t)_{,j}\, T^{ij} + G(t)T^{ij}{}_{,j}). \tag{14}$$

Equations (12) and (14) are the key equations to study cosmological models with a perfect fluid in the presence of time-varying G and Λ. After including the effect of viscosity in the above, the perfect fluid pressure in eq. (14) is replaced by an effective pressure p_{eff}, which is given by $p_{\text{eff}} = p + \Pi$, where p is the isotropic pressure and Π is the bulk viscous stress.

In the extended irreversible thermodynamics, the bulk viscous stress Π satisfies a transport equation [48–51] given by

$$\Pi + \tau\dot{\Pi} = -3\xi H - \frac{\epsilon}{2}\tau\Pi\left[3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right], \tag{15}$$

where ξ is the coefficient of bulk viscosity, τ is the relaxation coefficient for transient bulk viscous effects and $T \geq 0$ is the absolute temperature of the Universe. The parameter ϵ takes the value 0 or 1. Here $\epsilon = 0$ represents the truncated Israel–Stewart theory and $\epsilon = 1$ represents the full Israel–Stewart (FIS) causal theory. One recovers the non-causal Eckart theory for $\tau = 0$.

The covariant conservation equation (14) including viscous fluid is given by

$$\dot{\rho} + 3(\rho + p + \Pi)H = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G}\right). \tag{16}$$

This equation reduces to the usual continuity equation for a barotropic fluid in the case of constants G and Λ and $\Pi = 0$.

Consider an equation of state for the barotropic fluid pressure given by

$$p = \gamma\rho, \tag{17}$$

where γ is a constant and $\gamma \in [0, 1]$.

The deceleration parameter q is related to H as

$$q = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right) - 1. \tag{18}$$

The deceleration parameter $q<0$ represents the accelerating phase of the Universe and $q>0$ represents the decelerating phase of the Universe. The temperature of the Universe is defined through the Gibbs equation, which is given by

$$T\mathrm{d}s = \mathrm{d}\left(\frac{\rho}{\eta}\right) + p\mathrm{d}\left(\frac{1}{\eta}\right). \tag{19}$$

Gibbs integrability condition for the behaviour of temperature in the Universe is obtained as

$$\eta\frac{\partial T}{\partial \eta} + (\rho+p)\frac{\partial T}{\partial \rho} = T\frac{\partial p}{\partial \rho}. \tag{20}$$

For a barotropic fluid, the temperature follows a power law which is $T \sim \rho^{\frac{\gamma}{1+\gamma}}$. The above temperature may be determined using the Gibbs integrability condition also:

$$\frac{\dot{T}}{T} = -3H\left[\left(\frac{\partial p}{\partial \rho}\right)_\eta + \frac{\Pi}{T}\left(\frac{\partial T}{\partial \rho}\right)_\eta\right]. \tag{21}$$

3. Cosmological solutions

In most of the investigations involving bulk viscosity, it is a widely accepted relation [52] that bulk viscosity is a power function of the energy density [53–55], given by

$$\zeta = \beta'\rho^s, \quad \tau = \beta'\rho^{s-1}, \tag{22}$$

where $\beta'(\geq 0)$ and $s(\geq 0)$ are constants.

Now eq. (13) for viscous fluid is reduced to

$$\dot{\rho} + 3(\rho + p + \Pi)H = 0. \tag{23}$$

From eqs (17) and (23), we get

$$\dot{\rho} + 3(1+\gamma)\rho H + 3\Pi H = 0. \tag{24}$$

From eqs (24) and (16), we get

$$8\pi\dot{G}\rho + \dot{\Lambda} = 0. \tag{25}$$

The variation law $\Lambda \sim H^2$ has been proposed by several researchers [56–59] and Berman [60] has given the relation between the energy density and Hubble parameter as $\rho G \propto H^2$. Using $\rho \sim H^m$ and $\Lambda \sim H^2$ in eq. (25), we obtain

$$G = AH^b, \tag{26}$$

where A and b $(= 2-m)$ are constants.

3.1 *Power-law model*

Consider a power law of the Universe, given by

$$a(t) = a_0 t^D, \tag{27}$$

where a_0 and D are constants which are to be determined from the field equation.

