Pramana – J. Phys. (2018) 91:76
https://doi.org/10.1007/s12043-018-1651-9

Aharonov–Bohm effect in the ghost interference

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MS received 20 February 2018; revised 19 April 2018; accepted 27 April 2018; published online 26 September 2018

Abstract. In the ghost interference experiment, a pair of entangled particles is sent in the opposite directions; one of the particles passes through a Young double-slit while the other continues its way freely. It turns out that the particles passing through the slits do not show any first-order interference while those propagating freely constitute an interference pattern when they are detected in coincidence with those which pass through the slits and detected at a fixed position. In this work, we consider that the particles are charged and the effect of a confined magnetic field is analysed between the slits in an Aharonov–Bohm configuration.

Keywords. Ghost interference; Aharonov–Bohm effect; entanglement; non-locality.

PACS Nos 03.65.Ta; 03.65.Ud; 03.75.−b

1. Introduction

Quantum mechanics (QM) contrary to classical physics, is a non-local theory. After interacting, two quantum systems generally end up in a non-separable (entangled) state. The properties of each system cannot be described independently of those of the others. This occurs irrespective of the distance between the components of the entangled state. Entanglement thus naturally leads to the non-locality (Einestein–Podolski–Rosen (EPR) non-locality) [1]. This notion tells us that physics at one place cannot be described independently of what goes on in another disconnected part of the Universe. This was initially thought by some to be an oddity restricted to the realm of thought experiments. However, Bell's inequalities [2] characterising local behaviour and the experimental demonstration of their violation [3] made it clear that the non-local properties of the pure quantum states are more than an intellectual curiosity.

Entanglement manifestations are seen in tremendous physical situations (for a review, see [4] and [5] for a recent application), but a puzzling experiment which gave a very clear demonstration of the non-local nature of the quantum correlations that exist in spatially separated entangled particles was first reported by Strekalov *et al* and has come to be known as the ghost interference (see §2) [6] (see also [7]). In this experiment, the (signal and idler) photons of a pair, generated by parametric down-conversion, are spatially separated, propagate in opposite directions and detected by two points like photon counting detectors for coincidences. A Young double-slit is inserted in the path of the signal photon. A (ghost) interference pattern has been observed in the coincidence counts by scanning the idler photon detector. Moreover, no first-order interference pattern behind the slits has been observed for the signal photons. Due to the wave properties of the matter, it is evident that such a phenomenon may, in principle, be possible also for momentum-entangled massive particles. Theoretical calculations in such a case have been done in [8–10].

Another aspect of non-locality of the QM is the Aharonov–Bohm (AB) effect (AB non-locality) [11]. This is one of the most fascinating and still controversial issues in physics. A charged quantum particle may acquire an observable phase shift by circling around a completely shielded magnetic flux. This remarkable effect is purely non-classical as the magnetic field vanishes at the location of the particle, which thereby does not experience Lorentz force. Many experiments confirmed this effect, particularly in interferometry experiments where the effect of the magnetic field is manifested as a displacement of the interference fringes (see [12] and references therein).

In this paper, we analyse, by general considerations and with specific calculations, the effect of a magnetic field on an 'experiment' with charged particles in the ghost interference configuration: the magnetic field of a long thin solenoid is placed behind the double slits without touching signal particles. We find that the non-local effect of the magnetic field in the signal particles is non-locally transmitted to the idler particles, i.e. a shift in the ghost interference is observed.

2. Entangled wave function in the presence of a confined magnetic field

In the experimental scheme, we consider (figure 1) that a source S sends pairs of charged particles (1) and (2) to an entangled state. Particles of a pair travel in the opposite Ox directions. Motion along this axis will be treated classically. Particle (1) passes through two Young slits, between which is inserted a long solenoid carrying an isolated magnetic flux so that the particles never go through. Particles (1) and (2) are collected on two screens E_1 and E_2 placed symmetrically on S. Slits along the z-axis are long enough to ignore diffraction effects in this direction. So, we consider only vertical deflections along the y-axis.

