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Effect of two-loop correction in the formation of quark–gluon plasma droplet

S SOMORENDRO SINGH[1,*] and G SAXENA[2]

[1]Department of Physics and Astrophysics, University of Delhi, Delhi110 007, India
[2]Department of Physics, Government Women Engineering College, Ajmer 305 002, India
*Corresponding author. E-mail: sssingh@physics.du.ac.in

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Abstract. The effect of two-loop correction in quark–gluon plasma (QGP) droplet formation is studied by introducing the two-loop correction factor in the mean-field potential. The correction factor leads to the stability in the droplet formations of QGP at different parametrisation factors of the QGP fluid. This also shows that the gluon parameter factor shifts to a larger value from its earlier value of one-loop correction in attaining the stability of the droplets. The results show a decrease in the observable QGP droplet sizes which are found to be 1.5–2.0 fm radii with the two-loop correction. It indicates that the dynamics of the QGP droplet and the stability of the droplet with the two-loop correction factor can be controlled by the fluid parameter in the model.

Keywords. Quark–gluon plasma; surface tension.

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1. Introduction

Lattice theory indicates the phase transition [1,2] from a deconfined phase of free quarks and gluons to a confined phase of hadrons. The deconfined state of matter is broadly known as quark–gluon plasma (QGP), that was probably obtained in the formation of early Universe. The study has made the theorists and experimentalists very busy in the last two/three decades searching and identifying the formation of QGP droplets. During these decades, many highly sophisticated laboratories are set up around the globe which are focussing on finding out how the early Universe started to form. If matter is formed in the earlier time, then it is made of free quarks and gluons in such a way that its hydrodynamical expansion and subsequent cooling leads to the formation of confined colourless matter of quarks of hadrons. This indicates the complicated nature of the formation of early Universe and due to its complication it has become an exciting field in heavy-ion collider physics [3–5]. Searching for such complicated phenomena and solving mysteries are the nature of human beings. In this aspect, there are a number of phenomenological methods which try to solve the phenomena. We also try to solve the problem by considering a phenomenological potential model based on the dynamical parameter. The model tries to create free energy evolution through different quark and gluon flow parameters forming droplets of various sizes. The formation of droplet differs with the change of temperature and somehow with the parametrisation values. Depending on these parametrisation values, the droplets are determined to be stable in nature. This indicates that droplet formation is dependent on both the parametrisation value of quark and gluon and the temperature. It also determines the critical size of the droplets when transforming the phase from a quark–gluon to a confined phase of hadron droplets. So this transformation is classified as either first order, second order or cross-over phase. The first-order phase transition is obtained at very low temperature and at high chemical potential. Then, there is the second-order phase transition after it reaches a certain end point of temperature and quark chemical potential. At very low chemical potential, the system experiences cross-over phase transition. In addition to this, many researchers also reported about phase transitions depending on the number of quark flavours and its mass [6]. Here we deal with only quark–gluon phase transition and its droplet formation depending on the critical size of the droplets during the formation of QGP phase. On the basis of the critical radius of the droplet, we also calculate the surface tension considering it to be a parameter making a sharp boundary between these two phases. Moreover, the calculation of surface tension gives another important property of liquid drop model to determine the stability of droplet formation in the system.

In this paper, we focus on the effect of two-loop correction in mean-field potential in QGP droplet formation, which is an extension of our work on one-loop correction [7], through different quark and gluon parameters. Due to the effect of this additional term of two loop to the one-loop correction, the stability of the QGP droplet is modified on the scale of parameters. To evaluate the droplet formation, thermodynamic partition function is incorporated with the correction of two loop through the mean-field potential of the Hamiltonian of the system. The partition function is correlated through Gibbs free energy which is developed through the density of states [8,9]. The density of state can be established using the earlier process of Thomas and Bethe model incorporating one- and two-loop correction in the potential. The correction in the potential with loops affects the droplet size and impacts on the stability of droplet formation with the variation of dynamical quark and gluon flow parameters.

The paper is organised as: In §2, we briefly try to construct the Hamiltonian of the system incorporating one-loop correction extending to the two-loop correction factor in the potential and set up the Gibbs free energy with the effect of the two-loop correction. In §3 the free energy and surface tension of the system symbolising stable droplet formation of QGP is explained. In §4, the analytical solutions as well as results are discussed. In §5, the conclusion with details of stable droplet formation of QGP with different flow parametrisation values are presented.

