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Test of isospin conservation in thermal neutron-induced fission of ^{245}Cm

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Abstract. We have recently shown that the general trends of partition-wise fission fragment mass distribution in heavy-ion-induced compound nuclear (CN) fission of heavy nuclei can be reproduced reasonably well by using the concept of isospin conservation, hence providing a direct evidence of isospin conservation in neutron-rich systems [Jain *et al, Nucl Data Sheets* **120**, 123 (2014); Garg and Jain, *Phys. Scr.* **92**, 094001 (2017); Jain and Garg, *EPJ Web of Conference* **178**, 05007 (2018); Garg *et al, Phys. Scr.* **93**, 124008 (2018)]. In this paper, we test the concept of isospin conservation to reproduce the fission fragment mass distribution emerging from thermal neutron-induced CN fission reaction, ^{245}Cm(n_{th}, f). As earlier, we use Kelson's conjectures [I Kelson, Proceedings of the Conference on Nuclear Isospin (Academic Press, New York, 1969)] to assign isospin to neutron-rich fragments emitted in fission, which suggest the formation of fission fragments in isobaric analogue states. We calculate the relative yields of neutron-rich fragments using the concept of isospin conservation and basic isospin algebra. The calculated results reproduce the experimentally known partition-wise mass distributions quite well. This highlights the usefulness of isospin as an approximately good quantum number in neutron-rich nuclei. This also allows us to predict the fragment distribution of the most symmetric Cd–Cd partition and the heavier mass fragment distributions, both not measured so far.

Keywords. Isospin conservation; isobaric analogue states; neutron-rich nuclei; thermal neutron fission; fission fragment distribution.

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1. Introduction

Isobaric spin, or isospin, depicts different electromagnetic states of a particle such as a nucleon, and is a fundamental tool for studying various nuclear processes [1–6]. In nuclear physics, one generally assigns isospin projection $T_3 = +1/2$ for neutron and $T_3 = 1/2$ for proton, which are two states of a nucleon having total isospin $T = 1/2$. Isospin behaves in the same way as spin and follows the SU(2) algebra for nucleons. In particle physics, isospin can have values other than $1/2$ also according to the various sets of particles involved; for example for pions, isospin $T_3 = +1$ for π^+, 0 for π^0 and 1 for π^-, together forming an isospin triplet for $T = 1$. However, isospin can assume very large values in nuclei, particularly in heavy nuclei and neutron-rich systems, where $N > Z$.

An early review by Robson [7] presented the details of isospin algebra and also the selection rules involving isospin quantum number. Generally, isospin is considered to be very useful for light nuclei as it is a conserved quantity there. It is also relatively easy to assign isospin to these nuclei [8]. In heavy mass nuclei, the Coulomb interaction becomes large and isospin mixing is thought to be significant, suggesting that isospin is not a good and useful quantum number.

A very lucid and succinct discussion of various aspects of isospin impurity in heavy nuclei was presented by Soper as early as 1969 [9], who underlined that until 1961, hardly any physicist would have taken isospin seriously as a quantum number beyond $A = 60$. These assumptions were soon questioned by the discovery of isobaric analogue states (IAS) in (p, n) reactions on nuclei near $A = 90$ [10].

In our recent works, we have been focussing on heavy mass nuclei which are natural $N > Z$ systems. Lane and Soper [11] obtained a very interesting and useful result by using the perturbation method which indicates that mixing of ground-state isospin with the states having one unit higher isospin value decreases as the neutron excess increases. They considered the nucleus to be made up of a $(N = Z)$ core and $(N - Z)$ excess neutrons and calculated the impurity generated by the Coulomb potential of protons. It was shown that the impurity decreases with neutron enrichment, making isospin nearly a good quantum number in neutron-rich systems. Sliv and Kharitonov [12] also calculated the isospin mixing of $T = T_0 + 1$ into the ground state having $T = T_0$ using the Coulomb potential as a perturbation and eigenfunctions of harmonic oscillator. They estimated an isospin impurity of about 2% for ^{16}O which rises up to 7% on reaching ^{40}Ca. The impurity, however, starts decreasing as we move towards the heavier nuclei with $N > Z$ along the β-stability line, eventually reducing to 2% for ^{208}Pb. Bohr and Mottelson [13] have discussed the role of isospin in heavy nuclei and also calculated the isospin mixing using the hydrodynamical model and concluded that isospin mixing is indeed very small for neutron-rich nuclei compared to $N = Z$ nuclei. Auerbach [14] in his review has compared the results of isospin mixing obtained from various approaches such as the shell model, hydrodynamical model, random phase approximation (RPA), etc. and concluded that isospin impurity continues to decrease with increasing neutron excess.

