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New stellar models generated using a quadratic equation of state

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Abstract. We obtain new regular exact solutions to the field equations for uncharged relativistic stellar objects with vanishing pressure anisotropy. We assume a quadratic equation of state and a choice of measure of anisotropy and a metric function defining one of the gravitational potentials. In our exact models, we regain anisotropic and isotropic results generated by other researchers as a special case. It is interesting that our results are in agreement with Minkowski space–time and earlier Einstein models. The physical analysis of the plots reveals that the gravitational potentials and matter variables are well behaved in the stellar interior. Using our model, we generate finite relativistic stellar masses which are consistent with the astronomical objects previously found by other researchers.

Keywords. Einstein field equations; vanishing pressure anisotropy; uncharged stellar object; quadratic equation of state.

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1. Introduction

The behaviour of the relativistic neutral or charged stellar objects is enhanced by the nonlinear Einstein field equations. These field equations are widely used for modelling relativistic compact stellar objects such as neutron stars, quark stars, gravastars, dark energy stars and black holes. Einstein field equations are generated by equating the energy–momentum tensor and the Einstein tensor involving the gravitating stellar objects with or without an electric field distribution. According to Schwarzschild [1], the relativistic stellar models have been studied ever since the first solution to the Einstein field equations for the interior of a compact object in hydrostatic equilibrium was obtained in 1916. Since then, the search for exact solutions describing the models of isotropic and anisotropic neutral or charged stellar objects has been attracting the interest of physicists and mathematicians. Ishak [2] explained that the constructive models developed for stellar objects and solutions to Einstein field equations in astrophysics are governed by the assumption that they are either static or non-static spherically symmetric on the metric and other simplifying restrictions. On the other hand, Mak and Harko [3] generated models for isotropic stars with astrophysical significance. Therefore, the study of relativistic compact objects is of great importance in astrophysics and other related fields.

The first solution to the field equations opened the door for researchers to construct a lot of physical predictions with high precision. According to Leijon [4], Einstein field equations are of great importance in astrophysics in the sense that they improve physical theories such as the light spectral shift and light deflection for massive bodies, the perihelion advance of planets in the solar system and many others. Apart from the Schwarzschild solution, there are other well-known solutions to Einstein field equations as stated by Leijon [4]. These include the Reissner–Nordstrom solution to the space–time surrounding a charged body, the Kerr–Newman solution to the space–time surrounding a charged and rotating body and the Kerr solution to the space–time surrounding a rotating body.

The most important element in the modelling of compact stellar objects is pressure anisotropy. Pressure anisotropy is an important quantity in the sense that it enables researchers to investigate the structures and properties of the stellar objects. The effects of pressure anisotropy on the stellar objects are evident in different investigations for the neutral or charged stellar objects. The study on anisotropic spheres in general relativity conducted by Bowers and Liang [5] widely discusses how the pressure anisotropy affects anisotropic bodies in general relativity. The study by Dev and Gleiser [6] shows that the presence of pressure anisotropy has significant effects on the structure and properties of the stellar objects. Investigations by Ruderman [7] on realistic stellar models show that the radial pressure may not be equal to the tangential pressure within the stellar interior. The study conducted by Karmakar *et al* [8] on the role of pressure anisotropy shows that the redshifts, maximum compactness and mass may increase due to the presence of anisotropic pressures. On the other hand, Pant *et al* [9] constructed a spherically symmetric charged anisotropic fluid model for superdense stars in isotropic coordinates. The study shows that the maximum mass decreases with the increase of pressure anisotropy.

Furthermore, the study on static space–time containing matter in spherically symmetric stellar objects conducted by Mak and Harko [10] shows that the energy density, radial and tangential pressures are finite and positive inside the anisotropic star. The models on anisotropic strange quark stars found by Panahi *et al* [11] show that pressure anisotropy increases the maximum mass and the corresponding radius of a typical strange quark star. It was also discovered that anisotropy can be more effective than the electric charge in increasing the maximum mass of a strange quark star. The investigation by Herrera and Barreto [12] on Newtonian polytropes for anisotropic matter shows that anisotropy has great impact on the structure and behaviour of stellar objects.

Different neutral anisotropic and isotropic stellar models have been generated in order to obtain exact solutions to Einstein field equations. The treatments by Mak and Harko [13] describe the model for an anisotropic quark matter in static spherically symmetric space–time. Anisotropic fluid star model was proposed by Pant *et al* [14]. The study on the existence and stability of self-gravitating spheres with anisotropic pressure performed by Dev and Gleiser [15] utilised Chandrasekhar's variational model for radial perturbations to investigate their stability. Furthermore, in the investigation conducted by Kalam *et al* [16], an anisotropic star model under the relativistic framework of Krori and Barua space–time was generated. On the other hand, the anisotropic charged stellar models include the models on the quark stars found by Sunzu *et al* [17], the models developed by Malaver [18] within the framework of MIT-Bag, the investigation conducted by Murad and Fatema [19], models developed by Mafa Takisa and Maharaj [20] in a core envelope setting, regular models generated by Maharaj and Mafa Takisa [21], the investigation conducted by Malaver [22], the study conducted by Matondo and Maharaj [23]. Most of the anisotropic models do not regain isotropic models as a special case and always have anisotropy, which is not physical. It is also found that most of the charged models with vanishing anisotropy have the presence of an electric field always which is also not physical. It is important to have neutral models with vanishing anisotropy.

