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Thermodynamics analysis of Ricci dark energy models in bouncing Universe

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Abstract. Ricci dark energy (RDE) model in the framework of bouncing Universe is considered. Both the interacting and the non-interacting cases of the Ricci model of dark energy have been studied. Expressions for important cosmic parameters are reconstructed for the assumed model. It is noticed that the Universe undergoes a continuous expansion with a negative deceleration parameter for two scenarios. Also, Om diagnostic parameter has been established showing a type of quintessence-like behaviour over the given time range. The total entropy of the system is calculated and the validity of the generalised second law of thermodynamics is studied.

Keywords. Dark energy; bouncing Universe; second law of thermodynamics.

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1. Introduction

The idea of accelerating expansion of the Universe that is supported by recent observations shows that the Universe expands without limit [1,2]. Researchers believe that this behaviour is due to a type of negative pressure energy known as dark energy (DE). This energy plays the main role of the cosmic acceleration [3–5]. Actually, some recent models are proposed to understand the nature of DE [6–13].

Li [14] studied DE by assuming that the infrared cut-off is represented by the Universe future event of horizon (the largest distance which the emitted light can ever reach the future observer). This outstanding work gives a solution for accelerated expansion behaviour of the Universe. In [15] Ricci dark energy (RDE) model is studied using the Ricci scalar curvature $L = R^{-1/2}$ as infrared cut-off. This model makes a great contribution in that the coincidence problem is completely solved in the context of this model. Granda [16] assumed what is known as a modified Ricci DE model. This model assumes that the DE density is a function of the curvature of Universe, and the Hubble parameter H and its first cosmic time derivative are calculated [17].

The idea of oscillating Universe through a series of expansions and contractions is discussed in [18,19]. In this model, the Universe reaches a state of maximum expansion which is known as turnaround, then it starts to recollapse. Once it reaches its smallest extent at the bounce, it begins again to expand. Actually, the idea of the bouncing Universe differs from the model of cyclic Universe, in which the phantom DE model is responsible for cosmic acceleration [20–22].

In this work, we shall consider the RDE model together with the bouncing Universe to study the evolution of our Universe.

The structure of this paper is as follows. In §2, the behaviour of the RDE model in the bouncing Universe framework is considered, then some cosmological parameters are deduced. In §4, we study thermodynamics related to our model. Finally, the conclusion is presented in §5.

2. RDE model in bouncing Universe

In the model of oscillating cosmology, the Big Bang is emerging from a bounce. The Universe at this point is assumed to have its smallest extent and largest energy density somewhere near the Planck density. Then the Universe expands and its density decreases, then it goes through the radiation and matter-dominated phases, with the usual primordial nucleosynthesis, microwave background and formation of large structure.

In the theory of standard cosmology, there is no method to avoid a singularity for small radius or scale factor a. For extra dimension models, one may have a bounce at finite a in which singularities can be avoided. Actually, a non-singular bounce is reached if the Friedmann equation is modified by adding a new negative term on the right-hand side, namely [21,22]

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_{\rm p}^2}(\rho_{\rm tot} - f(\rho_{\rm tot})). \quad (1)$$

Here, the Hubble parameter is represented by $H = \dot{a}(t)/a(t)$, the dot represents the first derivative with respect to cosmic time and the total energy density of the Universe $\rho_{\rm tot} = \rho_{\rm D} + \rho_{\rm m}$, $\rho_{\rm D}$ and $\rho_{\rm m}$ are the energy densities of DE and DM, respectively, $M_{\rm p} = 1/\sqrt{8\pi G}$ is the Planck mass, where G is the Newton gravitational constant and the curvature constants $k = +1, 0, -1$ represent, respectively, the closed, flat and open Universes. We can write $f(\rho_{\rm tot}) = \rho_{\rm tot}/2\sigma$ and by using the natural value from Randall–Sundrum-like models $\sigma = M_{\rm p}$, we get [21,22]

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_{\rm p}^2}\left(\rho_{\rm tot} - \frac{\rho_{\rm tot}^2}{2M_{\rm p}}\right). \quad (2)$$

Assuming a system of units in which G is taken to be equal to 1, one can write

$$H^2 + \frac{k}{a^2} = \frac{(8\pi)^2}{3}(\rho_{\rm D} + \rho_{\rm m})\left(1 - \frac{\sqrt{8\pi}}{2}(\rho_{\rm D} + \rho_{\rm m})\right). \quad (3)$$

The fractional energy densities for DE, DM and curvature parameter are given, respectively, by the following equations [23]:

$$\Omega_{\rm D} = \frac{\rho_{\rm D}}{\rho_{\rm cr}} = \frac{8\pi\rho_{\rm D}}{6H^2}, \quad (4)$$

$$\Omega_{\rm m} = \frac{\rho_{\rm m}}{\rho_{\rm cr}} = \frac{8\pi\rho_{\rm m}}{6H^2}, \quad (5)$$

$$\Omega_k = \frac{\rho_k}{\rho_{\rm cr}} = \frac{k}{a^2H^2}, \quad (6)$$

where the critical energy density of the Universe is represented by $\rho_{\rm cr}$.