The accelerating mode of expansion $(q<0)$ of the Universe is obtained for $D>1$.

From eqs (5), (26) and (27), the Hubble's parameter and gravitational parameter are given by

$$H = \frac{D}{t}, \tag{28}$$

$$G = A\left(\frac{D}{t}\right)^b. \tag{29}$$

From eqs (12) and (28) we have

$$\frac{3(2n+1)}{(n+2)^2}\left(\frac{D}{t}\right)^2 + \frac{54\alpha(n^2+2n+3)(-n^2+2n-1)}{(n+2)^4}\left(\frac{D}{t}\right)^4 - 6\alpha\left(\frac{D}{t^2}\right)^2 + 36\alpha\left(\frac{D^3}{t^4}\right) - 24\alpha\left(\frac{D^2}{t^4}\right) = \frac{\Lambda}{3} - \frac{8\pi G\rho}{3}. \tag{30}$$

From eqs (25) and (29), we obtain

$$\rho = -\frac{\dot{\Lambda}}{8\pi X t^{-(1+b)}}, \tag{31}$$

where $X = -AbD^b$.

Putting the value of ρ from eq. (31) into eq. (30), we get

$$\frac{9(2n+1)}{(n+2)^2}\left(\frac{D}{t}\right)^2 + \frac{162\alpha(n^2+2n+3)(-n^2+2n-1)}{(n+2)^4}\left(\frac{D}{t}\right)^4 - 18\alpha\left(\frac{D^2}{t^4}\right)^2 + 108\alpha\left(\frac{D^3}{t^4}\right) - 72\alpha\left(\frac{D^2}{t^4}\right) = \Lambda - \frac{\dot{\Lambda}t}{b}. \tag{32}$$

Solving the differential equation (32), we get

$$\Lambda = \frac{x_1 D^2 b}{b+3}t^{-2} + \frac{b(x_2D^4 + x_3D^2 + x_4D^3 + x_5D^2)}{(b+5)}t^{-4} + \frac{C'}{t^{-b}}, \tag{33}$$

where

$$x_1 = \frac{9(2n+1)}{(n+2)^2},$$

$$x_2 = \frac{162\alpha(n^2+2n+3)(-n^2+2n-1)}{(n+2)^4},$$

$x_3 = -18\alpha$, $x_4 = 108\alpha$, $x_5 = -72\alpha$ and C' is an integration constant.

From eq. (8) we get

$$\rho = -\frac{1}{8\pi X}\left\{\frac{-2x_1D^2b}{b+3}\frac{1}{t^{2-b}} - \frac{4b(x_2D^4+x_3D^2+x_4D^3+x_5D^2)}{(b+5)}\frac{1}{t^{4-b}} + \frac{bC'}{t^{-2}}\right\}. \tag{34}$$

Now from eq. (24) the bulk viscous stress is given by

$$\begin{aligned}\Pi =& -\frac{1}{3H}\dot{\rho} - (1+\gamma)\rho \\ =& \frac{t}{24\pi DX}\left(\frac{-2x_1D^2b}{b+3}\frac{(b-2)}{t^{3-b}} - \frac{4b(x_2D^4+x_3D^2+x_4D^3+x_5D^2)}{(b+5)}\frac{(b-4)}{t^{5-b}} + 2tbC'\right) - (1+\gamma)\left(-\frac{1}{8\pi X}\left(\frac{-2x_1D^2b}{b+3}\frac{1}{t^{2-b}} - \frac{4b(x_2D^4+x_3D^2+x_4D^3+x_5D^2)}{(b+5)} \times\frac{1}{t^{4-b}} + \frac{bC'}{t^{-2}}\right)\right).\end{aligned} \tag{35}$$

For physically realistic solution, the bulk viscous stress is essentially negative. For FIS theory we use $\epsilon = 1$, then eq. (15) reduces to the following differential equation:

$$\frac{\dot{T}}{T} = 3H - \frac{\dot{\rho}}{\rho} + \frac{6H\rho}{\Pi} + \rho^{1-s}\frac{2}{\beta'} + 2\frac{\dot{\Pi}}{\Pi}. \tag{36}$$

Integrating eq. (36) we get

$$T = T_0\frac{\Pi^2a^3}{\rho}e^{\int\frac{6H\rho}{\Pi}dt}e^{\frac{2}{\beta'}\int\rho^{1-s}dt}, \tag{37}$$

where T_0 is an integration constant.