2. Hamiltonian and mean-field potential with two-loop correction

The dynamical behaviours of quarks, antiquarks and gluons in QGP force us to identify interacting potentials among quark–quark, quark–antiquark, quarks and gluons, which in turn, give bulk thermodynamical and hydrodynamical properties of the particles of the system. The computation of interaction mean-field potential in perturbed QCD is earlier considered among the massive quarks and its extension is considered among the massless quarks also. We considered this concept of perturbed QCD as not only Coulombs potential but also as a tree-level potential arising from gluon exchange. So, one-loop correction and two-loop correction among the internal quarks are required for the calculation of actual interacting potential. Moreover, many theorists have given importance to the two-loop corrections leading several investigations. These are reported in [10–12]. So to look at the properties of QGP formation we use the effective mean-field potential in which one-loop and two-loop correction factors are involved. Due to the one-loop correction factor we have already obtained all the properties in our earlier papers [13,14] and as an extension to this one-loop correction we again look at the improvement of thermodynamic parameters obtained through the two-loop correction in the mean-field potential. Due to the two-loop correction we get improved results in all the parameters we calculated. Thus, the effective mean-field potential for QGP is calculated through the thermal mass formalism and the thermal mass is obtained in the following through the corresponding Hamiltonian of the confining/deconfining potentials among them. This leads to the thermal Hamiltonian as [15–17]

$$\begin{aligned} H(k,T) &= [k^2 + m^2(T)]^{1/2} \\ &= k + m^2(T)/2k \text{ for large } k \end{aligned} \tag{1}$$

$$H(k,T) = k + m_0^2/2k - \{m_0^2 - m^2(T)\}/2k, \tag{2}$$

where

$$m^2(T) = \frac{16\pi}{k} \frac{\sqrt{(}\gamma_q^2 + \gamma_g^2)}{\gamma_q \gamma_g} \alpha_s(k) T^2 \times \left[1 + \frac{\alpha_s(k)}{4\pi} a_1 + \frac{\alpha_s^2(k)}{16\pi^2} a_2\right]. \tag{3}$$

The thermal mass obtained after one- and two-loop corrections are introduced in the potential. The coefficients used in the thermal mass a_1 and a_2 are the one- and two-loop correction factors which are obtained through the interactions among the constituent particles by these loop corrections. They are defined numerically depending on the number of quark flavours and they are given as

$$a_1 = 2.5833 - 0.2778\, n_l, \tag{4}$$

$$a_2 = 28.5468 - 4.1471 n_l + 0.0772 n_l^2, \tag{5}$$

where n_l is the number of light quark elements [18–21], k is the quark (gluon) momentum, m_0 is the dynamic rest mass of the quark and T is the temperature. $\alpha_s(k)$ is the QCD running coupling constant defined as

$$\alpha_s(k) = \frac{4\pi}{(33 - 2n_f)\ln(1 + k^2/\Lambda^2)}, \tag{6}$$

where Λ is the QCD parameter having the value of 0.15 GeV. n_f is the degree of freedom of quark and gluon. So the interacting mean-field potential $V_{\rm conf}(k)$ is now obtained by including two-loop correction factor from simple confining potential obtained through the Hamiltonian and it is modified from the earlier potential of one-loop correction. The modified potential is now expressed through the expansion of strong coupling constants of two-loop factor within the perturbation theory as [22–26]

$$V_{\rm conf}(k) = \frac{8\pi}{k}\frac{\sqrt{(\gamma_q^2+\gamma_g^2)}}{\gamma_q\gamma_g}\,\alpha_s(k)T^2\left[1+\frac{\alpha_s(k)a_1}{4\pi}+\frac{\alpha_s^2(k)a_2}{16\pi^2}\right]-\frac{m_0^2}{2k}, \quad (7)$$

where the loop coefficients a_1 and a_2 play roles in the creation of interacting potential and in the formation of QGP droplet. The potential is calculated within the limits of perturbation theory and it can be expressed as the expansion of strong coupling constant, α_s. Due to this, we found that the quark and gluon parametrisation factors are correlated through the coupling constant. In addition, the parametrisation factors are defined as $\gamma_q = 1/14$ and $\gamma_g = (48–60)\gamma_q$. These factors play many functional roles in the creation of droplets. First it determines the critical droplet formation. It then increases the dynamics of QGP flow and it also enhances the process of transforming QGP droplets to hadron droplets. It overall controls the formation of stable droplets. Its value varies from non-loop correction to loop correction. With the incorporation of these parameters, the density of states in phase space with loop corrections in the interacting potential is modified and obtained through a generalised Thomas–Fermi model as [7,27–30]