A variety of mathematical techniques are being used by researchers when determining solutions to Einstein field equations for stellar objects, and it is important to use equation of state together with field equations. The forms of equations of the state include polytropic, van der Waals, linear and quadratic equations. The study on anisotropic models with a polytropic equation of state includes the study conducted by Thirukkanesh and Ragel [24], models generated by Spaans and Silk [25], Kinasiewicz and Mach [26], recent relativistic isotropic models found by Harko and Mak [27] and anisotropic neutral models found recently by Sunzu [28]. Anisotropic models with van der Waals equation of state include compact anisotropic models developed by Thirukkanesh and Ragel [29], charged star models developed by Malaver [30] and recent findings of Sunzu and Mahali [31]. Recent star models obeying Chaplygin's equation of state are demonstrated by Bhar *et al* [32] and Das *et al* [33].

Anisotropic models with linear equation of state include the charged models developed by Sunzu and Danford [34], generalised compact star models by Malaver [35], the investigation by Escupi and Aloma [36] on the conformal anisotropic relativistic charged fluid spheres and the work of Sharma and Maharaj [37] that describes the model for a class of relativistic stars. Other linear models are those found by Harko and Cheng [38], Sotani *et al* [39], the investigation by Mafa Takisa and Maharaj [40] on compact models with regular charge distribution and the study by Thirukkanesh and Maharaj [41] on a charged anisotropic matter. The anisotropic models with a quadratic equation of state include the study conducted by Feroze and Siddiqui [42] on charged anisotropic matter, the study by Ngubelanga *et al* [43] on models for compact stars and the investigation on a strange quark star model conducted by Malaver [44]. We observe that most of the models generated with a quadratic equation of state are charged and have non-vanishing anisotropy. Therefore, in our study, we establish neutral models with a quadratic equation of state, which have vanishing anisotropy.

The objective of our paper is to generate new exact solutions to the system of Einstein field equations for the neutral anisotropic matter in static spherically symmetric space–time using the quadratic equation of state. We consider the model developed by Sunzu *et al* [17], which has an electric field always present with a linear equation of state. In our work, we apply the same choice of measure of anisotropy and metric function as in Sunzu *et al* [17]; however, we consider a neutral star and a quadratic equation of state. Thus, we present our work in the following sequence. In §2, we give basic and field equations. In §3, we present the transformation of the field equations using the new variables proposed by Durgapal and Bannerji [45]. The transformed field equations are incorporated with the quadratic equation of state, thereafter we specify the metric function and the measure of anisotropy which is physically reasonable. The formulation of the differential equation that governs the neutral anisotropic model is presented in §4. In §5, we obtain solutions to the first non-singular model that regains the earlier Einstein model with isotropic pressures, and the model also agrees with Minkowski space–times. In §6, we generate the second class of exact solutions for the second non-singular model. In §7, we give discussion on the generated plots and stellar masses.

2. Basic and field equations

In this section, we formulate the model that is devoted to a neutral stellar object with an anisotropic matter. We model the interior of the stellar object with anisotropic matter in general relativity by considering the space–time geometry that is static and spherically symmetric. The interior space–time is expressed by the line element

$$\mathrm{d}s^2 = -\mathrm{e}^{2\nu(r)}\mathrm{d}t^2 + \mathrm{e}^{2\lambda(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \tag{1}$$

where $\nu(r)$ and $\lambda(r)$ are variables defining the gravitational potentials. The exterior space–time is described by Schwarzschild [1] and is represented by a line element

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \left(1-\frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \tag{2}$$

where M is the total mass of the neutral stellar object. For the neutral anisotropic stellar object, the energy–momentum tensor is expressed by the matrix

$$\tau_{ij} = \mathrm{diag}(-\rho, p_{\mathrm{r}}, p_{\mathrm{t}}, p_{\mathrm{t}}), \tag{3}$$

where quantity ρ is the energy density, p_{r} and p_{t} are the radial and tangential pressures, respectively, measured relative to a comoving unit time-like fluid four-velocity.