The entangled state of the pair of charged particles sent at $t = 0$ is denoted by $\psi(y_1, y_2, 0)$. A particular form of this wave function will be chosen later. The evolution of ψ will be first generally determined in comparison with the evolution of the corresponding wave function $\psi^0(y_1, y_2, 0)$ in the absence of the magnetic field. We shall use the method of Green's functions for this purpose. At time t of the arrival of the particles to the observation screens, their wave function may be written as [13]

$$\psi(y_1, y_2, t) = \int \mathrm{d}y_1'\mathrm{d}y_2' K(y_1, y_2, t; y_1', y_2', 0)\psi(y_1', y_2', 0). \quad (1)$$

The propagator K represents the probability amplitude for the particles to go from the positions y_1', y_2' at $t = 0$ to the positions y_1, y_2 at time t. The two particles travel without mutual interaction, then the propagator K is the product of the two propagators K_1 and K_2, each one related to one of the particles:

$$K(y_1, y_2, t; y_1', y_2', 0) = K_1(y_1, t; y_1', 0) \times K_2(y_2, t; y_2', 0). \quad (2)$$

Particle (2) moves freely (in the presence of the magnetic field); its propagator acquires a global phase and takes the form [13]

$$K_2(y_2, t;\ y_2', 0) = \mathrm{e}^{\mathrm{i}(q/c\hbar)_{\gamma_2}\vec{A}\cdot\vec{\mathrm{d}l}} K_2^0(y_2, t;\ y_2', 0), \quad (3)$$

where K_2^0 is its propagator in the absence of the field and γ_2 is any curve connecting S and the point y_2 on the screen E_2 (figure 2), $\vec{A}$ is a potential vector associated with the magnetic field and q is the charge of the particles. For particle (1), one can write [13]

$$K_1(y_1, t; y_1', 0) = \int_{\Delta_A} \mathrm{d}y_p K_{1A}(y_1, t; y_p, t_0)K_{1A}(y_p, t_0; y_1', 0) + \int_{\Delta_B} \mathrm{d}y_p K_{1B}(y_1, t; y_p, t_0)K_{1B}(y_p, t_0; y_1', 0). \quad (4)$$

In this equation

$$\Delta_A = \left[y_0 - \left(\frac{\varepsilon}{2}\right), y_0 + \left(\frac{\varepsilon}{2}\right)\right]$$

and

$$\Delta_B = \left[-y_0 - \left(\frac{\varepsilon}{2}\right), -y_0 + \left(\frac{\varepsilon}{2}\right)\right],$$

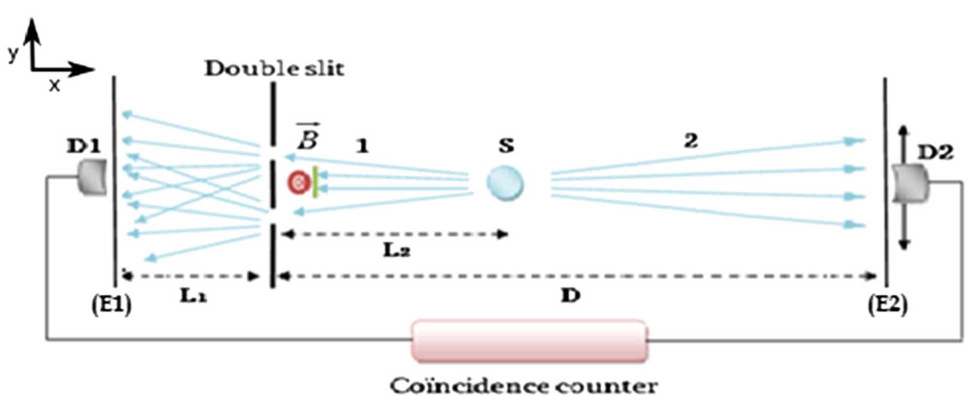

Figure 1. Schematic diagram of the two-slit ghost interference experiment. Entangled charged particles (1) and (2) emerge from a source S and travel in opposite directions along the x-axis.