For a suitable choice of variables $\alpha = 0$, $b = 1$, $s = 0$ and $C' = 0$, eqs (34) and (35) reduce to

$$\rho = \frac{-9(2n+1)D}{16\pi A(n+2)^2t} \tag{38}$$

and

$$\Pi = \frac{9(2n+1)}{16\pi A(n+2)^2t}\left(-\frac{1}{3} + D(1+\gamma)\right). \tag{39}$$

From eqs (38) and (39) in eq. (37), we get

$$T = T_0\frac{-9(2n+1)}{16\pi AD(n+2)^2}\left(-\frac{1}{3} + D(1+\gamma)\right)^2 \times a_0^3t^{-\frac{18D^2}{3D(1+\gamma)-1} - \frac{9(2n+1)}{16\pi A(n+2)^2}\frac{D}{\beta'} + 3D - 1}. \tag{40}$$

In the absence of viscosity, the variation of temperature by using Gibbs condition, eq. (21), is given by

$$T = T_0t^{-3\gamma D}. \tag{41}$$

3.2 *Intermediate scenario*

In this section, the scale factor is written as

$$a(t) = e^{Bt^\beta}, \tag{42}$$

where $B > 0$ and $0 < \beta < 1$.

From eqs (42) and (5), the Hubble parameter is given as

$$H = B\beta t^{\beta-1}. \tag{43}$$

From eqs (26) and (43), we have

$$G = AB^b\beta^bt^{b(\beta-1)}. \tag{44}$$

From eqs (12) and (43) we have

$$X_1t^{2(\beta-1)} + X_2t^{4(\beta-1)} - X_3t^{2(\beta-2)} - X_4t^{(3\beta-4)} - X_5t^{2(\beta-2)} = \Lambda - 8\pi G\rho, \tag{45}$$

where

$$\begin{aligned}X_1 &= \frac{9(2n+1)}{(n+2)^2}B^2\beta^2, \\ X_2 &= \frac{162\alpha(n^2+2n+3)(-n^2+2n-1)}{(n+2)^4}B^4\beta^4, \\ X_3 &= 18\alpha B^2\beta^2(\beta-1)^2, \\ X_4 &= 108\alpha B^3\beta^3(\beta-1), \\ X_5 &= 36\alpha B^2\beta^2(\beta-1)(\beta-2).\end{aligned}$$

From eqs (25) and (44), we get the density parameter as

$$\rho = -\frac{\dot{\Lambda}}{8\pi B\beta b(\beta-1)^2(\beta-2)t^{b(\beta-1)-1}}. \tag{46}$$

From eqs (44)–(46), we get the cosmological constant in terms of t as

$$\Lambda = b(\beta-1)\left(\frac{X_1}{(b+2)(\beta-1)}t^{2(\beta-1)} + \frac{X_2}{(b+4)(\beta-1)}t^{4(\beta-1)} - \frac{X_3+X_5}{(b+2)(\beta-1)-2}t^{2(\beta-2)}\right.$$

$$-\frac{X_4}{(b+3)(\beta-1)-1}t^{(3\beta-4)}\Bigg) + Ct^{-b(\beta-1)}, \tag{47}$$

where C is an integration constant.

Now from eqs (46) and (47) we have

$$\rho = -\frac{1}{8\pi AB^b\beta^b}\Bigg(\frac{2X_1}{(b+2)}t^{(2\beta-b\beta+b-2)} + \frac{4X_2}{(b+4)}t^{(4\beta-b\beta+b-4)} - \frac{2(X_3+X_5)(\beta-2)}{(b+2)(\beta-1)-2}t^{(2\beta-b\beta+b-4)} - \frac{X_4(3\beta-4)}{(b+3)(\beta-1)-1}t^{(3\beta-b\beta+b-4)} - Ct^{-2b(\beta-1)}\Bigg). \tag{48}$$