According to Ishak [2], Einstein field equations are nonlinear partial differential equations in four independent variables. These equations describe the behaviour and influence of the gravitational field on the content of the matter. They depend on the nature of the fluid matter under consideration. However, charged or neutral fluids can either be anisotropic or isotropic. The Einstein field equations for neutral anisotropic stellar objects were generated by Krasinski [46] and are given as

$$\frac{1}{r^2}(1-\mathrm{e}^{-2\lambda}) + \frac{2\lambda'}{r}\mathrm{e}^{-2\lambda} = \rho, \tag{4a}$$

$$-\frac{1}{r^2}(1-\mathrm{e}^{-2\lambda}) + \frac{2\nu'}{r}\mathrm{e}^{-2\lambda} = p_{\mathrm{r}}, \tag{4b}$$

$$\mathrm{e}^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu'}{r} - \frac{\lambda'}{r}\right) = p_{\mathrm{t}}, \tag{4c}$$

where prime denotes differentiation of the parameters with respect to radial coordinate r. In the above system of field equations, we are applying units where the coupling constant and the speed of light are unity. The mass contained in the sphere is expressed as

$$M(r) = \frac{1}{2}\int_0^r \omega^2\rho(\omega)\mathrm{d}\omega. \tag{5}$$

The relationship between the radial pressure and energy density for the neutral stellar object is given by the equation of state. We consider the quadratic equation of state, which is given by

$$p_{\mathrm{r}} = \alpha\rho^2 + \beta\rho - \gamma, \tag{6}$$

where α, β and γ are arbitrary real constants. It is important to note that when $\alpha = 0$, eq. (6) becomes linear. Recent models with a linear equation of state include the one proposed by Mafa Takisa and Maharaj [20] and Matondo and Maharaj [23]. Furthermore, for a particular choice $\beta = \frac{1}{3}$ and $\gamma = \frac{4}{3}B$ in a linear equation of state, we have quark star models. Recent models that describe the properties and structure of quark stars are given by Malaver [35], Sunzu and Danford [34], Sunzu *et al* [17,47] and Maharaj *et al* [48]. Finally, we observe that when $\beta = \gamma = 0$ we have the equation of state $p_{\mathrm{r}} = \alpha\rho^2$. This describes the well-known Bose–Einstein condensate stars as highlighted in Chavanis and Harko [49].

3. Transformation of field equations

The transformation of field equations is enhanced by introducing new variables as given by Durgapal and Bannerji [45]. These new variables are

$$x = Cr^2, \quad Z(x) = \mathrm{e}^{-2\lambda(r)}, \quad A^2y^2(x) = \mathrm{e}^{2\nu(r)}, \tag{7}$$

where A and C are arbitrary constants. From (7) the line element (1) becomes

$$\mathrm{d}s^2 = -A^2y^2(x)\mathrm{d}t^2 + \frac{1}{Z(x)}\frac{1}{4xC}\mathrm{d}x^2 + \frac{x}{C}(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2). \tag{8}$$

The mass function (5) becomes

$$M(x)=\frac{1}{4C^{3/2}}\int_0^x\sqrt{\omega}\rho(\omega)\mathrm{d}\omega. \tag{9}$$

We can write the transformed Einstein field equations together with the quadratic equation of state in the form

$$\rho=\left(\frac{1-Z}{x}-2\dot{Z}\right)C, \tag{10a}$$
$$p_{\mathrm{r}}=\alpha\rho^2+\beta\rho-\gamma, \tag{10b}$$
$$p_{\mathrm{t}}=p_{\mathrm{r}}+\Delta, \tag{10c}$$
$$\Delta=\left[4xZ\frac{\ddot{y}}{y}+\left(2x\frac{\dot{y}}{y}+1\right)\dot{Z}+\frac{1-Z}{x}\right]C, \tag{10d}$$

where $\Delta=p_{\mathrm{t}}-p_{\mathrm{r}}$ is the measure of anisotropy.

System (10) is the model for the neutral stellar object with quadratic equation of state. The system involves six variables $(\rho, p_{\mathrm{r}}, p_{\mathrm{t}}, Z, y, \Delta)$ in four equations. We can specify any two variables among the six in order to obtain the solution to the system. We choose to specify the metric function y and the measure of anisotropy Δ. The choices for y and Δ are given as

$$y=\frac{1-ax^m}{1+bx^n}, \tag{11}$$
$$\Delta=a_1x+a_2x^2+a_3x^3, \tag{12}$$

where a, b, m, n, a_1, a_2 and a_3 are arbitrary real constants.

The choice of the metric function is made based on the fact that y is regular, finite and continuous inside the stellar object which is an important condition for gravitational potentials. When $a=b=0$ we have $y=0$. Physically, we expect matter variables to vanish when gravitational potentials remain constant. The gravitational behaviour of the anisotropic neutral stellar object is governed by system (10). The choice of anisotropy allows isotropic models to be generated. This is so when $\Delta=0$ (i.e. $a_1=a_2=a_3=0$). A good physical anisotropic model should contain isotropic pressure as a special case. At the centre of the stellar star (i.e. $x=0$), we observe that $\Delta=0$. This is physical as we expect the radial pressure and tangential pressure to be the same at the stellar core. This choice of y and Δ is used in the charged models by Sunzu *et al* [17] with a linear quark equation of state. We keep the same choice of these variables to generate new neutral anisotropic models with a general form of quadratic equation of state.