These findings, along with our earlier results on heavy-ion-induced fusion–fission reactions, have encouraged us to test the validity of isospin as a good quantum number in the relative yields of fission fragments in thermal neutron-induced compound nuclear (CN) fission of heavy nuclei. We use the same methodology which has been discussed earlier in [1–4]. We note that the availability of precise data, where partition-wise fragment mass distributions are known to the precision of one mass unit, is still very scarce. This, however, is a must to test the idea of isospin conservation.

We may emphasise that this paper does not calculate the fission fragment distributions from the first principles; our calculations rather take important inputs from the experimental data and show that the idea of conservation of isospin can reproduce the fission fragment distribution rather well, notwithstanding the crucial role that the shell effects play. In this paper, we analyse the fission fragment data from the reaction $^{245}\mathrm{Cm}(n_{\mathrm{th}}, \mathrm{f})$ as reported by Rochman *et al* [15]. We show that the experimental data of light mass fragments, available from Rochman *et al* [15], may be reproduced reasonably well, confirming the approximate validity of isospin in heavy nuclei. This also allows us to predict the mass distribution for heavy mass fragments and for the most symmetric partition, Cd–Cd.

2. Formalism

In thermal neutron-induced fission, a neutron is incident on a target X to form the CN which further fissions into two fragments F_1 and F_2 with the emission of n number of neutrons:

$$\mathrm{neutron}\left(\frac{1}{2}, \frac{1}{2}\right) + X(T_X, T_{3_X}) \rightarrow \mathrm{CN}(T_{\mathrm{CN}}, T_{3_{\mathrm{CN}}})$$
$$\rightarrow F_1(T_{F1}, T_{3_{F1}}) + F_2(T_{F2}, T_{3_{F2}}) + n.$$

In order to test the validity of the isospin as a good quantum number in the fission process, we use the same formalism as reported earlier in our works [2–4]. Our formalism is divided into two parts: the first part is to assign isospin to all the constituents present in the reaction and the second part is to calculate the relative yields of fission fragments emitted in fission based on the assigned isospin and related algebra.

2.1 *Assignment of isospin*

We start by assigning isospin to CN. The incident neutron has an isospin $T = T_3 = 1/2$. The target nucleus, assumed to be in its ground state, has minimum possible value of isospin $T = T_{3_X}$, where $T_{3_X} = (N - Z)/2$. Therefore, isospin of the CN, T_{CN}, should lie between $|T_{3_X} - 1/2|$ and $(T_{3_X} + 1/2)$. The third component of the isospin of CN obviously has the value, $T_{3_{\mathrm{CN}}} = (T_{3_X} + 1/2)$. As the third component of the isospin can have a value either less than or equal to the total isospin, $T_3 \leqslant T$, it implies that the only possible value for $T_{\mathrm{CN}} = T_{3_{\mathrm{CN}}} = (T_{3_X} + 1/2)$. For example, in the present case of thermal neutron-induced fission $^{245}\mathrm{Cm}(n_{\mathrm{th}}, \mathrm{f})$, the isospin of the target is $T(^{245}\mathrm{Cm}) = 26.5$. The isospin of the CN, ^{246}Cm, can have two possible values, $T_{\mathrm{CN}} = 26$ and 27, whereas $T_{3_{\mathrm{CN}}} = 27$. Therefore, the CN has a unique possible value of isospin, $T_{\mathrm{CN}} = T_{3_{\mathrm{CN}}} = 27$.

