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Operational criterion for controlled dense coding with non-trivial tripartite entangled states

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Abstract. In this paper, we provide an operational criterion for controlled dense coding (CDC) with a general class of three-qubit partially entangled states. A general three-qubit pure entangled state can be classified into two inequivalent classes according to their genuine tripartite entanglement. We claim that if a three-qubit state shows entanglement characteristic similar to Greenberger–Horne–Zeilinger (GHZ)-class, then such non-trivial tripartite states are useful in CDC whereas states belonging to the W-class are not useful for that. We start with a particular class of non-trivial partially entangled states belonging to the GHZ-class and show that they are effective in CDC. Then we cite several other examples of different types of tripartite entangled states to support our conjecture.

Keywords. Greenberger–Horne–Zeilinger-states; W-states; partially entangled states; maximal sliced states; symmetric states; controlled dense coding.

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1. Introduction

Ever since its inception, the study of entanglement has always been in the limelight. Entanglement plays a pivotal role in quantum information processing tasks. Among all such tasks, dense coding [1] is one such application of entanglement, first implemented experimentally with polarised entangled photons [2]. The fundamental idea of dense coding is that, with the help of shared entangled channel between the two parties, two bits of information can be transmitted from the sender to the receiver using only one qubit, provided the shared channel is maximally entangled. For security purpose, this transmission of bits from the sender to the receiver can, however, be controlled by another third party when the trio share some tripartite entangled state among them. Such a modified dense coding protocol is known as controlled dense coding (CDC) scheme. The scheme was first presented by using the three-qubit maximally entangled Greenberger–Horne–Zeilinger-(GHZ) state in [3]. The GHZ-state can be generated in the laboratory and has been demonstrated experimentally using two pairs of entangled photons and also one can make the optimal distillation of these states [4,5]. Another important tripartite entangled state is the W-state [6]. In real experimental situation, it has always been a challenge to obtain such multi-qubit maximally entangled resources [7]. Recently, Zang *et al* [8] have shown a fantastic way of transforming a bipartite non-maximally entangled states into a tripartite W-state in cavity quantum electrodynamics (QED). Also, a scheme for deterministic joint remote preparation of an arbitrary four-qubit W-type entangled state has been discussed by Fu *et al* [9]. Hence, it is of fundamental importance to identify the multi-qubit partially entangled systems which may be useful in tasks such as CDC. Controlled dense coding has been investigated with the four-qubit entangled states [10], three-qubit symmetric and non-symmetric states [11,12], three-qubit maximal sliced states [13] and with six-particle graph state [14]. The CDC has also been realised using a five-atom cluster state in cavity-QED [15]. However, it was shown in [16] that the W-class of states is not useful in CDC. Since the GHZ- and W-states are two inequivalent classes of tripartite maximally entangled states, our main motivation in this work is to put forward a conjecture where we claim that the states belonging to the GHZ-class will be useful in CDC whereas those belonging to the W-class will not be. This is the main impetus of this work which will be having incredible experimental relevance. For the sake of further discussion, in this paper, we shall use the term ‘trivial’ for GHZ- and W-states and ‘non-trivial’ for the tripartite states other than GHZ- and W-states.

We, therefore, classify the tripartite states into two inequivalent classes according to their genuine tripartite entanglement. An appealing feature of multipartite entanglement in quantum information science is the existence of two different kinds of genuine tripartite entanglements, viz., entanglement of GHZ-class and entanglement of W-class of states [4,17]. Entanglement properties of these two classes of states are very interesting. While the GHZ-class of states is fragile in nature with respect to loss of particles, the entanglement of the W-class of states is maximally robust under the loss of particles. The inequivalence of these two classes of states under the stochastic local operation and classical communication (SLOCC) has also been extensively discussed in [6]. This inequivalence implies that if two arbitrary states $|\Phi\rangle$ and $|\Psi\rangle$ can, respectively, be converted into states belonging to the GHZ-class or W-class then it will not be possible to transform $|\Phi\rangle$ into $|\Psi\rangle$ or vice versa. The entanglement of the GHZ-class and W-class of states is contrasting in nature. We, with some examples, in this paper, show that if a tripartite state shows entanglement characteristic similar to the GHZ-class then those states are useful in controlled dense coding under suitable choice of state parameters. Our main motivation in this work is therefore to classify the non-trivial classes of tripartite entangled states which are valuable in CDC. The focus in this work is to categorise the class of three-qubit non-trivial partially entangled states into either of the two genuine tripartite classes of states (GHZ and W) on the basis of their usefulness in CDC. This will provide our (conjectured) operational criteria based on which one can differentiate the non-trivial classes of tripartite entangled states that are useful for CDC from those that are not useful for CDC. This paper is thereby organised as follows. In §2, we begin our analysis of CDC with a class of general tripartite partially entangled states which were defined in [18] and consequently study their effectiveness in CDC. Section 3 discusses the entanglement properties of the class of partially entangled states of [18]. We introduce our (conjectured) operational criteria for states that are useful for CDC in §4. To validate our claim, we cite several other examples of non-trivial tripartite entangled states which are useful in CDC in §5. Finally, a conclusion is given in §6.