By inserting the fractional energy density equations (4)–(6) in (3), one can obtain the following relation [24]:

$$1 + \Omega_k = 16\pi(\Omega_{\rm D} + \Omega_{\rm m}) \times \left[1 - \frac{3}{\sqrt{8\pi}}H^2(\Omega_{\rm D} + \Omega_{\rm m})\right]. \quad (7)$$

Recent observations constrain the fractional energy density of the curvature parameter k as $-0.0133 < \Omega_k < -0.0084$ in [25]. The conservation of energy is represented by the continuity equation

$$\dot{\rho}_{\rm tot} + 4H(\rho_{\rm tot} + p_{\rm tot}) = 0, \quad (8)$$

where $p_{\rm tot} = p_{\rm D}$ is the total pressure and dot denotes the time derivative. We should point to the fact that DM is a pressure-less matter, and so $p_{\rm m} = 0$:

$$\dot{\rho}_{\rm D} + 4H\rho_{\rm D}(1 + \omega_{\rm D}) = 0, \quad (9)$$

$$\dot{\rho}_{\rm m} + 4H\rho_{\rm m} = 0, \quad (10)$$

where $\omega_{\rm D}$ is the equation of state (EoS) parameter of DE:

$$\dot{\rho}_{\rm D} + 4H\rho_{\rm D}(1 + \omega_{\rm D}) = -\xi, \quad (11)$$

$$\dot{\rho}_{\rm m} + 4H\rho_{\rm m} = +\xi, \quad (12)$$

where ξ represents the interaction term, which is assumed to be a function in both $\rho_{\rm D}$ and $\rho_{\rm m}$. One of the most common forms is $\xi = 4H\gamma(\rho_{\rm D} + \rho_{\rm m})$, where γ is the coupling constant which represents the decay from DE to DM. Different Lagrangians have been proposed to generate this interaction term. Cosmic microwave background observations show that the coupling constant should be less than 0.025 [2]. Actually, the thermodynamic laws will be violated unless we choose γ to have positive values.

Following the holographic principle [26], the number of degrees of freedom is proportional to the surface area rather than the bounded volume. That way one can write $L^3\rho_{\rm D} \leq LM_{\rm p}^2$, where L is the infrared (IR) cut-off. This inequality supports the study done by Karolyhazy [27]. In this work, the quantum mechanics was combined with general relativity in which this inequality is established as one of the aspects of this important combination. Actually, Heisenberg uncertainty relation and gravity are combined and Karolyhazy derived a quantitative limitation on the uncertainty of the structure of space–time. Then, he incorporated the resulting uncertainty in the space–time structure into the equations for the propagation of quantum-mechanical wave amplitudes. For the sake of harvesting deep structure, we highlight that the Karolyhazy relation utilises quantum fluctuations of space–time and estimates the distance measurement of Minkowski space–time. It conjectures that the DE links astrophysics to particle physics. While the earlier theory describes the diverse processes underlying the Universe, the later ones aim to describe the fundamental approaches of space, time and matter as well. Thus, the DE is conjectured to manifest crucial characteristics of fundamental theory, as particle physics does, but yet only relying on astrophysical observations. Later on, almost all holographic DE models are based on this pioneering combination [27].

Now, one can write the energy density expression as [27–29]

$$\rho_{\rm D} = 3n^2M_{\rm p}^2L^{-2}, \quad (13)$$

where n is some numerical constant and L is the infrared cut-off. L is chosen, in this work, to be represented by the Ricci scalar curvature given by [29]

$$R = \left(- 6\dot{H} - 12H^2 - 6\frac{k}{a^2} \right), \quad (14)$$

where H is the Hubble parameter as mentioned before and a is a dimensionless scale factor which is used to study the expansion of the Universe. k is equal to zero for the flat case, and hence one can write

$$R = (- 6\dot{H} - 12H^2). \quad (15)$$

By using the Ricci scalar as infrared cut-off $L = R^{-1/2}$, one finds

$$\rho_{\mathrm{D}} = 3n^2 M_{\mathrm{p}}^2 R. \quad (16)$$

By differentiating eq. (3) and then by dividing by $2H$, one finds after some algebraic steps:

$$\dot{H} - \frac{k}{a^2} = \frac{(8\pi)^2}{6H}(\dot{\rho}_{\mathrm{D}} + \dot{\rho}_{\mathrm{m}})(1 - \sqrt{8\pi}(\rho_{\mathrm{D}} + \rho_{\mathrm{m}})). \quad (17)$$