4. The differential equation governing the model

In this section, we derive the general differential equation governing the model by considering the choice of the metric function y and measure of anisotropy Δ made in eqs (11) and (12). Substituting eqs (11) and (12) into eq. (10d), we obtain

$$\dot{Z}+\left(\frac{D(x)-E(x)}{R(x)(1+bx^n)}\right)Z=\frac{\frac{\Delta}{C}-\frac{1}{x}}{\frac{T(x)+(1-ax^m)(1+bx^n)}{(1-ax^m)(1+bx^n)}},$$
$$=\frac{\left[\frac{\Delta}{C}-\frac{1}{x}\right](1-ax^m)(1+bx^n)}{T(x)+(1-ax^m)(1+bx^n)}, \tag{13}$$

where, for simplicity, we have set

$$\begin{aligned}D(x)&=(1+bx^n)[4ab(n-m)(m+n-1)x^{m+n}]\\&\quad+(1+bx^n)[-4am(m-1)x^m-4bn(n-1)x^n],\\E(x)&=8ab^2n(n-m)x^{m+2n}-8abnmx^{m+n}\\&\quad-8b^2n^2x^{2n}-(1-ax^m)(1+bx^n)^2,\\R(x)&=2ab(n-m)x^{m+n+1}-2amx^{m+1}\\&\quad-2bnx^{n+1}+x(1-ax^m)(1+bx^n),\\T(x)&=2ab(n-m)x^{m+n}-2amx^m-2bnx^n.\end{aligned}$$

Clearly, eq. (13) is a nonlinear differential equation with variable $Z(x)$. When eq. (13) is integrated, the solution is a function $Z(x)$ which is one of the gravitational potentials. Having obtained $Z(x)$, we can find the expression for the energy density ρ, radial pressure p_{r} and tangential pressure p_{t}. Exact solution to the differential equation is obtained when a suitable choice of values of the constants m and n is made which in turn makes the differential equation linear and easy to be integrated. The solution of the differential equation should exist and produce matter variables which are well behaved. Equation (13) can be traceable by choosing particular values for the constants m and n in the metric function y given in eq. (11).

5. Non-singular model I

We generate the first class of exact solutions to eq. (13) by taking $m=1$, $n=1$ and $a=b=0$. The metric function becomes $y=1$, and eq. (13) becomes

$$\dot{Z}-\frac{Z}{x}=\frac{a_1x+a_2x^2+a_3x^3}{C}-\frac{1}{x}. \tag{14}$$

Solving eq. (14), we obtain

$$Z=\frac{x}{C}\frac{(6a_1x+3a_2x^2+2a_3x^3+6Ck)}{6}+1,$$
$$=\frac{6a_1x^2+3a_2x^3+2a_3x^4+6Ckx+6C}{6C}, \tag{15}$$

where k is a constant of integration. Using eq. (15), the gravitational potentials and matter variables in system (10) become

$$e^{2\nu} = A^2, \tag{16a}$$

$$e^{2\lambda} = \frac{1}{Z} = \frac{6C}{x(6a_1x + 3a_2x^2 + 2a_3x^3 + 6Ck) + 6C}, \tag{16b}$$

$$\rho = -5a_1x - \frac{7}{2}a_2x^2 - 3a_3x^3 - 3Ck, \tag{16c}$$

$$p_r = \alpha\left(-5a_1x - \frac{7}{2}a_2x^2 - 3a_3x^3 - 3Ck\right)^2 + \beta\left(-5a_1x - \frac{7}{2}a_2x^2 - 3a_3x^3 - 3Ck\right) - \gamma, \tag{16d}$$

$$p_t = \alpha\left(-5a_1x - \frac{7}{2}a_2x^2 - 3a_3x^3 - 3Ck\right)^2 + \beta\left(-5a_1x - \frac{7}{2}a_2x^2 - 3a_3x^3 - 3Ck\right) - \gamma + a_1x + a_2x^2 + a_3x^3, \tag{16e}$$

$$\Delta = a_1x + a_2x^2 + a_3x^3. \tag{16f}$$

The mass function (5) becomes

$$M(x) = -\frac{x^{3/2}}{4C^{3/2}}\left(2a_1x + a_2x^2 + \frac{2}{3}a_3x^3 + 2Ck\right) \tag{17}$$

and the line element (8) corresponding to this solution becomes

$$ds^2 = -A^2dt^2 + \frac{6dx^2}{4x\left[x(6a_1x + 3a_2x^2 + 2a_3x^3 + 6Ck) + 6C\right]} + \frac{x}{C}(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{18}$$