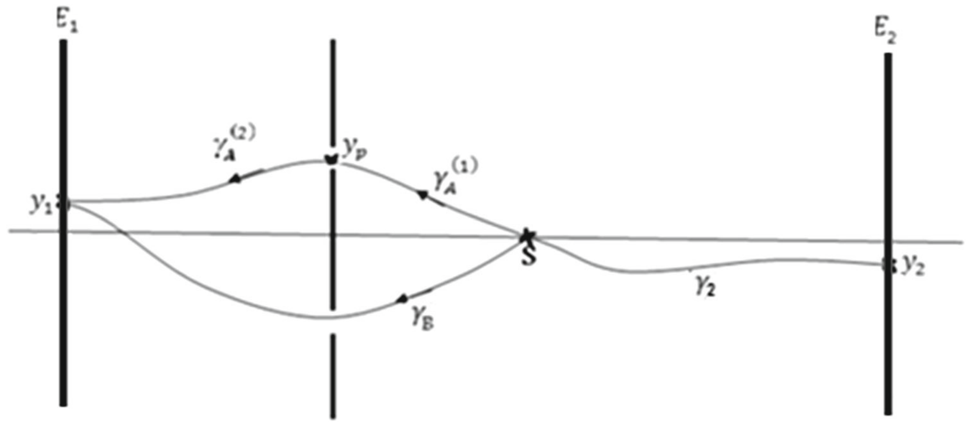

Figure 2. Integration paths.

where y_0 and $(-y_0)$ are the positions of the two slits, ε is their width and t_0 is the time of arrival of particle (1) at the slits.

Propagators in eq. (4) may be related to their corresponding propagators in the absence of the magnetic field. For example, for slit A we have

$$K_{1A}(y_1, t;\ y_p, t_0) = \mathrm{e}^{\mathrm{i}(q/c\hbar)\int_{\gamma_A^{(2)}} \vec{A}\cdot\vec{\mathrm{d}l}} K_{1A}^{(0)}(y_1, t;\ y_p, t_0) \quad (5)$$

and

$$K_{1A}(y_p, t_0;\ y_1', 0) = \mathrm{e}^{\mathrm{i}(q/c\hbar)\int_{\gamma_A^{(1)}} \vec{A}\cdot\vec{\mathrm{d}l}} K_{1A}^{0}(y_p, t_0;\ y_1', 0). \quad (6)$$

The product of these propagators is then

$$\mathrm{e}^{\mathrm{i}(q/c\hbar)\int_{\gamma_A} \vec{A}\cdot\vec{\mathrm{d}l}} K_{1A}^{0}(y_1, t;\ y_p, t_0)\ K_{1A}^{0}(y_p, t_0;\ y_1', 0). \quad (7)$$

In the above expressions, $\gamma_A^{(1)}$ and $\gamma_A^{(2)}$ are, respectively, arbitrary paths connecting the points S and y_p on the one hand, and y_1 and y_p on the the other hand; γ_A, their union, is an arbitrary path connecting the points S and y_1 and passing through slit A. K_{1A}^0 is the propagator in the absence of the magnetic field.