Now from eq. (24), the bulk viscosity stress for intermediate scenario is given by

$$\begin{aligned}\Pi &= -\frac{1}{3H}\dot{\rho} - (1+\gamma)\rho \\ &= \frac{1}{24\pi AB^{b+1}\beta^{b+1}t^{\beta-1}} \times \Bigg(\frac{2X_1(2\beta-b\beta+b-2)}{(b+2)}t^{(2\beta-b\beta+b-3)} \\ &\quad + \frac{4X_2(4\beta-b\beta+b-4)}{(b+4)}t^{(4\beta-b\beta+b-5)} \\ &\quad - \frac{2(X_3+X_5)(\beta-2)(2\beta-b\beta+b-4)}{(b+2)(\beta-1)-2}t^{(2\beta-b\beta+b-5)} \\ &\quad - \frac{X_4(3\beta-4)(3\beta-b\beta+b-4)}{(b+3)(\beta-1)-1}t^{(3\beta-b\beta+b-5)} \\ &\quad + 2Cb(\beta-1)t^{-2b(\beta-1)-1}\Bigg) \\ &\quad + (1+\gamma)\frac{1}{8\pi AB^b\beta^b}\Bigg(\frac{2X_1}{(b+2)}t^{(2\beta-b\beta+b-2)} + \frac{4X_2}{(b+4)}t^{(4\beta-b\beta+b-4)} \\ &\quad - \frac{2(X_3+X_5)(\beta-2)}{(b+2)(\beta-1)-2}t^{(2\beta-b\beta+b-4)} - \frac{X_4(3\beta-4)}{(b+3)(\beta-1)-1}t^{(3\beta-b\beta+b-4)} \\ &\quad - Ct^{-2b(\beta-1)}\Bigg).\end{aligned} \tag{49}$$

Here the initial condition is considered as $C=0, \alpha=0, b=2, s=0$ in eq. (16), and we get the density ρ and bulk viscous stress, respectively, as

$$\rho = -\frac{9(2n+1)}{16\pi A(n+2)^2} \tag{50}$$

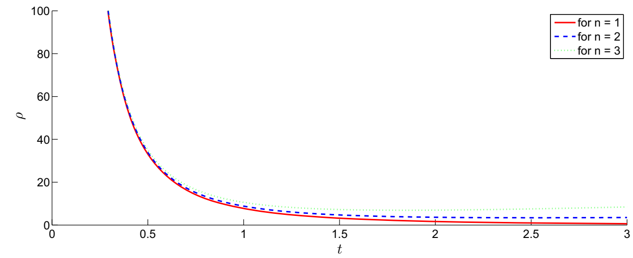

Figure 1. Density ρ vs. time t in power-law model for $n = 1, 2$ and 3.

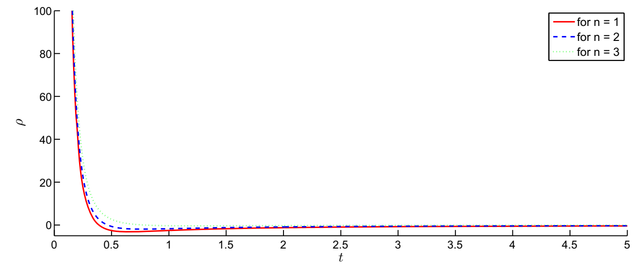

Figure 2. Density ρ vs. time t in intermediate scenario for $n = 1, 2$ and 3.

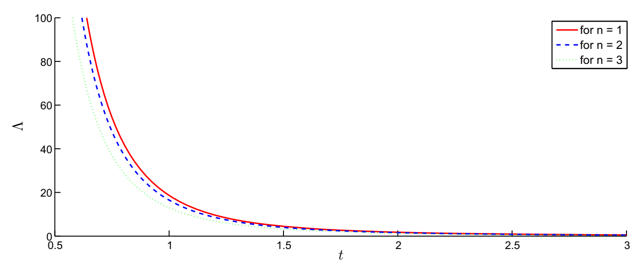

Figure 3. Cosmological constant Λ vs. time t in power-law model for $n = 1, 2$ and 3.