Assuming non-interacting scenario, the continuity equations for DE and DM take the form

$$\dot{\rho}_{\mathrm{D}} = - 4H\rho_{\mathrm{D}}\,(1 + \omega_{\mathrm{D}}). \quad (18)$$

$$\dot{\rho}_{\mathrm{m}} = - 4H\rho_{\mathrm{m}}. \quad (19)$$

Inserting eqs (18) and (19) into eq. (17), we can write

$$\dot{H} - \frac{k}{a^2} = -\frac{-2(8\pi)^2}{3}(\rho_{\mathrm{m}} + \rho_{\mathrm{D}}(1 + \omega_{\mathrm{D}})) \times (1 - \sqrt{8\pi}(\rho_{\mathrm{D}} + \rho_{\mathrm{m}})). \quad (20)$$

Adding eq. (3) to eq. (20), one obtains

$$\dot{H} + H^2 = \frac{64}{3}\pi^2(\sqrt{2}\sqrt{\pi}\,(\rho_{\mathrm{D}} + \rho_{\mathrm{m}}) \times (\rho_{\mathrm{D}}(4\omega_{\mathrm{D}} + 3) + 3\rho_{\mathrm{m}}) - \rho_{\mathrm{D}}(2\omega_{\mathrm{D}} + 1) - \rho_{\mathrm{m}}). \quad (21)$$

Inserting eqs (3) and (21) into eq. (14), we find

$$R = - 128\pi^2 \Big(\sqrt{2}\sqrt{\pi}(\rho_{\mathrm{D}}+\rho_{\mathrm{m}})(\rho_{\mathrm{D}}(4\omega_{\mathrm{D}} + 1) + \rho_{\mathrm{m}}) + \rho_{\mathrm{D}}(1 - 2\omega_{\mathrm{D}}) + \rho_{\mathrm{m}} \Big). \quad (22)$$

Using both eq. (22) and the definition of ρ_{D} given in eq. (16), we can derive a formula for the equation of state parameter as

$$\omega_{\mathrm{D}} = \frac{\alpha}{256(2\sqrt{2}\pi^{5/2}\rho_{\mathrm{D}}\rho_{\mathrm{m}} + 2\sqrt{2}\pi^{5/2}\rho_{\mathrm{D}}^2 - \pi^2\rho_{\mathrm{D}})}, \quad (23)$$

where

$$\alpha = - 256\sqrt{2}\pi^{5/2}\rho_{\mathrm{D}}\rho_{\mathrm{m}} - 128\sqrt{2}\pi^{5/2}\rho_{\mathrm{D}}^2 - 128\pi^2\rho_{\mathrm{D}} - 128\sqrt{2}\pi^{5/2}\rho_{\mathrm{m}}^2 - 128\pi^2\rho_{\mathrm{m}} - R. \quad (24)$$

By inserting eqs (18) and (19) into eq. (23), one finds

$$\omega_{\mathrm{D}} = \frac{\zeta}{48n^2\sqrt{\pi}\Omega_{\mathrm{D}}(2\sqrt{\pi} - 3\sqrt{2}H^2(\Omega_{\mathrm{D}} + \Omega_{\mathrm{m}}))}, \quad (25)$$

where

$$\zeta = 36\sqrt{2}H^2 n^2\sqrt{\pi}\,(\Omega_{\mathrm{D}} + \Omega_{\mathrm{m}})^2 + 48n^2\pi\,\Omega_{\mathrm{D}} + \Omega_{\mathrm{D}} + 48n^2\pi\,\Omega_{\mathrm{m}}. \quad (26)$$

For the interacting case, we follow the same mathematical treatment. By combining eqs (11) and (14), we get

$$R_{\mathrm{I}} = - 64\pi^2(6(\rho_{\mathrm{D}} + \rho_{\mathrm{m}})(1 - \sqrt{2}\sqrt{\pi}(\rho_{\mathrm{D}} + \rho_{\mathrm{m}})) - 4(1 - 2\sqrt{2}\sqrt{\pi}(\rho_{\mathrm{D}} + \rho_{\mathrm{m}}))(\rho_{\mathrm{D}}(\gamma + \omega_{\mathrm{D}} + 1) + \gamma\rho_{\mathrm{m}})). \quad (27)$$

By using the definition of ρ_{D} given in eq. (16), one finds

$$\omega_{\mathrm{DI}} = \frac{\eta}{48n^2\sqrt{\pi}\Omega_{\mathrm{D}}(2\sqrt{\pi} - 3\sqrt{2}H^2\,(\Omega_{\mathrm{D}} + \Omega_{\mathrm{m}}))}, \quad (28)$$

where

$$\eta = 36\sqrt{2}H^2 n^2\sqrt{\pi}\,(\Omega_{\mathrm{D}} + \Omega_{\mathrm{m}}) \times ((4\gamma + 7)\Omega_{\mathrm{D}} + (4\gamma + 3)\Omega_{\mathrm{m}}) - 96\gamma n^2\pi\,\Omega_{\mathrm{D}} - 240n^2\pi\,\Omega_{\mathrm{D}} - \Omega_{\mathrm{D}} - 96\gamma n^2\pi\,\Omega_{\mathrm{m}} - 144n^2\pi\,\Omega_{\mathrm{m}}. \quad (29)$$

Equation (28) is the interacting formula for the equation of state parameters.