Treating, in the same manner, the term relative to slit B, we obtain

$$\begin{aligned} K_1(y_1, t;\ y_1', 0) &= \mathrm{e}^{\mathrm{i}(q/c\hbar)\int_{\gamma_A} \vec{A}\cdot\vec{\mathrm{d}l}} \int_{\Delta_A} \mathrm{d}y_p K_{1A}^0(y_1, t;\ y_p, t_0) \\ &\quad \times K_{1A}^0(y_p, t_0;\ y_1', 0) + \mathrm{e}^{\mathrm{i}(q/c\hbar)\int_{\gamma_B} \vec{A}\cdot\vec{\mathrm{d}l}} \\ &\quad \times \int_{\Delta_B} \mathrm{d}y_p K_{1B}^0(y_1, t;\ y_p, t_0) K_{1B}^0(y_p, t_0;\ y_1', 0). \end{aligned} \quad (8)$$

From the above, we obtain finally the entangled wave function at time t (up to an inconsequential global phase)

$$\psi(y_1, y_2, t) = \psi_A^0(y_1, y_2, t) + \mathrm{e}^{\mathrm{i}q(\Phi/c\hbar)} \psi_B^0(y_1, y_2, t), \quad (9)$$

where ψ_A^0 and ψ_B^0 are the probability amplitudes for particle (1) to go, in the absence of the magnetic field, through slits A and B, respectively

$$\Phi = \int_{\gamma_B} \vec{A}\cdot\vec{\mathrm{d}l} - \int_{\gamma_A} \vec{A}\cdot\vec{\mathrm{d}l} = \int_C \vec{A}\cdot\vec{\mathrm{d}l} = \int_\Sigma \vec{B}\cdot\vec{\mathrm{d}S}$$

is the magnetic flux through a surface Σ that intersects the solenoid lying on the closed path $C \equiv \gamma_B - \gamma_A$.

Equation (9) shows that the confined magnetic field leads to a supplementary phase difference between the partial probability amplitudes ψ_A^0 and ψ_B^0. This phase difference depends on the magnetic flux in the same manner as for the single-particle Young double-slit experiment. Note (see figure 2) that, considered individually, particles (1), passing through the double slit and having the possibility to follow ways circling the magnetic field, seem, at first sight, to exhibit an AB effect, but not particles (2).

3. Ghost AB effect

Let us establish some general consequences of eq. (9) before performing explicit calculations with a particular choice of the entangled wave function. The partial wave functions $\psi_{A,B}^0(y_1, y_2) = |\psi_{A,B}^0|\mathrm{e}^{\mathrm{i}\varphi_{A,B}(y_1,y_2)}$. The probability density of the particles on the screens E_1 and E_2 is given by

$$|\psi(y_1,y_2)|^2 = |\psi_A^0|^2 + |\psi_B^0|^2 + 2|\psi_A^0\psi_B^0| \times\cos\left[(\varphi_B(y_1,y_2)-\varphi_A(y_1,y_2)) + q\frac{\Phi}{c\hbar}\right]. \quad (10)$$

(i) For a fixed value of y_1, this density presents sinusoidal variations as a function of y_2 (due to the cos term) and corresponds thus to an interference pattern. This corresponds to keeping the position of the detector D_1 fixed and varying the position of D_2. Even though particles (2) do not go through any slit, they exhibit an interference pattern, when they are detected in coincidence with particles (1) detected in a fixed position of the detector D_1. This is the so-called ghost interference.

(ii) For particles (1), no first-order interference is observed even though they go through a double slit. In fact, the probability density of particles (1) on E_1 is obtained by integrating (10) on y_2; this destroys the sinusoidal variations. In other words, for each fixed position of D_2 (fixed y_2), particles (1) detected in coincidence with detection of particles (2) in D_2, exhibit an interference pattern which depends on y_2. Interference patterns corresponding to different values of y_2 are displaced from each other, and their superposition will destroy the interference.

(iii) From (10) we see that the ghost interference pattern for particles (2) is displaced from the one in the absence of the magnetic field (for $\phi \neq k(hc/q)$; k is an integer). This is a 'ghost' non-local AB effect: the non-local action of the magnetic field on particles (1) is non-locally felt by particles (2).

(iv) Equation (8) shows that the interference displacement of the fringes takes place within an unshifted diffraction envelope. This is a fundamental characteristic of the AB effect which differentiates it from the local action of a magnetic field [14].