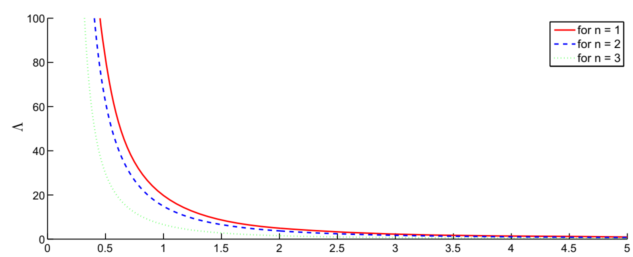

Figure 4. Cosmological constant Λ vs. time t in intermediate scenario for $n = 1, 2$ and 3.

and

$$\Pi = \frac{9(2n+1)}{16\pi A(n+2)^2}(1+\gamma). \tag{51}$$

From eqs (37), (50) and (51), we obtain

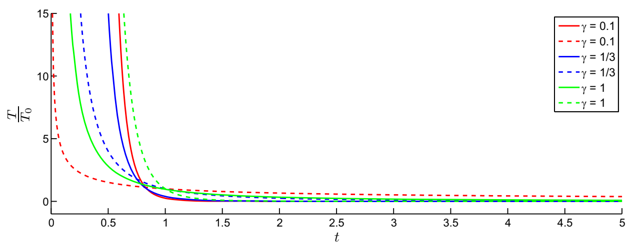

Figure 5. Solid and dashed lines represent the variation of T/T_0 with t in the presence and absence of viscosity, respectively, in power-law model for $\gamma = 0.1, 1/3, 1$ and $n = 0.001$ (where $A = -20$, $D = 2$, $a_0 = 4$, $\beta' = 2$).

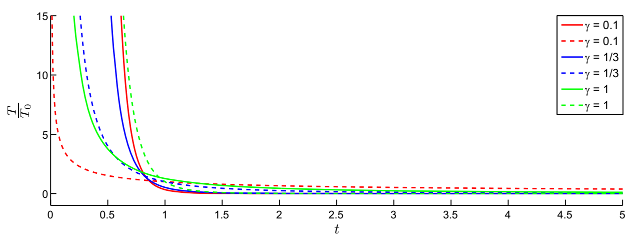

Figure 6. Solid and dashed lines represent the variation of T/T_0 with t in the presence and absence of viscosity, respectively, in power-law model for $\gamma = 0.1, 1/3, 1$ and $n = 1$ (where $A = -20$, $D = 2$, $a_0 = 4$, $\beta' = 2$).

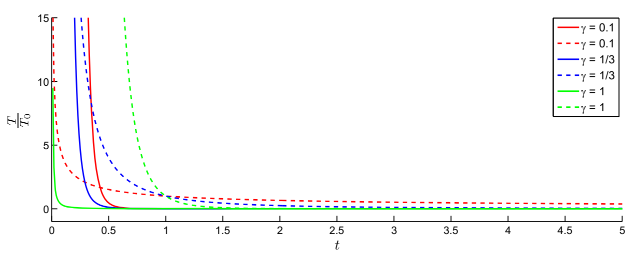

Figure 7. Solid and dashed lines represent the variation of T/T_0 with t in the presence and absence of viscosity, respectively, in power-law model for $\gamma = 0.1, 1/3, 1$ and $n = 1000$ (where $A = -20$, $D = 2$, $a_0 = 4$, $\beta' = 2$).

$$T = T_0 \frac{-9(2n+1)(1+\gamma)^2}{16\pi(n+2)^2 A} \mathrm{e}^{(Bt^\beta)^3 - \frac{6Bt^\beta}{1+\gamma} - \frac{9(2n+1)}{8\pi(n+2)^2 A\beta'} t}. \tag{52}$$

In the absence of viscosity the variation of temperature by using Gibbs condition, eq. (21), is given by

$$T = T_0 \mathrm{e}^{-3\gamma B t^\beta}. \tag{53}$$

4. Conclusions

We have studied the LRS BI Universe with variables Λ and G filled with viscous fluid in the $f(R)$ modified gravity. We have discussed the power law and the

