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Rogue wave dynamics in barotropic relaxing media

SERGE PAULIN T MUKAM[1,2,*], ABBAGARI SOULEYMANOU[1,2,3,4], VICTOR K KUETCHE[1,2,3,5] and THOMAS B BOUETOU[1,2,3,5]

[1]National Advanced School of Engineering, University of Yaounde I, P.O. Box 8390, Yaounde, Cameroon
[2]Laboratory of Mechanics, Materials and Structures, Department of Physics, University of Yaounde I, P.O. Box 812, Yaounde, Cameroon
[3]Centre d'Excellence en Technologies de l'Information et de la Communication (CETIC), University of Yaounde I, P.O. Box 812, Yaounde, Cameroon
[4]Department of Basic Science, Faculty of Mines, Law and Humanities, Faculty of Mines and Petroleum Industries, University of Maroua, P.O. Box 46, Maroua, Cameroon
[5]The Abdus Salam International Centre for Theoretical Physics, Strada Costiera, 11, 34151 Trieste, Italy
*Corresponding author. E-mail: spmukam@gmail.com

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Abstract. In this work, we deal with a nonlinear wave equation, namely the Vakhnenko equation, which models the propagation of nonlinear wave in the barotropic relaxing media. Based on the homoclinic breather limit method, we seek rogue wave solution to the above equation. The results show that rogue wave or giant wave can exist in such a medium.

Keywords. Barotropic relaxing media; homoclinic breather limit method; rogue wave.

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1. Introduction

Rogue waves, originally referred to as huge waves, are responsible for many marine disasters [1]. Many efforts have been made to study and understand the nature of this mysterious phenomenon, and it is found that the rogue wave appears not only in the oceanic condition but also in nonlinear physical systems such as plasma [2], Bose–Einstein condensates [3], nonlinear optics [4–7], hydrodynamics [8–11] etc. It was also seen from the study that rogue wave appears from nowhere and disappears without a trace [12]. Their amplitude is two or more times larger than the known common waves [13]. The expressions of a rogue wave solution are rational functions localised both in space and time, and these imply their unpredictability [14,15]. The simplest rogue wave solution was first obtained by Peregrine [15]. Moreover, the first-order rogue wave solution for the nonlinear Schrödinger equation was calculated by Akhmediev *et al* [12]. Analytical rogue wave solution has also been obtained for various physical models [16–29]. Recently, we performed a study on a system modelling the propagation of waves in nonlinear fibre optics, and the results showed that the rogue wave can interact with soliton-like wave without the destruction and the modification of the soliton wave [30]. Also, in our recent work, we have shown that rogue waves can be controlled during their propagation by changing the values of the parameter denoting the strength of higher-order nonlinearity [31].

In this work, we are concerned with a nonlinear wave equation, namely the Vakhnenko equation (VE), which describes the propagation of nonlinear waves in the barotropic relaxing media, given as follows:

$$\partial_x(\partial_t + v\partial_x)v + v = 0. \tag{1}$$

This equation was studied in [32] and its loop soliton solutions were obtained using the Hirota bilinear method. The quantities v, x and t scale the pressure, the distance and the time, respectively. The term 'barotropic' in meteorology means a case where the pressure of the atmosphere is dependent upon its density only. Using the homoclinic breather limit method [33], we provide rogue wave solution to the VE. Thus, the organisation of this work is as follows: In the next section, we show the procedure on how to obtain the breather wave for the VE starting from a homoclinic test function and the last section is devoted to a brief conclusion.

2. Homoclinic breather and rogue wave solution to the VE

The aim of this section is to construct rogue wave solution to eq. (1) using the homoclinic test approach. Here we follow the procedure given in [32], to get the bilinear form of the VE.

For this, we make some transformations by introducing variables ξ and τ given as

$$x=\varphi(\xi,\tau):=\tau+\int_{-\infty}^{\xi}U(\xi',\tau)\,\mathrm{d}\xi'+x_0 \quad t=\xi \quad (2)$$

with $v(x,t)=U(\xi,\tau)$ and x_0 is a constant. According to eq. (2)

$$\partial_\xi=\partial_t+v\partial_x,\quad \partial_\tau=\rho\partial_x, \quad (3)$$

with

$$\rho(\xi,\tau)=1+\int_{-\infty}^{\xi}U_\tau\,\mathrm{d}\xi', \quad (4)$$

so that

$$\rho_\xi=U_\tau.$$

Combining eqs (3) and (4), we obtain the following equation:

$$U_{\xi\tau}+\rho U=0. \quad (5)$$

If we eliminate the quantity ρ between (4) and (5), the new form of the VE is obtained which is written as follows:

$$UU_{\xi\xi\tau}-U_\xi U_{\xi\tau}+U^2U_\tau=0. \quad (6)$$

Introducing the quantity V defined by $V_\xi=U$ and assuming the condition $\xi\to-\infty$, then $\rho=1+V_\tau$ and (5) becomes

$$V_{\xi\xi\tau}+V_\xi V_\tau+V_\xi=0. \quad (7)$$

Then taking

$$V=6(\ln f)_\xi, \quad (8)$$

the bilinear form of eq. (7) is given as follows:

$$(D_\tau D_\xi^3+D_\xi^2)f\cdot f=0, \quad (9)$$

where the quantity D is the Hirota operator defined as follows:

$$D_t^m D_x^n f(t,x)g(t,x)=\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^m\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^n f(t,x)g(t',x')|_{t'=t,x'=x}. \quad (10)$$

We choose the homoclinic test function to be written as follows:

$$f(\xi,\tau)=\mathrm{e}^{-p_1(\xi-w_1\tau)}+c_1\cos(p_2(\xi+w_2\tau)) +c_2\mathrm{e}^{p_1(\xi-w_1\tau)}. \quad (11)$$

Inserting the above test function in the bilinear form given in eq. (9), we obtain the following set of equations:

$$\begin{aligned}&p_2^2-p_1^2+p_1^4w_1-p_2^4w_2+3p_1^2p_2^2w_2-p_2^2p_1^2w_1=0,\\&-2-p_1^2w_2-p_2^2w_1+3p_1^2w_1+3p_2^2w_2=0,\\&-2c_1^2p_2^2+8c_1^2p_1^4w_2+8p_1^2c_2-32p_1^4w_1c_2=0. \quad (12)\end{aligned}$$

Solving this set of equation, making the restriction $w_1=w_2$ (without loss of generality), yields

$$c_1=\pm2\sqrt{\frac{(4w_1p_1^2-1)p_1^2}{4p_1^4w_1-p_2^2}c_2},\quad p_1=\sqrt{\frac{4+w_1}{w_1(3w_1+4)}}, \quad (13)$$

$$p_2=\sqrt{\frac{4-w_1}{w_1(3w_1+4)}},\quad w_1<4.$$

Inserting the expressions given in eq. (13) into the homoclinic test function given in (11), we obtain the following:

$$f_1=2\sqrt{c_2}\cosh(\zeta+\ln\sqrt{c_2})+k_1\cos[\chi], \quad (14)$$

$$f_2=2\sqrt{c_2}\cosh(\zeta+\ln\sqrt{c_2})-k_1\cos[\chi], \quad (15)$$

where

$$k_1=2\sqrt{\frac{\left(4w_1p_1^2-1\right)p_1^2}{4p_1^4w_1-p_2^2}c_2}.$$

From the expressions of the test function given above, we derive the solution of eq. (6) as follows:

$$\begin{aligned}U_1=&\frac{1}{[2\sqrt{c_2}\cosh(\zeta)+k_1\cos(\chi)]^2}\\&\times(2k_1\sqrt{c_2}(p_1^2-p_2^2)\cosh(\zeta)\cos(\chi)\\&+4\sqrt{c_2}p_1p_2\sinh(\zeta)\sin(\chi)+4c_2^2p_1^2-k_1^2p_2^2), \quad (16)\end{aligned}$$

$$\begin{aligned}U_2=&\frac{1}{[2\sqrt{c_2}\cosh(\zeta)-k_1\cos(\chi)]^2}\\&\times(2k_1\sqrt{c_2}(p_1^2-p_2^2)\cosh(\zeta)\cos(\chi)\\&+4\sqrt{c_2}p_1p_2\sinh(\zeta)\sin(\chi)+4c_2^2p_1^2+k_1^2p_2^2),\end{aligned}$$

with $\zeta=p_1(\xi-w_1\tau)+\ln\sqrt{c_2}$ and $\chi=p_2(\xi+w_1\tau)$.

Figure 1 corresponds to the solution U_1.