If we set $\alpha = 0$ in system (16), we obtain the gravitational potentials and matter variables, which correspond to the linear equation of state

$$p_r = \beta\rho - \gamma. \tag{19}$$

For an isotropic case ($\Delta = 0$), we have $a_1 = a_2 = a_3 = 0$, and the gravitational potentials and matter variables in system (16) become

$$e^{2\nu} = A^2, \tag{20a}$$

$$e^{2\lambda} = \frac{1}{kx + 1}, \tag{20b}$$

$$\rho = -3Ck, \tag{20c}$$

$$p_r = p_t = 9C^2k^2\alpha - 3\beta Ck - \gamma, \tag{20d}$$

$$M = -\frac{x^{3/2}k}{2C^{1/2}}. \tag{20e}$$

The line element corresponding to the isotropic model becomes

$$ds^2 = -A^2dt^2 + \left(\frac{1}{4xC(kx+1)}\right)dx^2 + \frac{x}{C}(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{21}$$

The line element in (21) can be expressed in the form

$$ds^2 = -A^2dt^2 + \left(1 - \frac{r^2}{\delta^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{22}$$

where $\delta^2 = -(1/kC)$ and $k < 0$. The line element (22) is a familiar neutral isotropic Einstein model. When $k = 0$, $\gamma = 0$ and $\Delta = 0$, the gravitational potentials and matter variables in system (20) become

$$e^{2\nu} = A^2,\ e^{2\lambda} = 1,\ \rho = 0,\ p_r = p_t = 0,\ M = 0.$$

This reveals that the gravitational potentials remain constant when the matter variables vanish. This is true for the Minkowski space–time. It is important to note that when $\alpha = 0$ and $\Delta = 0$, we obtain an isotropic model with a linear equation of state.

6. Non-singular model II

Different exact solutions to the differential equation (13) can be obtained when the metric function is not kept constant. For this, we choose $m = 1, n = 1, a \neq 0$ and $b = 0$. Considering this choice of values, the metric function becomes

$$y(x) = 1 - ax. \tag{23}$$

After decomposition in partial fractions, the differential equation (13) becomes

$$\dot{Z} - \left(\frac{1}{x} + \frac{2a}{1-3ax}\right)Z = \frac{\left[\frac{(a_1x + a_2x^2 + a_3x^3)x}{C} - 1\right](1 - ax)}{x(1-3ax)}. \tag{24}$$

Solving eq. (24), we obtain

$$Z = -\frac{1}{Ca^3}\left[\left(\frac{2}{5} - \frac{1}{5}ax\right)a_1a^2x - F(x)a_2ax + G(x)a_3x - a^3C - \frac{Cka^3x}{(1-3ax)^{2/3}}\right]. \tag{25}$$

The gravitational potentials and matter variables in system (10) become

$$e^{2\nu} = A^2(1-ax)^2, \tag{26a}$$

$$e^{2\lambda} = -Ca^3\left[\left(\frac{2}{5}-\frac{1}{5}ax\right)a_1a^2x - F(x)a_2ax + G(x)a_3x + I(x)\right]^{-1}, \tag{26b}$$

$$\rho = -\frac{1}{a^3(1-3ax)}H(x), \tag{26c}$$

$$p_{\mathrm{r}} = \alpha\left[-\frac{1}{a^3(1-3ax)}H(x)\right]^2 + \beta\left[-\frac{1}{a^3(1-3ax)}H(x)\right] - \gamma, \tag{26d}$$

$$p_{\mathrm{t}} = \alpha\left[-\frac{1}{a^3(1-3ax)}H(x)\right]^2 + \beta\left[-\frac{1}{a^3(1-3ax)}H(x)\right] - \gamma + \Delta, \tag{26e}$$

$$\Delta = a_1x + a_2x^2 + a_3x^3 \tag{26f}$$

For simplicity, we have set

$$F(x) = \left(\frac{-3}{40} - \frac{3}{20}ax + \frac{1}{8}a^2x^2\right),$$

$$G(x) = \left(\frac{1}{55} + \frac{2}{55}ax + \frac{1}{11}a^2x^2 - \frac{1}{11}a^3x^3\right),$$

$$H(x) = \left(\frac{-6}{5} - \frac{23}{5}ax + 3a^2x^2\right)a^2a_1 + \frac{Ca^3k(3-5ax)}{(1-3ax)^{2/3}} - \left(\frac{9}{40} + \frac{3}{40}ax - \frac{25}{8}a^2x^2 + \frac{21}{8}a^3x^3\right)aa_2 - \left(\frac{3}{55} + \frac{1}{55}ax + \frac{1}{11}a^2x^2 - \frac{30}{11}a^3x^3 + \frac{27}{11}a^4x^4\right)a_3,$$

$$I(x) = -a^3C - \frac{Ca^3kx}{(1-3ax)^{2/3}}.$$

The line element (1) corresponding to this model becomes

$$ds^2 = -A^2(1-ax)^2dt^2 - Ca^3\left[\left(\frac{2}{5}-\frac{1}{5}ax\right)a_1a^2x - F(x)a_2ax - G(x)a_3x - a^3C + I(x)\right]^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{27}$$