By setting the present values for cosmological parameters $\Omega_{\mathrm{m}} = 0.27$, $\Omega_{\mathrm{D}} = 0.73$ and $H = H_0 = 1$, one finds $\omega_{\mathrm{D}} < -1$ indicating a phantom DE model [26]. Studying the EoS plays an important role in understanding the stability of different DE systems [30] and the stability of the modified gravity models is considered in [24,31]. An exciting work was done by Myung [32] to study the stability of the holographic Universe by imposing the future event horizon as infrared cut-off claiming that holographic models are not stable.

An important estimation for the specific heat of the Universe has been made by Bhandari *et al* [33] showing that the cosmic fluid should reach thermodynamic stability although the phantom nature of the DE fluid is not thermodynamically stable.

Now, the behaviour of the deceleration parameter q is considered. The most popular form of q is [34]

$$q = -1 - \frac{\dot{H}}{H^2}. \quad (30)$$

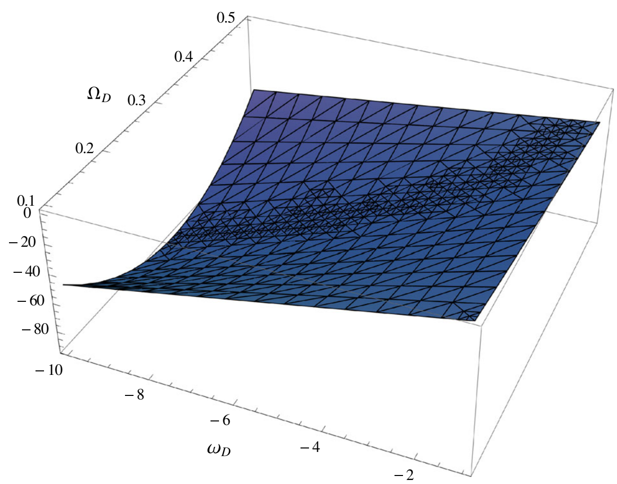

Figure 1. The growth of deceleration parameter for the non-interacting case.

By considering $\dot{H}$ given in eq. (17), $\dot{H}/H^2$ can be written as follows:

$$\frac{\dot{H}}{H^2} = \Omega_k - 32\pi(\Omega_m + \Omega_D(1+\omega_D)) \times \left(1 - \frac{6H^2}{\sqrt{8\pi}}(\Omega_m + \Omega_D)\right). \tag{31}$$

Now by substituting in eq. (30), one finds

$$q = -1 - \Omega_k + 32\pi(\Omega_m + \Omega_D(1+\omega_D)) \times \left(1 - \frac{6H^2}{\sqrt{8\pi}}(\Omega_m + \Omega_D)\right). \tag{32}$$

By using eq. (7), one can write

$$q = -16\pi(\Omega_D + \Omega_m)\left(1 - \frac{3H^2(\Omega_D+\Omega_m)}{2\sqrt{2}\sqrt{\pi}}\right) + 32\pi((\omega_D+1)\,\Omega_D + \Omega_m) \times \left(1 - \frac{3H^2(\Omega_D+\Omega_m)}{\sqrt{2}\sqrt{\pi}}\right). \tag{33}$$

For the present values of cosmological parameters [26] and for phantom-dominated Universe, i.e. $\omega_D < -1$, we study in figure 1 the growth of q as a function in both Ω_D and ω_D, which leads to $q < 0$ showing an accelerating expansion for the Universe.

Now, we are going to study the evolution of fractional energy density Ω'_D. By using the continuity equation, we have

$$\dot{\rho}_D = -4H\rho_D(1+\omega_D). \tag{34}$$

By using eq. (4) together with eq. (34), one can write

$$\dot{\Omega}_D = -2H\Omega_D\left[\frac{\dot{H}}{H^2} + 2(1+\omega_D)\right]. \tag{35}$$

By dividing eq. (35) by Hubble parameter H, one obtains the following expression:

$$\frac{\dot{\Omega}_D}{H} = -2\Omega_D\left[\frac{\dot{H}}{H^2} + 2(1+\omega_D)\right]. \tag{36}$$

Now by inserting the expression of $\dot{H}/H^2$ from eq. (31), one can write

$$\Omega'_D = -2\Omega_D(2\omega_D(24\sqrt{2}H^2\sqrt{\pi}\Omega_D(\Omega_D+\Omega_m) - 16\pi\Omega_D + 1) + 4\sqrt{\pi}(\Omega_D+\Omega_m) \times (9\sqrt{2}H^2(\Omega_D+\Omega_m) - 4\sqrt{\pi}) + 1), \tag{37}$$

where $\Omega'_D = \dot{\Omega}_D/H$. Here, the prime indicates the first derivative with respect to $\ln a$.