We now proceed to assign isospin values to various fission fragments. Before proceeding further, we introduce an auxiliary concept of the residual compound nucleus (RCN) which is formed after the emission of n number of neutrons from CN. Here, we have assumed that all the neutrons are emitted in one go, and no distinction is being made between pre-scission and post-scission neutrons, an approximation that seems to work well for our purpose. This simplifies the problem of many-body system to a two-body system. The third component of the isospin of RCN is, therefore, given by

$$T_{3_{\mathrm{RCN}}} = T_{3_{\mathrm{CN}}} - n/2 = T_{3_{F1}} + T_{3_{F2}}.$$

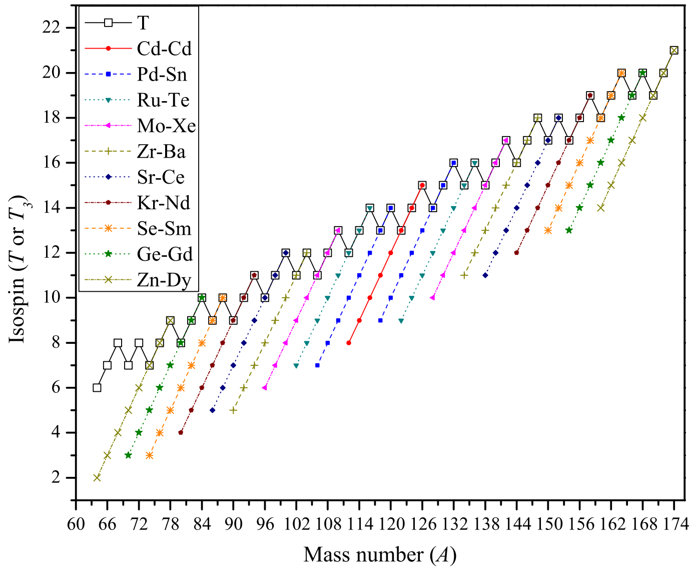

Figure 1. Assigned values of isospin T or T_3 vs. mass number A of the fission fragments emitted in the reaction $^{245}Cm(n_{th}, f)$. Open squares connected by the solid line show the isospin T assigned to each mass number. Other symbols show the T_3 values for fragments of different partitions. One particular type of symbol is used to connect T_3 values for the fragments of a distinct partition. Out of the two lines connecting the same type of symbols, the one on the right-hand side is for the heavier and the one on the left-hand side is for the lighter fragments.

The isospin value of RCN, therefore, may have a range of values given by

$$|T_{CN} - n/2| \leq T_{RCN} \leq (T_{CN} + n/2). \quad (1)$$

Alternatively, it should also satisfy

$$|T_{F1} - T_{F2}| \leq T_{RCN} \leq (T_{F1} + T_{F2}). \quad (2)$$

We fix the value of T_{RCN} by using what we now term as Kelson's conjectures [5]. Kelson [5] considered the role of isospin and IAS in the fission phenomenon, and found it to be very useful and important in assigning isospin values to the fission fragments. We have presented detailed arguments in favour of these conjectures in our previous paper [4]. These two conjectures are: (i) as more and more neutrons are emitted in fission, the probability for the formation of highly excited states with $T > T_3$ increases. (ii) The fission fragments are preferably formed in IAS.

Using Kelson's first conjecture, we assign the isospin value of RCN as $T_{RCN} = T_{F1} + T_{F2}$ with the ridding condition that it lies within the range given in eq. (1). We then proceed to assign isospin values to the neutron-rich fission fragments for which we use Kelson's second conjecture. We choose three isobars corresponding to each mass number. These have same mass number but differ in T_3 values by two units, e.g. T_3, $T_3 + 2$ and $T_3 + 4$. As per Kelson's second conjecture, the fission fragments are preferably formed in IAS and, therefore, we consider the IAS of these three isobars for each mass number. We assign each mass number the isospin value T which is maximum among the three T_3 values, i.e. $T_3 + 4$, as this is the minimum value needed to generate all the members of any isobaric multiplet. For example, for mass number $A = 94$, we have ^{94}Kr, ^{94}Sr and ^{94}Zr, which have T_3 values 11, 9 and 7, respectively. Therefore, we assign isospin $T = 11$ to $A = 94$ which is the maximum of the three T_3 values. We assign isospin value T to each mass number in a similar fashion. The assignments so made are illustrated in figure 1.