$$\rho_{q,g}(k) = \frac{v}{3\pi^2}\frac{{\rm d}V_{\rm conf}^3(k)}{{\rm d}k} \quad (8)$$

or

$$\rho_{q,g}(k) = \frac{v}{\pi^2}\left[\frac{(\gamma_q^2+\gamma_g^2)^{3/2}T^2}{2\gamma_q^3\gamma_g^3}\right]^3 \times\left[1+\frac{\alpha_s(k)a_1}{4\pi}\right]^3 g^6(k)B, \quad (9)$$

where

$$B = \left[1+\frac{\alpha_s(k)a_1}{\pi}+\frac{\alpha_s^2(k)a_2}{\pi^2}\right]^2 \times\left[\frac{(1+\alpha_s(k)a_1/\pi+\alpha_s(k)^2a_2/\pi^2)}{k^4} +\frac{2(1+2\alpha_s(k)a_1/\pi+3\alpha_s(k)^2a_2/\pi^2)}{k^2(k^2+\Lambda^2)\ln(1+(k^2/\Lambda^2))}\right] \quad (10)$$

and v is the volume occupied by the QGP and $g^2(k) = 4\pi\alpha_s(k)$.

3. The free energy and surface tension with two-loop correction

The free energy of quarks and gluons after incorporating the two-loop correction in the density of state is defined as follows [28,31]:

$$F_i = -\eta T g_i \int {\rm d}k\rho_{q,g}(k)\ln(1+\eta{\rm e}^{-(\sqrt{m_i^2+k^2})/T}), \quad (11)$$

where a positive value of η gives the contribution from the bosonic particle and a negative value of η gives the contribution from the fermionic particles. The extreme in the potential obtained by minimising the confining potential in terms of momentum is given as

$$V(k_{\rm min}) = \left[\frac{8a_1\sqrt{(\gamma_q^2+\gamma_g^2)}N^{1/3}T^2\Lambda^4}{27\pi^2\gamma_q\gamma_g}\right]^{1/6}, \quad (12)$$

where $N = (4/3)[12\pi/(33-2n_f)]$, which is different from the earlier value of minimum potential. It is much smaller so that the free energy has larger contribution in the entire calculation. So the minimum cut-off in the model leads the integral to more accurate finite value, just avoiding the infra-red divergence while taking the magnitude of Λ and T as of the same order of lattice QCD. g_i is the degeneracy factor (colour and particle–antiparticle degeneracy) which is six for quarks and eight for gluons. The interfacial energy obtained through a scalar Weyl surface in Ramanathan *et al* [22,32] with a suitable modification to take care of the hydrodynamic effects is given as

$$F_{\rm interface} = \frac{\sqrt{(\gamma_q^2+\gamma_g^2)}}{4\gamma_q\gamma_g}R^2T^3. \quad (13)$$

The interfacial energy replaces the bag energy of MIT model minimising the drawback produced by MIT model. Here, we use light hadrons and their energies are defined as [33]

$$F_h = (d_iT/2\pi^2)v\int_0^\infty k^2{\rm d}k\ln(1-{\rm e}^{-\sqrt{m_h^2+k^2}/T}), \quad (14)$$

where d_i is the degeneracy factor for different light hadronic particles and m_h is the corresponding light hadron masses. We considered only light hadrons as they are produced the maximum in the reaction plane. To calculate the total free energies, the particle masses are taken as quark masses $m_u = m_d = 0$ MeV and $m_s = 0.15$ GeV. Now we can compute the total modified free energy $F_{\rm total}$ as

$$F_{\text{total}} = \sum_j F_j + F_h, \tag{15}$$

where j stands for u, d, s quark, interface and gluon.