One can write for the interacting case using the above procedure:

$$\frac{\dot{H}}{H^2} = 4(3\sqrt{2}\sqrt{\pi}(\Omega_D+\Omega_m)((4\omega_D+3)\Omega_D + 3\Omega_m) - 4\pi((2\omega_D+1)\Omega_D+\Omega_m)+\gamma(\Omega_D+\Omega_m)) - 1. \tag{38}$$

Hence, q for the interacting case takes the form

$$q_I = -16\pi(\Omega_D+\Omega_m)\left(1 - \frac{3H^2(\Omega_D+\Omega_m)}{2\sqrt{2}\sqrt{\pi}}\right) - 4\gamma(\Omega_D+\Omega_m) + 32\pi((\omega_D+1)\,\Omega_D+\Omega_m) \times \left(1 - \frac{3H^2(\Omega_D+\Omega_m)}{\sqrt{2}\sqrt{\pi}}\right). \tag{39}$$

Using the present values of cosmic parameters and by taking the interaction parameter $\gamma = 0.02$ [2], we notice that $q < 0$ showing the same behaviour as the non-interacting case.

For the interacting case, we have

$$\dot{\rho}_{ID} = -4H\rho_D(1+\omega_D) - 4\gamma H\rho_D. \tag{40}$$

By using eqs (4) and (40), we have

$$\dot{\Omega}_{ID} = -2H\Omega_D\left[\frac{\dot{H}}{H^2} - 2((1+\omega_D)+\gamma)\right]. \tag{41}$$

Dividing eq. (41) by the Hubble parameter H and by using $\Omega'_D = \dot{\Omega}_D/H$, we obtain the following expression:

$$\Omega'_{ID} = 4(\Omega_D+\Omega_m)(9\sqrt{2}\sqrt{\pi}\,(\Omega_D+\Omega_m) - 4\pi + \gamma) + \omega_D\big(48\sqrt{2}\sqrt{\pi}\Omega_D\,(\Omega_D+\Omega_m) - 32\pi\Omega_D + 2\big) + 1. \tag{42}$$

For phantom field $\omega < -1$, we notice that $\Omega_D > 1$ for the non-interacting equation (37) and interacting equation (42) cases which agree with the recent cosmological observations [14]. This behaviour indicates that Ω_D was lower in the past than in the present.

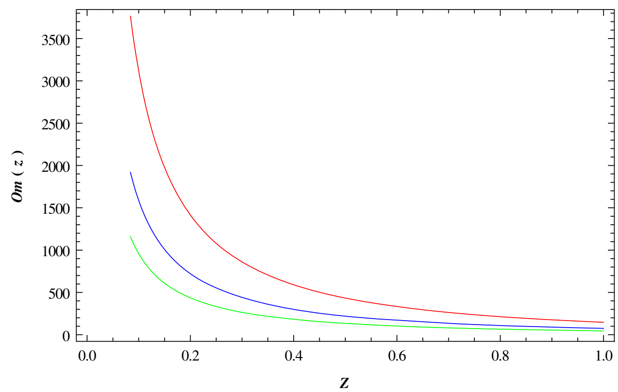

Figure 2. Evolution of Om(z) as a function of red-shift z for different values of n.

3. Om-diagnostic parameter

A diagnostic parameter called Om [35], proposed in 2008, helps to distinguish between the energy densities of various DE models. The advantage of Om is that it involves only the first derivative of the scale factor, and is easier to reconstruct from the observational data. The Om diagnostic is defined as

$$\mathrm{Om}(z) = \frac{(H(z)^2/H_0^2) - 1}{(z+1)^3 - 1}. \tag{43}$$

The cosmic time can be represented in terms of the red-shift as [36]

$$t = \frac{2}{H_0\big((z+1)^2+1\big)}. \tag{44}$$

Using the field equation (3) together with eq. (44) in eq. (43) and for DE-dominated Universe, one can get, after some algebraic steps, the Om diagnostic in Einstein gravity as follows:

$$\mathrm{Om}(z) = \frac{\dfrac{1.3\times10^6\tan^2\left(\dfrac{152\left(735n^2-\dfrac{30}{H_0\left((z+1)^2+1\right)}\right)}{n}\right)}{H_0^2n^2} - 1}{(z+1)^3-1}. \tag{45}$$

In figure 2, a plot studying the evolution of Om(z) against red-shift z is presented corresponding to $n = 0.5$ (red line), $n = 0.7$ (blue line) and $n = 0.9$ (green line).

Following [37], the positive slope of Om(z) suggests the phantom $\omega < -1$ and the negative slope of Om(z) suggests the non-phantom $\omega > -1$. In figure 2, we note that Om(z) is characterised by a negative slope indicating quintessence-like behaviour for our considered model.