We note that the experimental data are known only for the light mass fragments in a pair of fission fragments [15]. Therefore, to perform the complete calculations, we must also consider the corresponding heavy mass fragments. This gives us nine partitions, namely Pd–Sn, Ru–Te, Mo–Xe, Zr–Ba, Sr–Ce, Kr–Nd, Se–Sm, Ge–Gd and Zn–Dy (from the symmetric combination to the most asymmetric combination). In addition, we also consider the most symmetric combination Cd–Cd to complete three members for each mass number, although we do not have any experimental data on this partition. The assigned T values for each mass number are shown by open squares in figure 1. We can see from the figure that around the central partition Cd–Cd, isospin assignment is symmetric which is similar to what we obtained in our earlier work [4] for $^{208}Pb(^{18}O, f)$ and $^{238}U(^{18}O, f)$ reactions.

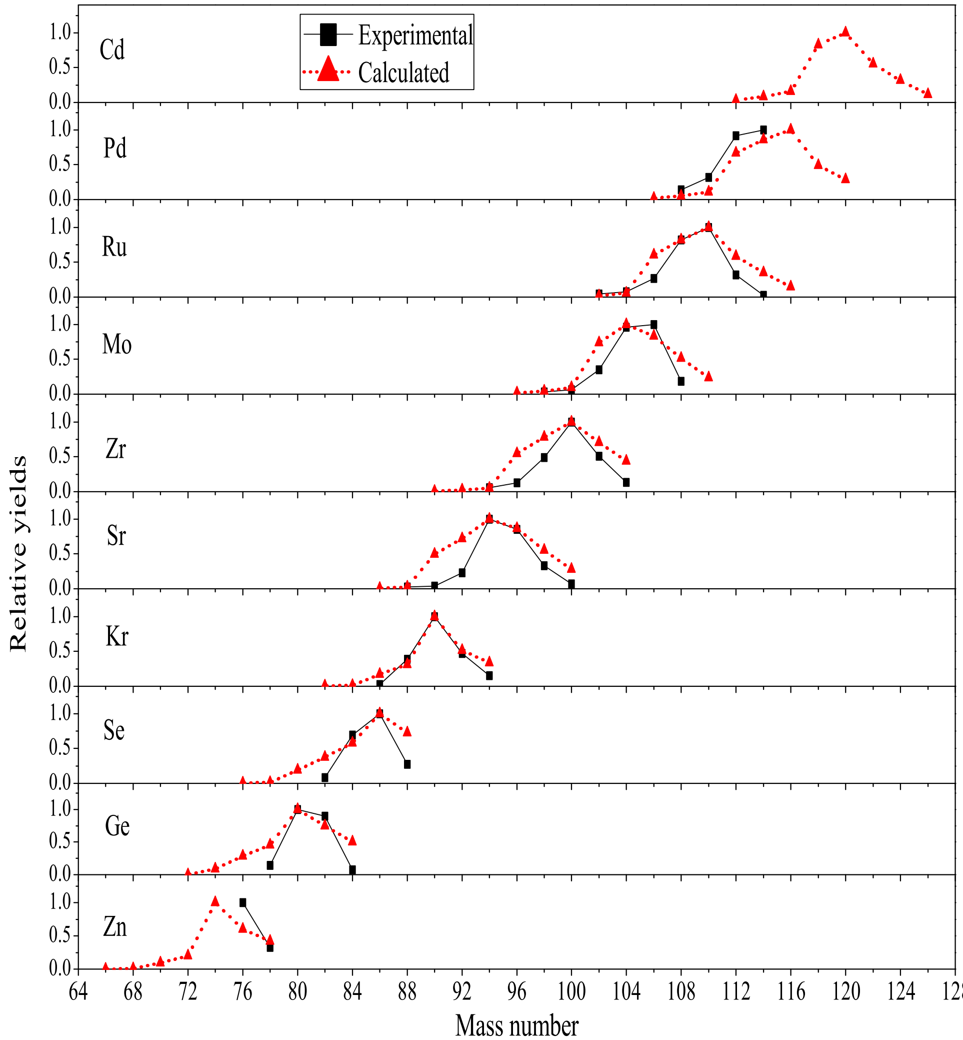

Figure 2. Comparison of the calculated and experimental relative yields of light mass fission fragments vs. mass number A for all the 10 partitions formed in ^{245}Cm(n_{th}, f). Experimental data are taken from Rochman *et al* [15]. Note that there are no observed data for the Cd–Cd partition and also many additional fragments in all the partitions.