Now, we can also calculate the surface tension with two-loop correction in the potential and the calculation is done through difference relation between free energy of the QGP phase and the light element hadrons phase. It means that there is a sharp boundary between hadron and quark phases, balancing the pressure and keeping chemical equilibrium in the mixed phase. However, the calculation of surface tension can be highly affected by the mixed phase in which large finite size effects are included [34,35]. To exclude the effects of finite size, we remove the mixed phase system in the present calculation. So the difference in energy of the two phases define critical phase transition of the liquid drop model after neglecting the finite size effects and shape contribution. It is therefore given as

$$\Delta F = -\frac{4\pi}{3} R^3 [P_{\text{had}}(T) - P_{q,g}(T)] + 4\pi R^2 \sigma, \tag{16}$$

where the first term represents pressure difference and the second term represents the contribution from the surface tension. The surface tension is calculated by minimising the above expression with respect to the droplet size R. So, the surface tension formula is obtained as

$$R_c = \frac{2\sigma}{\Delta p} \quad \text{or} \quad \sigma = \frac{3\Delta F}{4\pi R_c^2}, \tag{17}$$

where ΔF is the change in the free energy and R_c is the corresponding critical radius obtained at the transition point from quark droplet to hadron droplet.

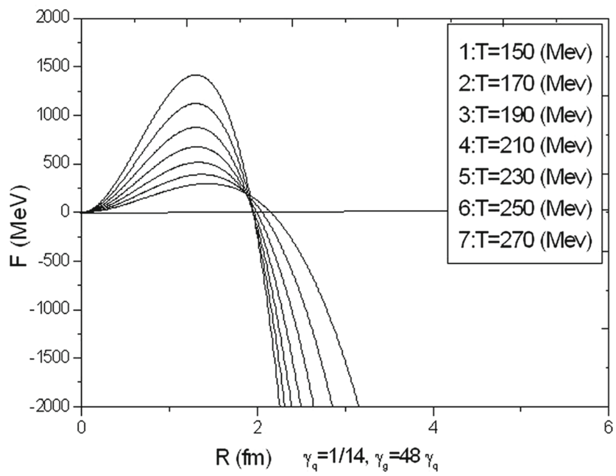

Figure 1. The free energy vs. R when $\gamma_q = 1/14$ and $\gamma_g = 48\gamma_q$ for various temperatures.

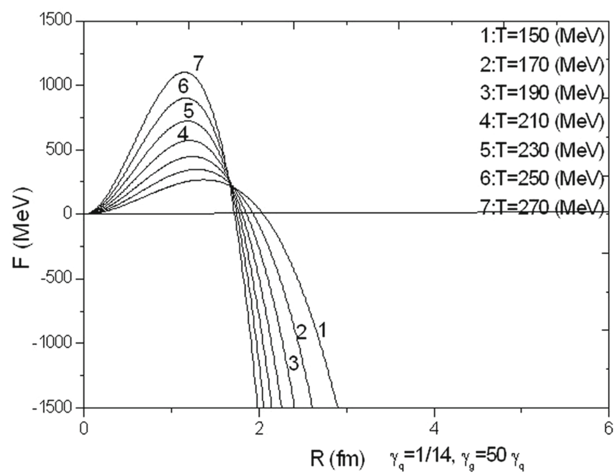

Figure 2. The free energy vs. R when $\gamma_q = 1/14$ and $\gamma_g = 50\gamma_q$ for various temperatures.

4. Results

The effects of two-loop correction factor in QGP droplet formation in the interacting mean-field potential is numerically calculated. Due to the inclusion of the two-loop correction, the QGP droplet changes a lot from one-loop correction and without loop correction. The modifications in the droplet sizes are replicated in the figures showing changes in the amplitude in the free energies. The results in the free energies are modified by the quark and gluon flow parameters involved in the two-loop correction. These parameters really play the role of regularising the stability of the droplets and acting as phenomenological parameters in heavy-ion collision. In figure 1, we can see the stability evolution of the droplet at the particular quark and gluon flow parametrisation factors $\gamma_q = 1/14$, $\gamma_g = 48\gamma_q$. There is stable droplet formation at all the temperatures forming a droplet of 2.0 fm radius under the effect of two-loop correction. Figure 2 shows the increase in the gluon flow parameter keeping the quark flow parameter fixed. We get stable droplets for all the temperatures at another gluon flow parameter $\gamma_g = 50\gamma_q$. The size of the droplet is found to be around $R \leq 2.0$ fm when gluon parametrisation $\gamma_g = 50\gamma_q$. It means that stability of the droplet is really observed for different temperatures at certain range of gluon parameters and at a fixed quark parameter. In these droplets, the obtained free energy amplitudes are less than 2.0 GeV and at lesser gluon parameter the energy amplitude is larger with large stable droplet size. In figure 3, we further increase the gluon flow parameter. We obtain a slightly stable droplet with the increase of gluon flow parameter, $\gamma_g < 52\gamma_q$, and the amplitude of the free energy is found to be lesser. This indicates that by adopting the flow parameter in the range $\gamma_q = 1/14$ and $\gamma_g <= 52\gamma_q$, stability of the QGP droplet increases