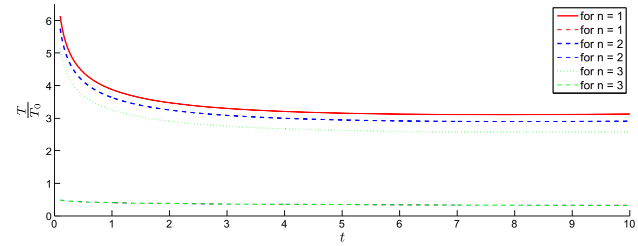

Figure 8. Solid and dashed lines represent the variation of T/T_0 with t in the presence and absence of viscosity, respectively, in intermediate scenario for $n = 1, 2, 3$ (where $A = -0.25$, $B = 0.5$, $\beta = 0.5$, $\beta' = 1$).

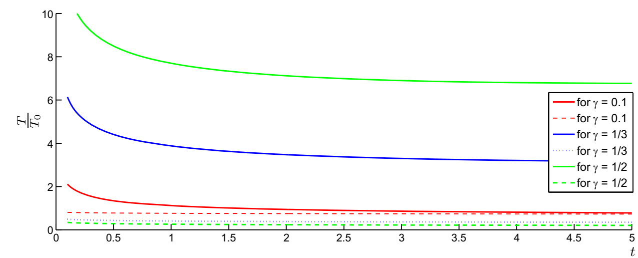

Figure 9. Solid line represents graphs between T/T_0 and t in the presence of viscosity for $A = -0.25$, $n = 2$, $B = 0.5$, $\beta = 0.5$, $\beta' = 1$ and dashed line, in the absence of viscosity.

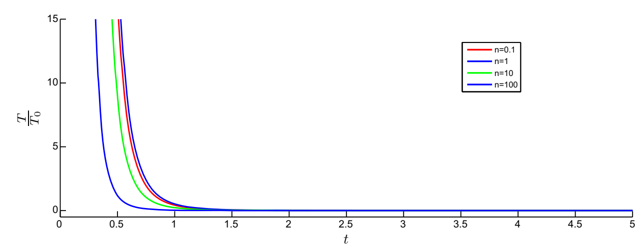

Figure 10. Variation of T/T_0 with t in the presence of viscosity in power-law model for $n = 0.1, 1, 10, 100$ and $\gamma = 1/3$ (where $A = -20$, $D = 2$, $a_0 = 4$, $\beta' = 2$).

intermediate scenario for the scale factor which is an essential feature for the dynamics of the Universe. From figures 1 and 2 it is noticed that energy density ρ deceases with the evolution of time in both power-law model and intermediate scenario. In figures 3 and 4, we have plotted the cosmological constant $\Lambda(t)$ against the cosmic time. It is noticed that $\Lambda(t)$ approaches zero with the evolution of the Universe in both the scenarios. It is also observed that the cosmological constant $\Lambda(t)$ in the anisotropic model (i.e. when $n \neq 1$) has lower value than the isotropic model (i.e. when $n = 1$) at a given instant of time.

Figures 5–9 show the variation of temperature for different values of γ and n in the presence or the absence of viscosity in both the scenarios. It is evident that higher value of γ leads to a Universe with

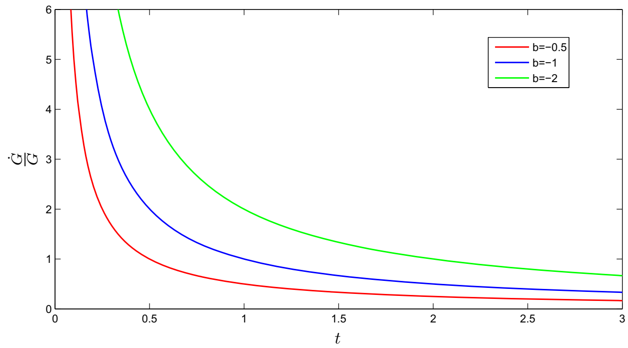

Figure 11. Variation of $\dot{G}/G$ with t in power-law model for $b = -0.5, -1, -2$.