In the following, a particular choice of the wave function of the entangled particles will be made. This choice has the advantage that time evolution has already been determined in the absence of exterior fields and it reproduces correctly the main aspects of the ghost interference [8].

At $t = 0$, the state of the particles (1) and (2) is described by the wave function

$$\psi(y_1,y_2,0) = \psi^0(y_1,y_2,0) = C\int_{-\infty}^{+\infty} \mathrm{d}p\,\mathrm{e}^{-p^2/4\sigma^2}\mathrm{e}^{-\mathrm{i}py_2/\hbar}\mathrm{e}^{\mathrm{i}py_1/\hbar}\mathrm{e}^{-((y_1+y_2)^2/4\Omega^2)}, \quad (11)$$

where C is the normalisation constant. This is a momentum-entangled wave function which, upon integration over p, takes the form

$$\psi(y_1,y_2,0) = \sqrt{\frac{\sigma}{\pi\hbar\Omega}}\mathrm{e}^{-\frac{(y_1-y_2)^2\sigma^2}{\hbar^2}}\mathrm{e}^{-\frac{(y_1+y_2)^2}{4\Omega^2}}. \quad (12)$$

Uncertainties in positions and momenta are given by

$$\Delta y_1 = \Delta y_2 = \sqrt{\Omega^2 + \frac{1}{4\sigma^2}}, \quad (13)$$

$$\Delta p_{1y} = \Delta p_{2y} = \frac{1}{2}\sqrt{\sigma^2 + \frac{1}{4\Omega^2}}. \quad (14)$$

The pair of charged particles propagates freely (without mutual interaction but in the presence of the magnetic field). The evolution of their wave function is characterised by three stages:

- free evolution during time t_0 until particle (1) reaches the slits,
- reduction of the wave function due to the passage of particle (1) through the slits,
- free evolution, during time τ until the two particles reach the observation screens.

The wave function at time $t = t_0 + \tau$ in the absence of the magnetic field has been calculated in [8] and is given by

$$\psi^0(y_1,y_2,t_0+\tau) = \psi_A^0(y_1,y_2,t_0+\tau) + \psi_B^0(y_1,y_2,t_0+\tau), \quad (15)$$

where

$$\psi_A^0(y_1,y_2,t_0+\tau) = C_\tau\mathrm{e}^{-\frac{(y_1-y_0)^2}{\varepsilon^2+2\mathrm{i}\hbar\tau/m}}\mathrm{e}^{-\frac{(y_2-y_0')^2}{\Gamma^2+2\mathrm{i}\hbar\tau/m}}, \quad (16)$$

and

$$\psi_B^0(y_1,y_2,t_0+\tau) = C_\tau\mathrm{e}^{-\frac{(y_1+y_0)^2}{\varepsilon^2+2\mathrm{i}\hbar\tau/m}}\mathrm{e}^{-\frac{(y_2+y_0')^2}{\Gamma^2+2\mathrm{i}\hbar\tau/m}}. \quad (17)$$

The wave function in the presence of the field is then

$$\psi(y_1,y_2,t_0+\tau) = C_\tau\left[\mathrm{e}^{-\frac{(y_1-y_0)^2}{\varepsilon^2+2\mathrm{i}\hbar\tau/m}}\mathrm{e}^{-\frac{(y_2-y_0')^2}{\Gamma^2+2\mathrm{i}\hbar\tau/m}} + \mathrm{e}^{\mathrm{i}q\frac{\Phi}{c\hbar}}\mathrm{e}^{-\frac{(y_1+y_0)^2}{\varepsilon^2+2\mathrm{i}\hbar\tau/m}}\mathrm{e}^{-\frac{(y_2+y_0')^2}{\Gamma^2+2\mathrm{i}\hbar\tau/m}}\right]. \quad (18)$$