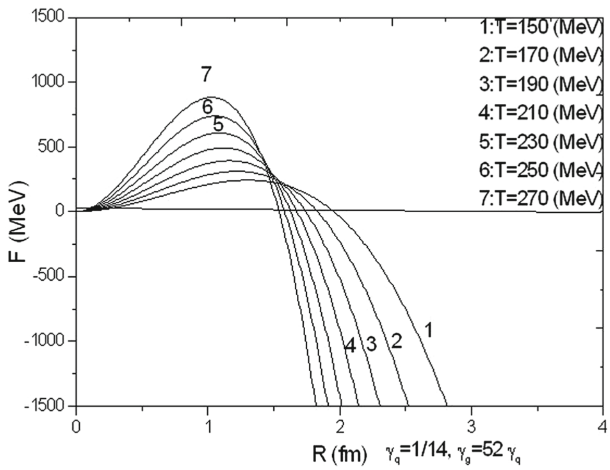

Figure 3. The free energy vs. R when $\gamma_q = 1/14$ and $\gamma_g = 52\gamma_q$ for various temperatures.

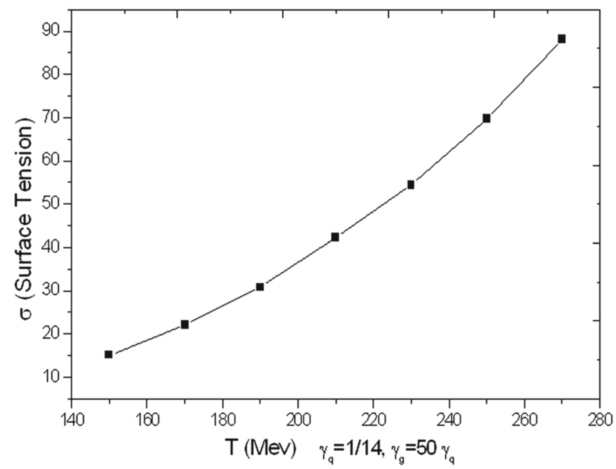

Figure 5. The surface tension vs. T when $\gamma_q = 1/14$ and $\gamma_g = 50\gamma_q$.

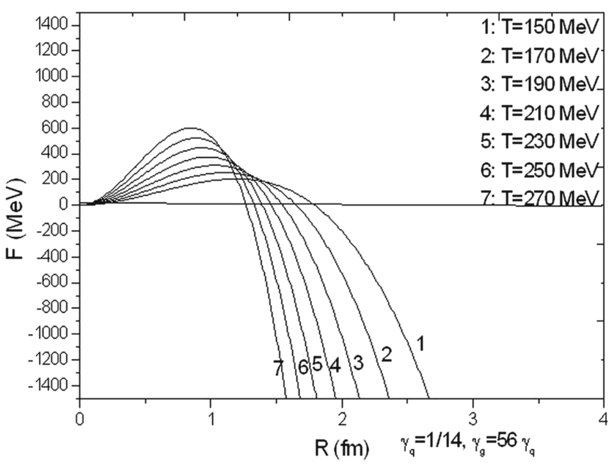

Figure 4. The free energy vs. R when $\gamma_q = 1/14$ and $\gamma_g = 56\gamma_q$ for various temperatures.

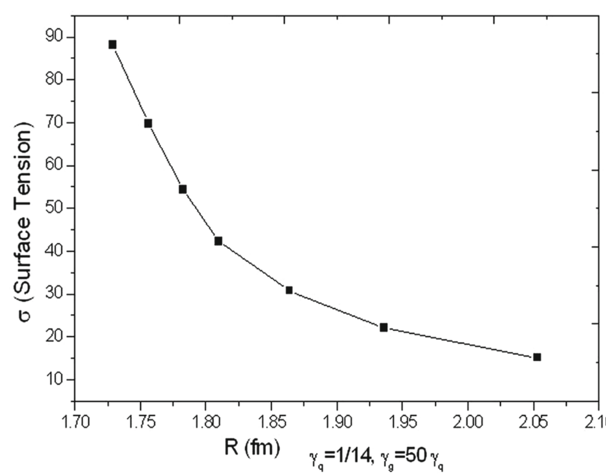

Figure 6. The surface tension vs. R_c when $\gamma_q = 1/14$ and $\gamma_g = 50\gamma_q$.