Now, we suppress the periodicity of the function U_1 by taking $p_2\to0$. Hence, we obtain the following expression:

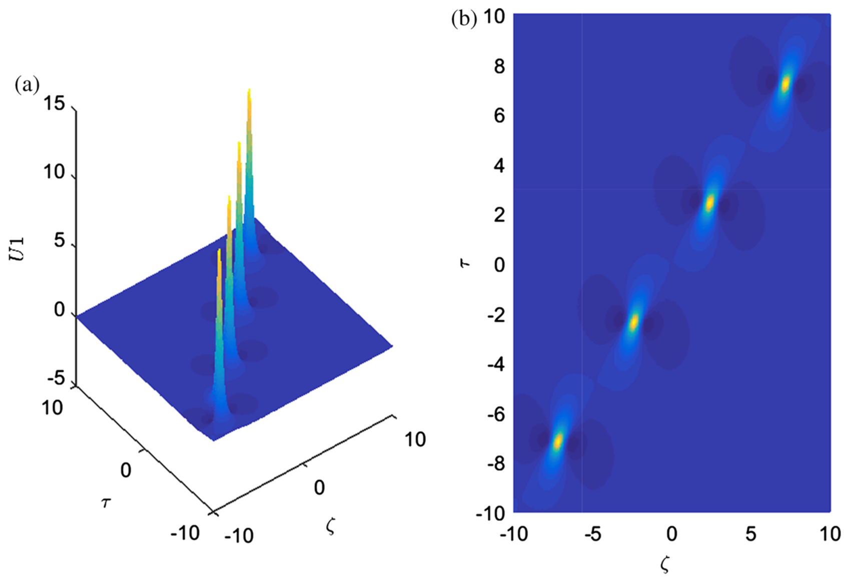

Figure 1. Breather wave solution U_1 for $w_1 = 1$ and $c_2 = 1$: (**a**) three-dimensional (3D) representation and (**b**) density plot.

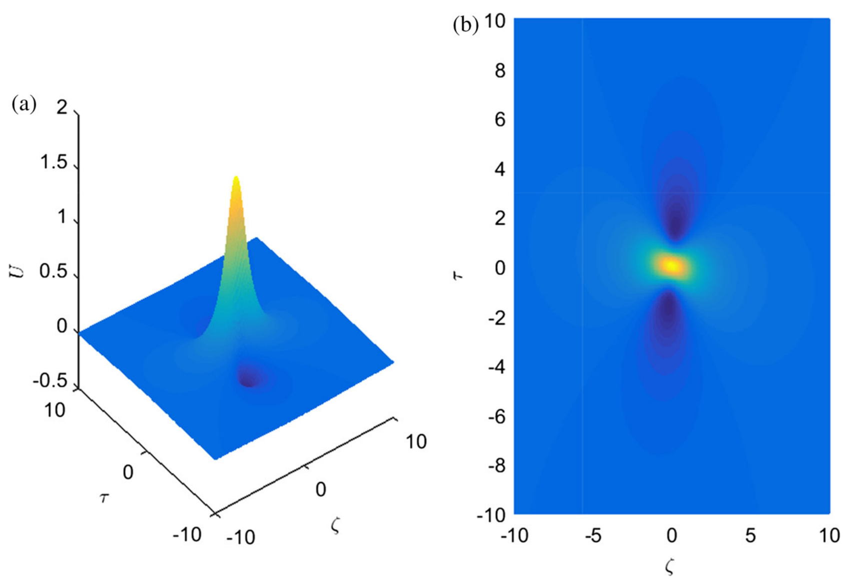

Figure 2. Rogue wave solution U for $w_1 = 1$ and $c_2 = 1$: (**a**) 3D representation and (**b**) density plot.

$$U = \frac{4\left(\frac{4}{3w+4}(1-2\xi w\tau) + 2\left(\xi^2 - w^2\tau^2\right) + 1\right)}{\left((w\tau-\xi)^2 + (w\tau+\xi)^2 + 2\right)^2}. \tag{17}$$

U is the rational homoclinic rogue wave solution to eq. (6). The corresponding feature is given in figure 2.

It is important to point out that $U \to 0$ when x and t go to infinity. U is a rogue wave solution. Indeed it is formed in a short time, it has a zero background and has an amplitude higher than those of the common known waves. It is also important to note that the VE uses a real function and so the rogue wave obtained in this work is a real-valued one. It differs from the complex-valued ones which sit on a non-zero background [27,30]. Rogue wave solution similar to the one presented here was recently reported in [34].

3. Conclusion

In this work, we have applied homoclinic breather limit method to the VE. As a result, we have obtained the solitary breather wave of the VE. Meanwhile, considering a limit behaviour of the breather wave, we obtain rogue wave solution to the equation under consideration. In our future work, we aim to use other methods such as the Darboux transformation and the Wronskian determinant to construct a higher-order rogue wave solution to the VE.

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