In the above equations, we have

$$C_\tau = [(2\pi)(\varepsilon + 2\mathrm{i}\hbar\tau/m\varepsilon)(\Gamma + 2\mathrm{i}\hbar\tau/m\Gamma)]^{-1/2}, \quad (19)$$

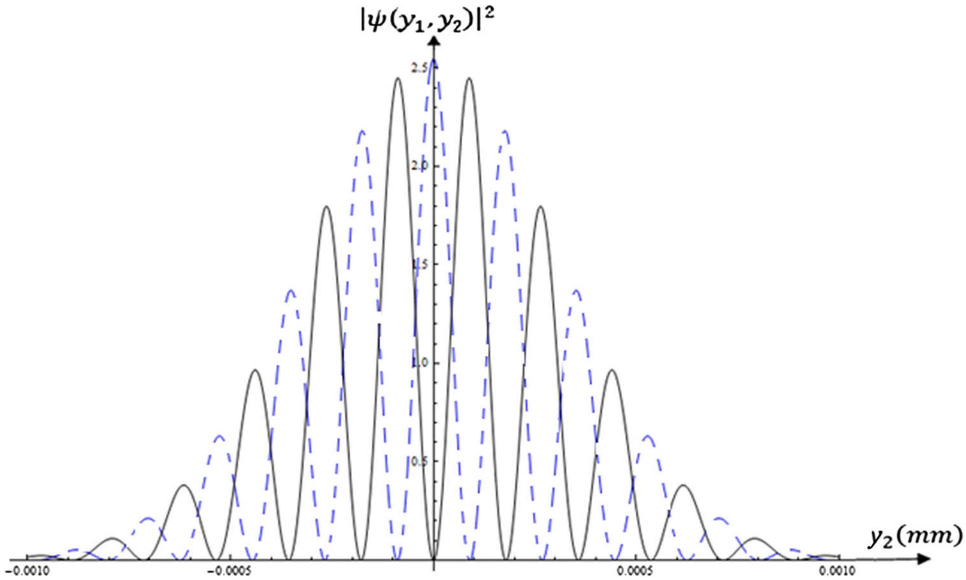

Figure 3. Probability density of particle (2) as a function of the position of detector D_2 for the fixed position $y_1 = 0$ of detector D_1. The dashed line is for $\phi = 0$ and the solid one is for $(\phi/\phi_0) = 1/2$; $\lambda_d = 0.05$ nm, $y_0 = 140$ nm, $\varepsilon = 25$ nm, $L_1 = 30$ cm, $D = 50$ cm.

$$\Gamma^2 = \left[\frac{\hbar^2}{\sigma^2}\left(1 + \frac{\varepsilon^2 + 2i\hbar t_0/m}{4\Omega^2}\right) + \varepsilon^2 + 2i\hbar t_0/m\right] \times \left[1 + \frac{\varepsilon^2 + 2i\hbar t_0}{\Omega^2} + \frac{\hbar^2}{4\Omega^2\sigma^2}\right]^{-1} + \frac{2i\hbar t_0}{m}, \quad (20)$$

ε is the width of the slits and

$$y_0' = \left[\frac{4\Omega^2\sigma^2/\hbar^2 + 1}{4\Omega^2\sigma^2/\hbar^2 - 1} + \frac{4\varepsilon^2}{4\Omega^2 - \hbar^2/\sigma^2}\right]^{-1} y_0. \quad (21)$$

Before going further, let us make some simplifications. In the limit (of the large extent of the wave function) $\Omega \gg \varepsilon$ and $\Omega \gg \hbar/\sigma$, we have $\Gamma^2 \approx \gamma^2 + 4i\hbar t_0/m$, where $\gamma^2 \approx \varepsilon^2 + \hbar^2/\sigma^2$ and $y_0' \approx y_0$. Furthermore, the following transformation (from times to distances) will be needed:

$$\hbar(\tau + 2t_0)/m = \hbar\upsilon(\tau + 2t_0)/p = \lambda_d \upsilon(\tau + 2t_0)/2\pi = \lambda_d D/2\pi,$$

where p and υ are, respectively, the x-axis momentum and velocity of particle (2) and λ_d is its de Broglie wavelength. D is the distance from the detector D_2 to the slits (see figure 1).