The mass function (5) becomes

$$M(x) = \frac{x^{3/2}}{a^3(1-3ax)C^{3/2}} \times \left[J(x) - L(x) + N(x) - \frac{1}{2}a^3Ck(1-3ax)^{1/3}\right], \tag{28}$$

where, for simplicity, we have set

$$J(x) = \left(\frac{1}{5} - \frac{7}{10}ax + \frac{3}{10}a^2x^2\right)a^2a_1,$$

$$L(x) = \left(\frac{-3}{80} + \frac{3}{80}ax + \frac{23}{80}a^2x^2 - \frac{3}{16}a^3x^3\right)aa_2,$$

$$N(x) = \left(\frac{1}{110} - \frac{1}{110}ax - \frac{1}{110}a^2x^2 - \frac{2}{11}a^3x^3 + \frac{3}{22}a^4x^4\right)a_3.$$

The gravitational potentials and matter variables obtained are regular, finite and continuous inside the stellar object.

If we set $\alpha = 0$ and $a < 0$ in system (26), we obtain a linear equation of state. The exact solutions to Einstein field equations for the neutral anisotropic model above are finite, regular and continuous inside the stellar object. For the case of isotropic neutral model ($\Delta = 0$), we have $a_1 = a_2 = a_3 = 0$. The line element corresponding to the neutral isotropic stellar model becomes

$$ds^2 = -A^2(1-ax)^2dt^2 + \left(\frac{1}{1+\frac{kx}{(1-3ax)^{\frac{2}{3}}}}\right)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{29}$$

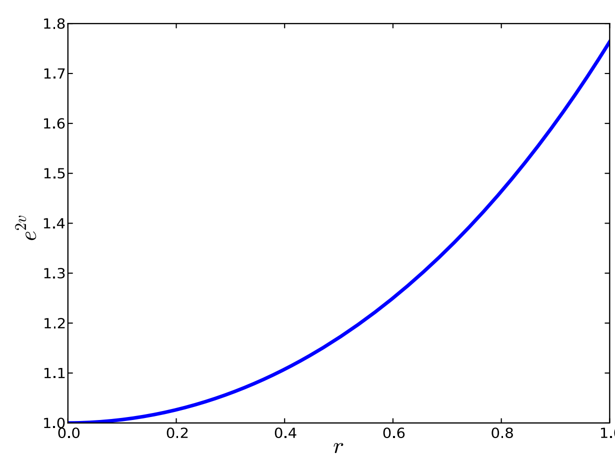

Figure 1. The gravitational potential $e^{2\nu}$ against radial distance r.

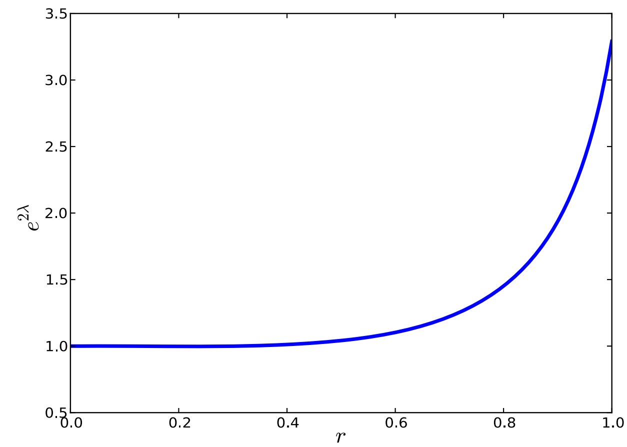

Figure 2. The gravitational potential $e^{2\lambda}$ against radial distance r.

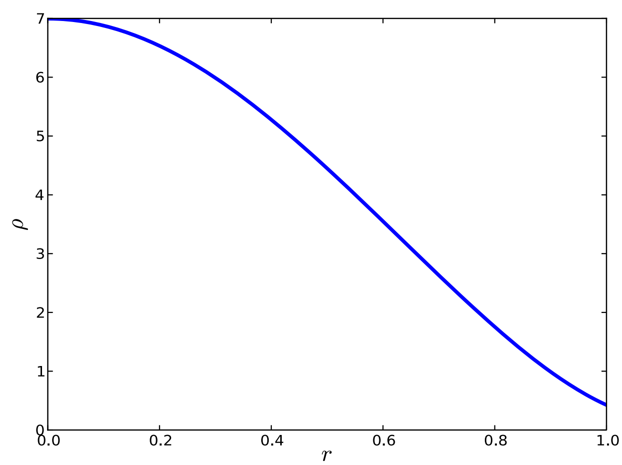

Figure 3. The energy density ρ against radial distance r.