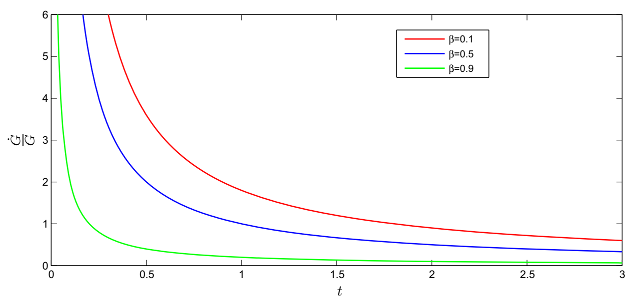

Figure 12. Variation of $\dot{G}/G$ with t in intermediate scenario for $\beta = 0.1, 0.5, 0.9$ and $b = 2$.

lower temperature at a given instant of time. The evolution of temperature of a viscous Universe is found to be more than that in a Universe without viscosity. From figure 10, it is noticed that the higher the value of n ($\neq 1$ i.e. anisotropic Universe) leads to a Universe with lower temperature at a given instant of time. Hence, the anisotropic model has lower temperature in comparison to isotropic model at a given instant of time. Here we see from figure 8 an interesting solution which suggests that the present temperature of the Universe $T \sim 3$ K in an intermediate scenario, which is in good agreement with the observed value $T \sim 2.72$ K from CMBR.

From figures 11 and 12 it is clear that the variations of $\dot{G}/G$ finally tend to a constant that satisfied the constraints from Viking Landers on Mars data [61]. $\dot{G}/G \leq 6$ in our scale and also constraints from the pulsar system PSR B1913 + 16 and PSR B1855 + 09 [62]. $\dot{G}/G \leq 9$ in our scale.

Acknowledgements

The authors express their sincere thanks to referees for their valuable comments and suggestions. The authors (RC and RR) express their sincere thanks to CSIR, New Delhi, for the financial assistance under Project No. 25(0259)/17/EMR-II. One of the authors (TS) would like to thank the Indian Institute of Advanced Study, Shimla, Himachal Pradesh, India, for financial support.

References

[1] S Perlmutter *et al*, *Astrophys. J.* **517**, 565 (1999)
[2] A G Riess *et al*, *Astron. J.* **116**, 1009 (1998)
[3] D N Spergel *et al*, *Astrophys. Ser.* **148**, 175 (2003)
[4] E Komatsu *et al*, *Astrophys. J. Suppl. Ser.* **180**, 330 (2009)
[5] M Tegmark *et al*, *Phys. Rev. D* **69**, 103501 (2004)
[6] D J Eisenstein *et al*, *Astrophys. J.* **633**, 560 (2005)
[7] B Jain and A Taylor, *Phys. Rev. Lett.* **91**, 141302 (2003)
[8] E J Copeland, M Sami and S Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006)
[9] S M Carroll, V Duvvuri, M Trodden and M S Turner, *Phys. Rev. D* **70**, 043528 (2004)
[10] T P Sotiriou and V Faraoni, *Rev. Mod. Phys.* **82**, 451 (2010)
[11] O Bertolami, G Boehmer, T Harko and F S N Lobo, *Phys. Rev. D* **75**, 104016 (2007)
[12] P A M Dirac, *Nature* **139**, 323 (1937)
[13] Abdusattar and R G Vishwakarma, *Class. Quant. Grav.* **14**, 945 (1997)
[14] A Bonanno and M Reuter, *Phys. Rev. D* **65**, 043508 (2002)
[15] E Bentivegna, A Bonanno and M Reuter, *J. Cosmol. Astropart. Phys.* **0401**, 001 (2004)
[16] C J Copi, A N Davis and L M Krauss, *Phys. Rev. Lett.* **42**, 171301 (2004)
[17] L Smolin, *Nucl. Phys. B* **160**, 253 (1979)
[18] S Adler, *Phys. Rev. Lett.* **44**, 1567 (1980)
[19] A Zee, *Phys. Rev. Lett.* **42**, 1567 (1979)
[20] C L Bennett *et al*, *Astrophys. J. Suppl.* **148**, 97 (2003)
[21] C W Misner, *Astrophys. J.* **151**, 431 (1968)
[22] G W Gibbons and S W Hawking, *Phys. Rev. D* **15**, 2738 (1977)
[23] G F R Ellis and H van Elst, *NATO ASIC Proc.* **541**, 1 (1999)
[24] E W Kolb and M S Turner, *The early Universe* (Addison-Wesley, 1990)
[25] C W Misner, K S Thorne and J A Wheeler, *Gravitation* (W.H. Freeman, New York, 1973)
[26] B L Hu and L Parker, *Phys. Rev. D* **17**, 933 (1978)
[27] S W Hawking and G F R Ellis, *The large scale structure of space–time* (Cambridge University Press, UK, 1973)
[28] V A Belinskii, I M Khalatnikov and E M Lifshits, *Adv. Phys.* **19**, 525 (1970)
[29] M A H MacCallum, *Anisotropic and inhomogeneous relativistic cosmologies in general relativity* (Cambridge University Press, UK, 1979) Chapter 11
[30] G Hinshaw *et al*, *Astrophys. J. Suppl.* **148**, 135 (2003)
[31] G Hinshaw *et al*, *Astrophys. J. Suppl.* **288**, 170 (2007)
[32] G Hinshaw *et al*, *Astrophys. J. Suppl.* **180**, 225 (2009)
[33] J Jaffe *et al*, *Astrophys. J.* **629**, L1 (2005)