4. Thermodynamics

Now we are going to study the thermodynamic analysis and it is known that the entropy of a black hole is proportional to the area of its event horizon $S = \pi R^2$. This has a deep meaning in physics. Actually, how the entropy is related to a cosmological event horizon is not completely understood, similar to the case of the de Sitter horizon [38]. The generalised second law (GSL) for both the interacting and non-interacting scenarios is considered. This law shows that the total entropy of the matter system is always positive with time growth. The importance of this type of thermal study is considered in [39], which shows that the Friedman equations emerge from the first law of thermodynamics.

Let us first establish the mathematical formula for the entropy by using the first law of thermodynamics [40]:

$$T\,\mathrm{d}S = p\mathrm{d}V + \mathrm{d}E, \tag{46}$$

where T, S, P, V and E represent the temperature, entropy, pressure, volume and internal energy of the considered system, respectively. The volume of the considered system is given by [40]

$$V = \frac{4\pi L^3}{3} \tag{47}$$

and the differential form is

$$\mathrm{d}V = 4\pi L^2\mathrm{d}L. \tag{48}$$

Applying eq. (46) for both DE and DM, we get

$$T\,\mathrm{d}S_\mathrm{D} = p_\mathrm{D}\mathrm{d}V + \mathrm{d}E_\mathrm{D}, \tag{49}$$

$$T\,\mathrm{d}S_\mathrm{m} = p_\mathrm{m}\mathrm{d}V + \mathrm{d}E_\mathrm{m}. \tag{50}$$

The internal energy equations for DE and DM are given by [40]

$$E_\mathrm{D} = \frac{4\pi L^3}{3}\rho_\mathrm{D}, \tag{51}$$

$$E_\mathrm{m} = \frac{4\pi L^3}{3}\rho_\mathrm{m}. \tag{52}$$

The horizon temperature [40]

$$T = \frac{1}{2\pi L} \tag{53}$$

and the horizon entropy is given by [26]

$$S_\mathrm{H} = 8\pi^2L^3. \tag{54}$$

Differentiating eq. (54) with respect to cosmic time:

$$\dot{S}_\mathrm{H} = 16\pi^2L^2\dot{L}, \tag{55}$$

$$\dot{S}_\mathrm{D} = \frac{p_\mathrm{D}\dot{V} + \dot{E}_\mathrm{D}}{T}, \tag{56}$$

$$\dot{S}_\mathrm{m} = \frac{\dot{E}_\mathrm{m}}{T}. \tag{57}$$

Adding the above equations, one can write the total entropy of the system as

$$\dot{S} = \dot{S}_{\mathrm{H}} + \dot{S}_{\mathrm{D}} + \dot{S}_{\mathrm{m}} = \frac{4\pi L^2}{T}(\dot{L}(\rho_{\mathrm{D}}(1+\omega_{\mathrm{D}}) + \rho_{\mathrm{m}}) + \frac{L}{3}(\dot{\rho_{\mathrm{m}}} + \dot{\rho_{\mathrm{D}}}) + 4\pi T). \tag{58}$$

Taking the first time derivative of the IR cut off $L = R^{-1/2}$, one finds

$$\dot{L} = -\frac{1}{2}\dot{R}R^{-3/2}, \tag{59}$$

where $\dot{R}$ for both the non-interacting and interacting cases are given by

$$\begin{aligned}\dot{R} =& -128\pi^2(-4\sqrt{2}\sqrt{\pi}(H(\rho_{\mathrm{D}}(5\omega_{\mathrm{D}}+3)\rho_{\mathrm{m}} \\ &+\rho_{\mathrm{D}}^2(\omega_{\mathrm{D}}+1)(4\omega_{\mathrm{D}}+1) + 2\rho_{\mathrm{m}}^2) \\ &+H\rho_{\mathrm{D}}(\omega_{\mathrm{D}}+1)(4\omega_{\mathrm{D}}+1)(\rho_{\mathrm{D}}+\rho_{\mathrm{m}})) \\ &+4H\rho_{\mathrm{D}}(2\omega_{\mathrm{D}}^2+\omega_{\mathrm{D}}-1) - 4H\rho_{\mathrm{m}})\end{aligned} \tag{60}$$