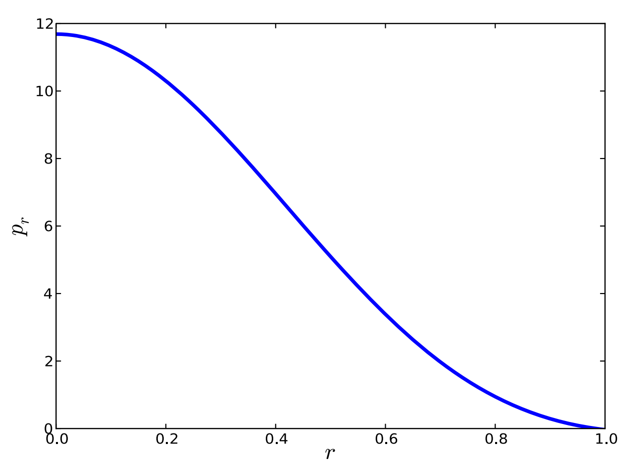

Figure 4. The radial pressure p_{r} against radial distance r.

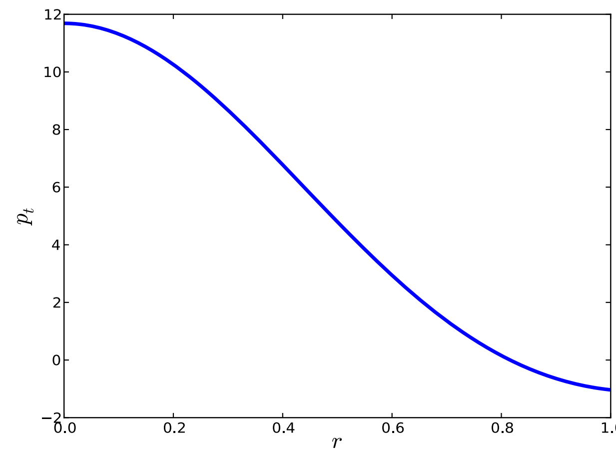

Figure 5. The tangential pressure p_{t} against radial distance r.

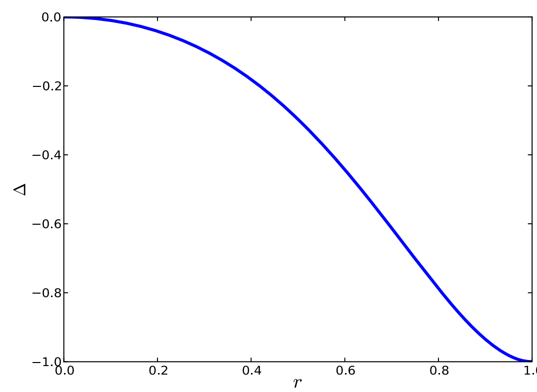

Figure 6. The measure of anisotropy Δ against radial distance r.

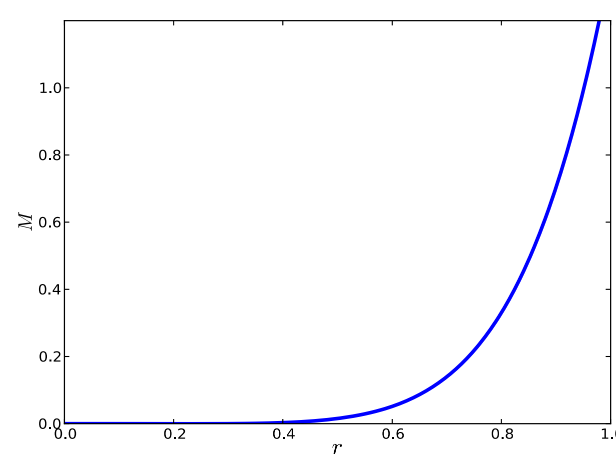

Figure 7. The mass M against radial distance r.

When $k = 0$, $\gamma = 0$ and $\Delta = 0$, we obtain

$$e^{2\nu} = A^2(1 - ax)^2,\ e^{2\lambda} = 1,\ \rho = 0,$$
$$p_{\mathrm{r}} = p_{\mathrm{t}} = 0,\ M(0) = 0.$$

When $a = 0$, the gravitational potentials become constant while the matter variables vanish. This is true for Minkowski space–time. This is also the case of the centre (i.e. $x = 0$). It is important to note that when

Table 1. Relativistic stellar masses are consistent with the observation generated for several parameter values.

$\tilde{a}$	$\tilde{C}$	$\tilde{a_1}$	$\tilde{a_2}$	$\tilde{a_3}$	$\tilde{k}$	r (km)	$M(M_\odot)$	References
4.03	1.0	0.2	1.5	2.1	1.0	5.78	1.73	Sunzu *et al* [47]
4.27	1.0	0.2	2.5	1.0	1.5	7.61	1.60349	Sunzu *et al* [47]
5.50	1.0	2.0	2.5	2.0	1.0	9.46	2.86	Mak and Harko [51]
6.54	1.0	0.5	1.9	2.0	1.0	9.40	1.67	Freire *et al* [52]
7.15	1.0	0.1	0.8	2.0	2.0	10.99	2.02	Negreiros *et al* [53]
5.15	1.0	0.1	0.1	2.0	2.0	7.07	1.433	Dey *et al* [54]
6.42	1.0	1.0	0.1	2.0	2.0	9.10	1.58	Güver *et al* [55]
6.36	1.0	1.0	1.0	2.0	2.0	9.30	1.74	Güver *et al* [56]
6.35	1.0	1.0	0.1	2.0	2.0	9.69	1.97	Gangopadhyay *et al* [57]

$\alpha = 0$ and $\Delta = 0$, we obtain an isotropic model with a linear equation of state.