[34] J Jaffe *et al*, *Astrophys. J.* **643**, 616 (2006)
[35] J Jaffe *et al*, *Astron. Astrophys.* **460**, 393 (2006)
[36] A H Guth, *Phys. Rev. D* **23**, 347 (1981)
[37] A D Linde, *Phys. Lett. B* **108**, 389 (1982)
[38] A D Linde, *Phys. Lett. B* **129**, 177 (1983)
[39] A D Linde, *Phys. Lett. B* **259**, 38 (1991)
[40] A D Linde, *Phys. Lett. B* **49**, 748 (1994)
[41] V Fayaz, M R Setare and H Hossienkhani, *Can. J. Phys.* **91**, 153 (2013)
[42] M Khurshudyan, B Paurhassan and A Pasqua, *Can. J. Phys.* **93**(**4**), 449 (2015)
[43] R Chaubey, A K Shukla and R Raushan, *Pramana – J. Phys.* **88**: 61, 65 (2017)
[44] B C Paul and P S Debnath, arXiv:1105.3307 (2011)
[45] M Khurshudyan, B Paurhassan and E O Kahya, *Int. J. Geom. Methods Mod. Phys.* **11**, 1450061 (2014)
[46] J Sodeghi, M Khurshudyan, A Movsisyan and H Farahan, *J. Cosmol. Astropart. Phys.* **12**, 031 (2013)
[47] E O Kahya, M Khurshudyan, B Pourhassan, R Myrzakulov and A Pasqua, *Eur. Phys. J. C* **75**, 43 (2015)
[48] I Muller, *Z. Phys.* **198**, 329 (1967)
[49] W Israel, *Ann. Phys.* **100**, 329 (1967)
[50] W Israel and J M Stewart, *Ann. Phys.* **118**, 341 (1970)
[51] D Pavon, D Jou and J C Vozquez, *Ann. Inst. Henri Poincare A* **36**, 79 (1982)
[52] A I Arbab, *Gen. Rel. Grav.* **29**, 61 (1997)
[53] W Zimdahl, *Phys. Rev. D* **53**, 5483 (1996)
[54] D Pavon, J Bafaluy and D Jou, *Class. Quant. Grav.* **8**, 357 (1991)
[55] R Maartens, *Class. Quant. Grav.* **12**, 1455 (1991)
[56] T S Olson and T F Jordan, *Phys. Rev. D* **35**, 3258 (1987)
[57] D Pavon, *Phys. Rev. D* **43**, 375 (1991)
[58] M D Maia and G S Silva, *Phys. Rev. D* **50**, 7233 (1994)
[59] L F B Torres and I Waga, *Mon. Not. R. Astron. Soc.* **279**, 712 (1996)
[60] M S Berman, *Int. J. Theor. Phys.* **29**, 571 (1990)
[61] R W Hellings *et al*, *Phys. Rev. Lett.* **51**, 1609 (1983)
[62] V M Kaspi, J H Taylor and M F Ryba, *Astrophys. J.* **428**, 713 (1994)