and

$$\begin{aligned}\dot{R}_{\mathrm{I}} =& -64\pi^2(-4(1-2\sqrt{2}\sqrt{\pi}(\rho_{\mathrm{D}} \\ &+\rho_{\mathrm{m}}))(4\gamma^2 H(\rho_{\mathrm{D}}+\rho_{\mathrm{m}}) \\ &+(\gamma+\omega_{\mathrm{D}}+1)(-4\gamma H(\rho_{\mathrm{D}}+\rho_{\mathrm{m}}) \\ &-4H\rho_{\mathrm{D}}(\omega_{\mathrm{D}}+1)) - 4\gamma H\rho_{\mathrm{m}}) \\ &-4(\rho_{\mathrm{D}}(\gamma+\omega_{\mathrm{D}}+1)+\gamma\rho_{\mathrm{m}}) \\ &\times(1-2\sqrt{2}\sqrt{\pi}(-4H\rho_{\mathrm{D}}(\omega_{\mathrm{D}}+1) - 4H\rho_{\mathrm{m}})) \\ &+6(1-\sqrt{2}\sqrt{\pi}(\rho_{\mathrm{D}}+\rho_{\mathrm{m}})) \\ &\times(-4H\rho_{\mathrm{D}}(\omega_{\mathrm{D}}+1) - 4H\rho_{\mathrm{m}}) \\ &+6(\rho_{\mathrm{D}}+\rho_{\mathrm{m}})(1-\sqrt{2}\sqrt{\pi} \\ &\times(-4H\rho_{\mathrm{D}}(\omega_{\mathrm{D}}+1) - 4H\rho_{\mathrm{m}}))).\end{aligned} \tag{61}$$

By using eqs (4), (5), (11) and (12) together with eq. (58), we can write

$$\dot{S} = 8\pi L^3\left(\left(\dot{L} - \frac{4}{3}HL\right) \times \frac{6H^2}{8\pi}(\Omega_{\mathrm{D}}(1+\omega_{\mathrm{D}})+\Omega_{\mathrm{m}}) - \frac{1}{L}\right). \tag{62}$$

Now, we are going to test the validity of eq. (62) for the two cases:

Case (i). Non-interacting: $\dot{L}$ for the non-interacting case can be written by using eqs (59) and (60) as

$$\dot{L} = \frac{\Theta}{\Pi}, \tag{63}$$

$$\begin{aligned}\Theta =& H^2\sqrt{\pi}(\Omega_{\mathrm{D}}(\sqrt{\pi}(0.408248 - 0.816497\omega_{\mathrm{D}}) \\ &+H^2(1.73205\omega_{\mathrm{D}}+0.866025)\Omega_{\mathrm{m}}) \\ &+H^2(1.73205\omega_{\mathrm{D}}+0.433013)\Omega_{\mathrm{D}}^2 \\ &+\Omega_{\mathrm{m}}(0.433013H^2\Omega_{\mathrm{m}}+0.408248\sqrt{\pi}))\end{aligned} \tag{64}$$

and

$$\begin{aligned}\Pi =& (4H^2\pi((2\omega_{\mathrm{D}}-1)\Omega_{\mathrm{D}} - \Omega_{\mathrm{m}}) \\ &-3\sqrt{2}H^4\sqrt{\pi}(\Omega_{\mathrm{D}}+\Omega_{\mathrm{m}})((4\omega_{\mathrm{D}}+1)\Omega_{\mathrm{D}} \\ &+\Omega_{\mathrm{m}}))^{3/2}.\end{aligned} \tag{65}$$

Case (ii). Interacting: Following the same way for the non-interacting case, by using eqs (59) and (61), one can write

$$\dot{L} = \frac{\Delta}{\chi}, \tag{66}$$

where

$$\begin{aligned}\Delta =& H^2\sqrt{\pi}(H^3\Omega_{\mathrm{D}}^2(-20.78\gamma \\ &+\omega_{\mathrm{D}}(-20.78\gamma - 13.86\omega_{\mathrm{D}} - 17.30) - 3.461) \\ &+\Omega_{\mathrm{m}}(H^3\Omega_{\mathrm{m}}(-20.78\gamma - 6.93\gamma\omega_{\mathrm{D}} + 10.39) \\ &+\sqrt{\pi}(-0.82\gamma + 3.26\gamma H\omega_{\mathrm{D}} \\ &+6.53\gamma H - 4.89H + 1.23)) \\ &+\Omega_{\mathrm{D}}(H^3\Omega_{\mathrm{m}}(-41.56\gamma + \omega_{\mathrm{D}}(-27.70\gamma \\ &-6.922\omega_{\mathrm{D}} - 10.39) + 6.92) \\ &+\sqrt{\pi}(-0.82\gamma + \omega_{\mathrm{D}}(3.26H\omega_{\mathrm{D}} \\ &+6.54\gamma H + 1.63299H - 0.82) \\ &+(6.53\gamma - 1.63)H + 0.41)))\end{aligned} \tag{67}$$

and

$$\begin{aligned}\chi =& (4H^2\pi(\Omega_{\mathrm{D}}(2\gamma+2\omega_{\mathrm{D}}-1) + (2\gamma-3)\Omega_{\mathrm{m}}) \\ &-3\sqrt{2}H^4\sqrt{\pi}(\Omega_{\mathrm{D}}+\Omega_{\mathrm{m}})(\Omega_{\mathrm{D}}(4\gamma+4\omega_{\mathrm{D}}+1) \\ &+(4\gamma-3)\Omega_{\mathrm{m}}))^{3/2}.\end{aligned} \tag{68}$$