2.2 *Calculation of relative intensities of fission fragments in different partitions*

After assigning isospin values to all the fission fragments, we now proceed to calculate their relative yields. Cassen and Condon [16] introduced isospin in wave functions so that the nuclear wave function should be antisymmetric under space, spin and isospin coordinates. For our calculation, we consider only the isospin part of the total wave function involving isospin values of RCN and two fragments, F_1 and F_2. In a particular partition, for a given n-emission channel,

$$|T_{\text{RCN}}, T_{3\text{RCN}}\rangle_n = \langle T_{F1} T_{F2} T_{3_{F1}} T_{3_{F2}} | T_{\text{RCN}} T_{3_{\text{RCN}}} \rangle \; | T_{F1}, T_{3_{F1}}\rangle | T_{F2}, T_{3_{F2}}\rangle, \quad (3)$$

where n denotes a particular n-emission channel and the first part on the left-hand side $\langle T_{F1} T_{F2} T_{3_{F1}} T_{3_{F2}} | T_{\text{RCN}} T_{3_{\text{RCN}}} \rangle$ represents the Clebsch–Gordon coefficient (CGC). The square of this CGC is proportional to the intensity of that particular pair of fragments. The yield of a particular fission fragment in a given n-emission channel for a particular partition may, therefore, be written as

$$I_n = \langle \text{CGC} \rangle^2 = \langle T_{F1} T_{F2} T_{3_{F1}} T_{3_{F2}} | T_{\text{RCN}} T_{3_{\text{RCN}}} \rangle^2. \quad (4)$$

To calculate the total yield of a fragment, we take the sum of intensities from all the n-emission channels under consideration, $I = \sum_n I_n$. In Rochman *et al* [15], there is no information about the weight factors of various n-emission channels. The average value of neutron multiplicity is 3.83 as reported by Gonnenwein [17] in a