The probability density of the particles at positions y_1 and y_2 then becomes

$$\begin{aligned} P(y_1, y_2) &= |\psi(y_1, y_2, t_0 + \tau)|^2 \\ &= P_1(y_1, y_2) + P_2(y_1, y_2) + 2P_3(y_1, y_2). \\ &\quad \times [|\alpha|^2 \cos(\theta_1 y_1 + \theta_2 y_2 + \Phi) \\ &\quad + |\beta|^2 \cos(\theta_1 y_1 + \theta_2 y_2 - \Phi)], \quad (22) \\ P(y_1, y_2) &= P_1(y_1, y_2) + P_2(y_1, y_2) + 2P_3(y_1, y_2) \\ &\quad \times \sqrt{\cos^2\Phi + \cos^2\delta \sin^2\Phi}\cos(\lambda - \Phi), \end{aligned}$$

where

$$P_1(y_1, y_2) = |C_\tau|^2 e^{-\frac{2(y_1 - y_0)^2}{\varepsilon^2 + \left(\frac{\lambda_d L_1}{\pi\varepsilon}\right)^2}} e^{-\frac{2(y_2 - y_0')^2}{\gamma^2 + \left(\frac{\lambda_d D}{\pi\gamma}\right)^2}}, \quad (23)$$

$$P_2(y_1, y_2) = P_1(-y_1, -y_2), \quad (24)$$

$$P_3(y_1, y_2) = |C_\tau|^2 e^{-\frac{2(y_1^2 + y_0^2)}{\varepsilon^2 + \left(\frac{\lambda_d L_1}{\pi\varepsilon}\right)^2}} e^{-\frac{2(y_2^2 + y_0'^2)}{\gamma^2 + \left(\frac{\lambda_d D}{\pi\gamma}\right)^2}}, \quad (25)$$

and θ_1 and θ_2 are given by

$$\theta_1 = \frac{4y_0(\lambda_d L_1/\pi)}{\varepsilon^4 + (\lambda_d L_1/\pi)^2}, \quad \theta_2 = \frac{4y_0(\lambda_d D/\pi)}{\gamma^4 + (\lambda_d D/\pi)^2}, \quad \Phi_0 = \frac{q}{hc}. \quad (26)$$

In figure 3, the probability density (eq. (22)) is represented as a function of y_2 (for electrons) for $\phi = 0$ (dashed line) and $\phi \neq 0$, $k(c\hbar/q)$; k is an integer (solid line). These two curves are displaced from each other but are situated inside the same envelope. Note that the fringe width in the presence or absence of the magnetic flux is the same:

$$i_2 = \frac{2\pi}{\theta_2} \approx \frac{\lambda_d D}{2y_0} \qquad (\text{for } \gamma \ll \lambda_d D). \tag{27}$$

This is the fringe width in a Young double-slit experiment where the slits are separated by $2y_0$ and are located at distance D from the detector plane. It appears then as if particles (2) pass through a virtual double-slit located exactly at the real slits in the presence of a virtual confined magnetic flux. Note that this virtual field must be the opposite of the real one.

4. Conclusion

From the preceding analysis, we conclude that AB effect is not experienced individually by particles (1) even though these particles effectively pass through the double slit and have the possibility to follow ways circling the magnetic field. Particles (2), which do not go through any slits, present an AB effect on their ghost interference pattern. It looks like if the beam of particles (2) is sent by a source located at the detector D_1, then it is split by a virtual double slit located at the real one where a confined virtual flux, opposed to the real one, takes place. The non-local effect of the confined magnetic field on particles (1) is non-locally felt by particles (2) due to their entanglement.

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