We notice that $\dot{S}$ for both cases depends on H, L, R and $\dot{L}$. For expanding Universe, the horizon is an increasing function, and consequently its derivative is positive [40]. To examine the validity of GLS for phantom-like field where $\omega_{\mathrm{D}} < -1$ [1], for the present cosmic parameter values [26] in eqs (62), (63) and (66), we plot in figure 3 the growth of $\dot{S}$ as a function of EoS and fractional DE for the non-interacting case. The figure shows that $\dot{S} > 0$ and the GLS holds for the considered model. For the interacting case, we follow the same procedure as we did for the non-interacting case. We notice that in figure 4, $\dot{S} > 0$ like before. The difference with the non-interacting case is that the growth here is at a higher rate allowing the Universe to reach the

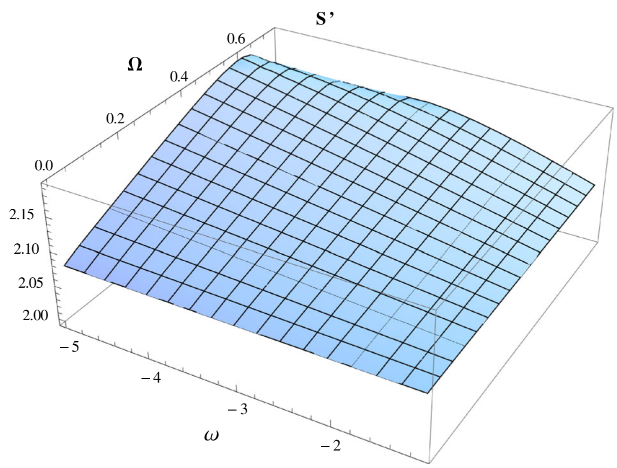

Figure 3. Growth of non-interacting $\dot{S}$ as a function of EoS and fractional DE density.

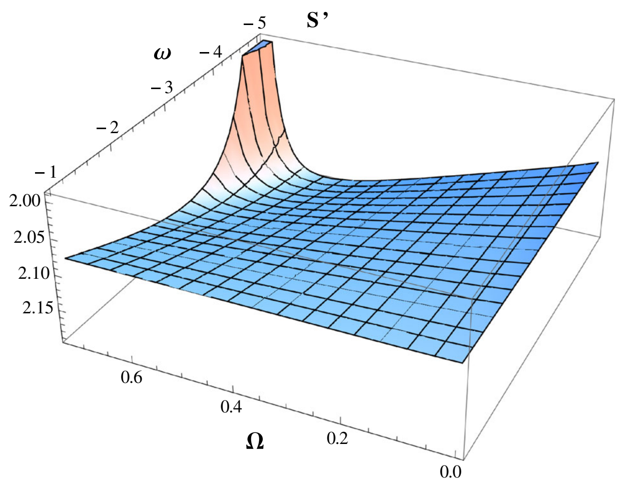

Figure 4. Growth of interacting $\dot{S}$ as a function of EoS and fractional DE density.

final state quickly. These results indicate that the GSL is fulfilled for a Universe filled with interacting DE for the considered IR cut-off. So, the GSL is verified for the considered model for the phantom-dominated Universe. Actually, one of the important features of this verification is that, it predicts the end of the Universe in what is known as heat death, in which everything is at the same temperature.

5. Conclusion

The RDE model is considered within the context of the bouncing cosmology model. Both the non-interacting and interacting scenarios are considered in this work by using the Friedman equation for our model together with the energy density of holographic dark energy (HDE) with Ricci scalar R as IR cut-off. The equation of state parameter ω_{D} is deduced. Inserting the present values of cosmic parameters for $\gamma = 0.02$ and by assuming a phantom DE Universe $\omega_{\mathrm{D}} < -1$, we established a mathematical formula for the deceleration parameter q. We notice that for Ω_{D} in the range from 0 to 0.75, $q < 0$ indicate the present expansion mode for our Universe. Using the cosmic parameter values in the expression of Ω'_{D}, we notice that both scenarios in the phantom regime $\omega_{\mathrm{D}} < -1$ lead to $\Omega'_{\mathrm{D}} > 1$, indicating a growth of DE density with time, which agrees with the observation.

We study the Om diagnostic parameter because of its importance in differentiating between different DE models and we note that by assuming a DE-dominated Universe Om has a negative slope behaviour indicating quintessence-like behaviour [37].

In this work, we assume that the total entropy is the sum of the horizon entropy, DE entropy and DM entropy. Here, the calculations have been done for both the non-interacting and interacting cases. For the phantom-dominated Universe $\omega_{\mathrm{D}} < -1$, we find $\dot{S} > 0$ showing that GSL is valid for both the cases throughout the evolution of the Universe.

In summary, we can say that the thermal properties of DE, especially the GSL of thermodynamics, play an important role in understanding the nature of the possible DE candidates.

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