and the amplitude of the free energies drop down with increasing gluon flow parameter. In figure 4, we further increase the gluon parameter up to $\gamma_g = 56\gamma_q$ and we observe unstable droplet formation as we increase the gluon parameter from $\gamma_g = 52\gamma_q$ to $56\gamma_q$. It shows that the instability of the droplets starts from the gluon parameter $\gamma_g = 52\gamma_q$ with QGP droplet formation under the effect of two-loop correction in the potential. This effect of obtaining instability droplet may be a characteristic factor for nucleation and transformation to hadronic matter. Further, we get the cross-over phase transition in which nucleation part of the system is completely stopped due to uncertainty in distinction of two phases of quark–gluon and hadronic matter. Such effects are also obtained earlier in one-loop correction when the droplet sizes are bigger. By adding two-loop correction in the potential, the size of the droplet decreases and stable droplet formations are tightly bound in comparison to the earlier droplet formation of one-loop correction. So stable droplet formation is specially found in the range of parametrisation factor $48\gamma_q \leq \gamma_g \leq 52\gamma_q$ with the loop correction and the magnitude of the free energy with the stability is modified by these quark and gluon flow parameters.

We again calculate the surface tension at these particular quark and gluon flow parameters where more stable droplets are obtained as the surface tension of the droplet shows the characteristic feature of fluid to determine the stability of the droplet. On the basis of surface tension, the stable droplet features are shown in figures 5–7. Figure 5 indicates the increasing order of surface tension with increasing temperature at the parametrisation values of $48\gamma_q \leq \gamma_g \leq 52\gamma_q$. In figure 6, we observe the decreasing order of surface tension with the increase

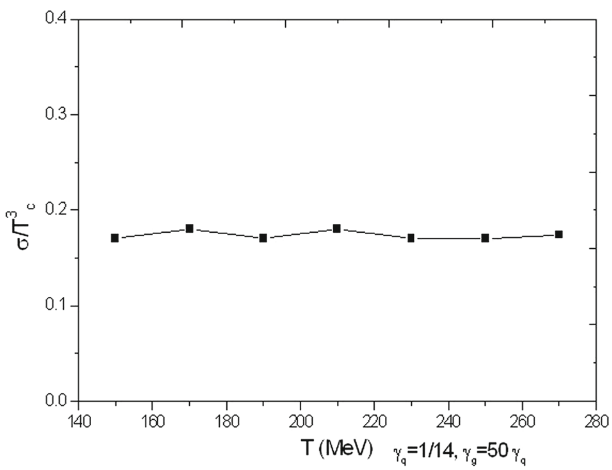

Figure 7. The surface tension/T_c^3 vs. T when $\gamma_q = 1/14$ and $\gamma_g = 50\gamma_q$.

in critical radius of the droplets. As the size of the droplet is smaller we get larger surface tension so that QGP droplets are tightly bound and more stable. By increasing the size of the droplet, the surface tension is bound to be lesser. In figure 7, we plot the ratio of surface tension to the cube of critical temperature showing constancy of σ/T_c^3 with the temperature [36] showing comparative result with the lattice data. The result is found to be $\sigma = 0.173T_c^3$ which is almost equal to the lattice result (=$0.2T_c^3$) [37–39]. So, the inclusion of two-loop correction in the mean-field potential with these parametrisations improve and enhance the stability of the QGP droplet. So, the parameter is like the Reynold's number which shows characteristic features of fluid dynamics. It means that quark and gluon parameters can have some other characteristic value to determine the stability of QGP droplets.

5. Conclusion

The results show the effects of two-loop correction in the mean-field potential on the stability of the droplet. The effects of stability is increased when the droplet size decreases as indicated by figure 6. The size of the droplet is more affected by the gluon flow parameter. If the gluon parameter is increased beyond $\gamma_g \geq 52\gamma_q$, then unstable droplet starts forming and it is difficult to predict the size of the droplet. In the range of the gluon flow parameters, say $48\gamma_q \leq \gamma_g \leq 52\gamma_q$, stable droplets are formed and the stability is more in the case of two-loop correction than in the case of one-loop correction and without loop correction [7,17]. It indicates that two-loop correction with the dynamical flow parameter can enhance stable droplet formation. This is another possible indication that evolution of QGP fireball is steady fluid dynamics depending on some kind of dynamical parameter which plays in forming the stable droplets.

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