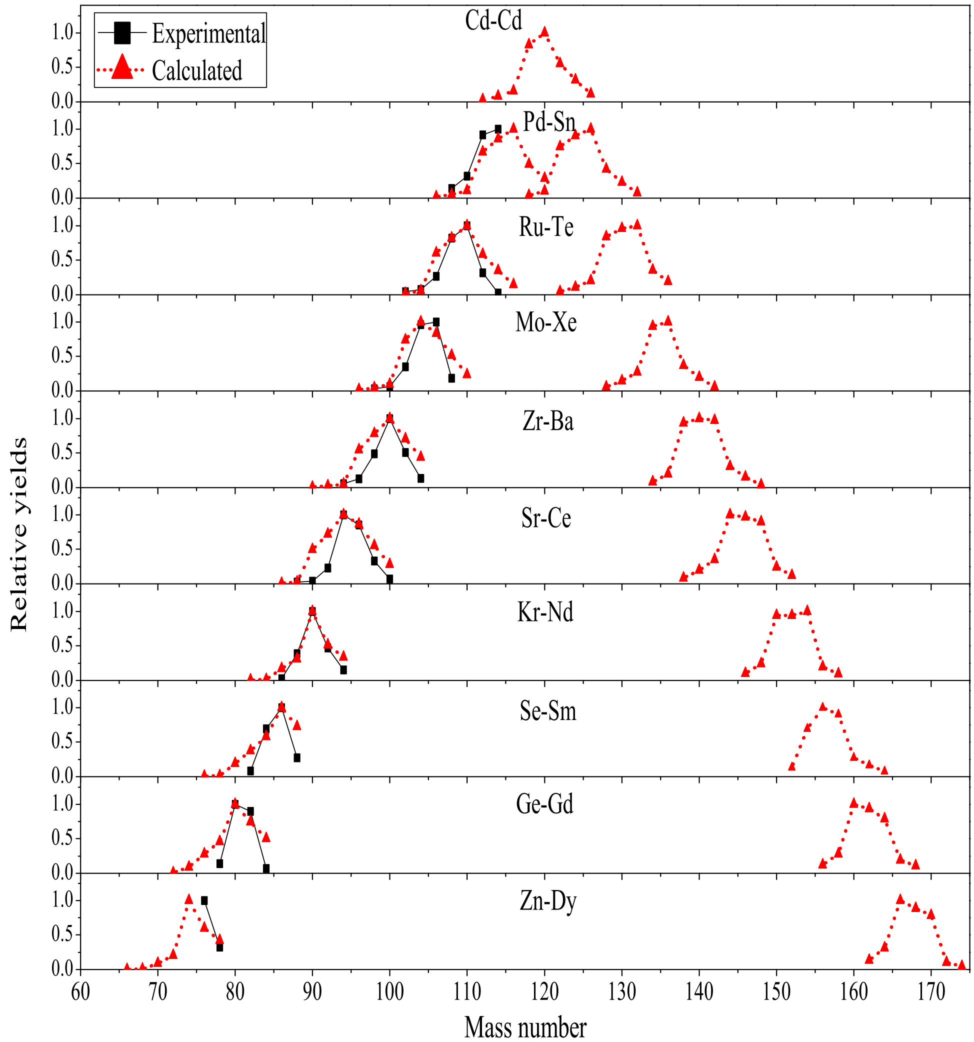

Figure 3. Comparison of the calculated and experimental relative yields of both light and heavy mass fission fragments vs. mass number *A* for all the 10 partitions formed in ^{245}Cm$(n_{\mathrm{th}}, \mathrm{f})$. Experimental data are taken from Rochman *et al* [15].

talk. We perform two sets of calculations where we first consider $4n$, $6n$ and $8n$ emission channels and then $2n$, $4n$ and $6n$ emission channels.

3. Results and discussion

We have performed the calculations of all the partitions using two combinations of n-emission channels, $4n$, $6n$, $8n$ emission channels and $2n$, $4n$, $6n$ emission channels. As these calculations provide only relative yields, we must normalise the yields of all the fragments of a partition with respect to the maximum yield fragment for both the calculated yields and experimental data [15]. Comparison with the experimental data shows that for the first six partitions, the $4n$, $6n$, $8n$ emission channel combination works very well, and for the last four partitions, the $2n$, $4n$, $6n$ emission channel combination gives quite good results. In figure 2, we plot our calculated relative yields with the experimental data for all the 10 partitions, first six from $4n$, $6n$, $8n$ emission channels and next four from $2n$, $4n$, $6n$ emission channels.

We can see that our calculated results match with the experimental data fairly well in figure 2. There are some deviations which may be due to the shell effects, presence of isomers and side feeding of levels as discussed in Danu *et al* [18,19]. The shell effects become prominent at closed shell configurations. The only closed shell configurations which may influence the data in the present case are $N = 50$ and 82. However, the total fission fragment distribution data available for light fragments only in Rochman *et al* [15], do not display any significant dips due to shell closures of the same nature as seen by Danu *et al* [18] and Bogachev *et al* [20]. Also, there will be at least 5–10% error in the data. Even then, the overall agreement is quite good. These calculations are done without the inclusion of any weight factors as these are not known from the data.

In figure 3, we plot the relative yields of both the light and heavy mass fragments. This is an approximate prediction of the heavy mass fragment distribution. We have also predicted the possible distribution of the most symmetric partition, i.e. Cd–Cd, which is plotted in figures 2 and 3.

4. Conclusion

We calculated the partition-wise relative yields of fission fragments emitted in thermal neutron-induced reaction ^{245}Cm(n_{th}, f) using the concept of conservation of isospin. For making the isospin assignments, we used Kelson's arguments who came up with the idea that the final fission fragments prefer to form in IAS with the emission of neutrons [5]. This idea helped us to assign isospin to all the fission fragments. The calculated results are in quite good agreement with the experimental data and also allowed us to predict the mass distribution of heavy fragments not known so far. We predicted the fragment distribution of the Cd–Cd partition. We also noted that there are many additional fragments in each partition for which no measurements are available. There are deviations at some points which may have many possible reasons like shell effects or presence of isomers. Also, we expect that there will be at least 5–10% error in the experimental data. But, if we look at the complete description presented here, then we can say that isospin plays a very significant role in fission. This also confirms Lane and Soper's idea of isospin purity in neutron-rich nuclei [11]. The predictions made for the heavier fission fragments and the Cd–Cd partition stand as a challenge for the experimentalists. We believe that the modern fragment separators and/or γ-ray tagging of fission fragments by using γ-ray arrays [18–21] may be the right approach to identify them. These ideas may also help to predict the fission fragment mass distribution more precisely if included in the theories of nuclear fission.

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