7. Discussion

In this section, we show that the exact solutions obtained in §6 for the second non-singular model are well behaved throughout its interior. For this, we generate graphs for the gravitational potentials and matter variables. The graphs are plotted by using Python programming language. We made specific choices of the values of the constants as follows: $A = 1.0, a = -0.328, C = 1.0, a_1 = -1.0, a_2 = -1.0, a_3 = 1.0, \alpha = 0.2, \beta = 0.3, \gamma = 0.2$ and $k = 0.1$. The graphs generated are for the gravitational potentials $e^{2\nu}$ (figure 1), gravitational potential $e^{2\lambda}$ (figure 2), energy density ρ (figure 3), radial pressure p_r (figure 4), tangential pressure p_t (figure 5), measure of anisotropy Δ (figure 6) and mass M (figure 7). The figures are plotted against the radial distance r. We find that the gravitational potentials (figures 1 and 2) are regular, finite and continuous throughout the stellar interior. This is physical. We observe that the energy density, radial pressure and tangential pressure in figures 3–5 are decreasing functions with radial coordinate r. This means that $\rho' < 0$, $p_r' < 0$ and $p_t' < 0$. We expect these variables to be maximum at the centre of the stellar object. Similar profiles for these quantities were obtained by Sunzu [50], Feroze and Siddiqui [42] and Mafa Takisa and Maharaj [21]. We also observe that the measure of anisotropy in figure 6 is finite and continuous. It decreases from the centre of the stellar object to the surface. This indicates that $p_r \geq p_t$ throughout in the interior of the stellar object. We observe that $\Delta = 0$ at the centre of the stellar object. This means that the radial pressure is the same as the tangential pressure. It is physical to have this behaviour. A similar profile is observed in the work by Sunzu *et al* [17], Sunzu [50], Karmakar *et al* [8] and Kalam *et al* [16]. In figure 7, we observe that the mass is a monotonically increasing function as the radial distance increases.

We also generate relativistic masses using the same exact solutions in §6. We transform mass function (28) using the transformation $\tilde{a} = aR^2, \tilde{a_1} = a_1R^2, \tilde{a_2} = a_2R^2, \tilde{a_3} = a_3R^2, \tilde{C} = CR^2$ and $\tilde{k} = kR^2$. For computation purpose, we set $R = 45.00$. We generate a star with radius $r = 9.46$ km and mass $M = 2.86M_\odot$ consistent with the object found by Mak and Harko [51]. Other stellar objects generated are consistent with the star PSR J1903+327 generated by Freire *et al* [52] with mass $M = 1.67M_\odot$ and radius $r = 9.4$ km, the stellar object obtained by Negreiros *et al* [53] with mass $M = 2.02M_\odot$ and radius $r = 10.99$ km, a stellar object obtained by Sunzu *et al* [47] with mass $M = 1.60349M_\odot$ and radius $r = 7.6$ km, the object obtained by Dey *et al* [54] with mass $M = 1.433M_\odot$ and radius $r = 7.07$ km, the body generated by Sunzu *et al* [17] with mass $M = 1.73268M_\odot$ and radius $r = 5.78$ km, the star 4U 1820-30 obtained by Güver*et al* [55] with mass $M = 1.58M_\odot$ and radius $r = 9.10$ km, the star 4U 1608-52 generated by Güver *et al* [56] with mass $M = 1.74M_\odot$ and radius $r = 9.30$ km and the stellar object with mass $M = 1.97M_\odot$ and radius $r = 9.69$ km obtained by Gangopadhyay *et al* [57]. The parameter values used to generate masses and radii in this model are summarised in table 1.

8. Conclusion

In this study, we have obtained two classes of exact solutions to the Einstein field equations for the space–time geometry that are static and spherically symmetric. The quadratic equation of state is incorporated into the transformed Einstein field equations to generate gravitational potentials and matter variables. Results generated in our models regain the well-known earlier Einstein model and are in agreement with the Minkowski space–time. We have also observed that when the anisotropic matter variables are set to vanish ($\Delta = 0$), we obtain the isotropic models as a special case. The graphs for gravitational potentials and matter variables plotted against the radial distance are well behaved and physically reasonable. Relativistic stellar objects with masses and radii consistent with the observations have been generated from our model. The masses generated are consistent with those found by Sunzu *et al* [17,47], Mak and Harko [51], Freire *et al* [52], Negreiros *et al* [53], Dey *et al* [54], Güver *et al* [55,56] and Gangopadhyay *et al* [57]. Further study on the behaviour and structure of the interior of the stellar objects can be conducted if this model is considered with different choices of measure of anisotropy, metric function and equation of state such as van der Waals and polytropic.

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