2. Class of partially entangled states in CDC

A class of partially entangled superposition of Bell states was constructed and was shown to be useful for perfect controlled teleportation by Li and Ghose [18]. The classes of states are defined as

$$|\chi\rangle_{abc} = \alpha\, |0\rangle_a\, |\phi^+\rangle_{bc} + \beta\, |1\rangle_a \sigma_{kc} |\phi^+\rangle_{bc}, \tag{1}$$

$$|\tilde{\chi}\rangle_{abc} = \alpha\, |0\rangle_a |\phi^-\rangle_{bc} + \beta\, |1\rangle_a \sigma_{kc} |\phi^-\rangle_{bc}, \tag{2}$$

where $|\alpha|^2 + |\beta|^2 = 1$ and $|\phi^\pm\rangle = (|0\,0\rangle \pm |1\,1\rangle)/\sqrt{2}$. The qubits a, b, c are distributed to Alice, Bob and Cliff, respectively, and σ_{kc} ($k = x, y, z$) are, respectively, three Pauli spin matrices acting on the qubit c, where

$$\sigma_{xc} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad \sigma_{yc} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},\quad \sigma_{zc} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

In this paper, we ask the question: ‘whether the states (1) and (2) are useful for CDC too!’. In the CDC scheme, when the trio share some entangled channels among them, then depending upon any one party’s von Neumann measurement outcomes, the remaining two parties are expected to share maximal Bell states so that the usual dense coding can be pursued. That ‘one party’ acts as the controller of the scheme and the controller can be any one of the trios. The remaining two parties play the roles of the sender and the receiver of the bits of information. To illustrate the scheme, we start with the state (1), and assume Alice, who possesses qubit a as the controller and Bob and Cliff possess, respectively, the qubits b and c. Now if Alice decides to measure with respect to measurement basis $\{|0\rangle_a, |1\rangle_a\}$, then if her von Neumann measurement results in $|0\rangle_a$, Bob and Cliff know that they would share the maximal Bell state $|\phi^+\rangle$, of course when Alice sends her measurement outcomes to both Bob and Cliff. On the other hand, if she gets her von Neumann outcome as $|1\rangle_a$, then Bob and Cliff would share any one of the maximally entangled Bell states for an appropriate choice of Pauli spin operators. Thus, depending upon Alice’s measurement result $|1\rangle_a$, Bob and Cliff know which Bell state they are going to share

$$\sigma_{kc} |\phi^+\rangle_{bc}|_{k=x} \rightarrow |\psi^+\rangle_{bc}, \tag{4}$$

$$\sigma_{kc} |\phi^+\rangle_{bc}|_{k=y} \rightarrow |\psi^-\rangle_{bc}, \tag{5}$$

$$\sigma_{kc} |\phi^+\rangle_{bc}|_{k=z} \rightarrow |\phi^-\rangle_{bc}, \tag{6}$$

where $|\psi^\pm\rangle = (|0\,1\rangle \pm |1\,0\rangle)/\sqrt{2}$. So we see that, in all these cases, Bob and Cliff will be sharing any one of the four Bell states between them and hence they can proceed with the usual dense coding scheme. Just as in eqs (1) and (2), one can also start with a similar class of partially entangled states as shown below and can proceed with the protocol mentioned above with these classes of states:

$$|\xi\rangle_{abc} = \alpha|0\rangle_a|\psi^+\rangle_{bc} + \beta|1\rangle_a\sigma_{kc}|\psi^+\rangle_{bc}, \quad (7)$$

$$|\tilde{\xi}\rangle_{abc} = \alpha|0\rangle_a|\psi^-\rangle_{bc} + \beta|1\rangle_a\sigma_{kc}|\psi^-\rangle_{bc}. \quad (8)$$

3. Entanglement of the class of partially entangled states

To analyse the entanglement characteristics of the partially entangled states (1), (2), (7) and (8), we use the three-tangle (τ) as a measure for genuine tripartite entanglement [19]. The three-tangle (τ) is defined as

$$\tau = C^2_{a(bc)} - C^2_{ab} - C^2_{ac}, \quad (9)$$

where $C^2_{a(bc)}$ measures the entanglement between qubit a and the joint state of qubits b and c and C^2_{ab} (or C^2_{ac}) measures the entanglement between qubits a and b (or between qubits a and c). C^2 here means 'concurrence squared'. For pure entangled state, one can show that $C_{a(bc)} = 2\sqrt{\det\rho_a}$ while ρ_{abc} is the corresponding three-qubit density matrix. The tripartite entangled state (1) can be explicitly expressed with the choices of state parameters $\alpha = \sin\epsilon$ and $\beta = \cos\epsilon$ as follows:

$$|\chi^1\rangle_{abc} = \frac{\sin\epsilon}{\sqrt{2}}(|0\,0\,0\rangle_{abc} + |0\,1\,1\rangle_{abc}) + \frac{\cos\epsilon}{\sqrt{2}}(|1\,1\,0\rangle_{abc} + |1\,0\,1\rangle_{abc}), \quad (10)$$

$$|\chi^2\rangle_{abc} = \frac{\sin\epsilon}{\sqrt{2}}(|0\,0\,0\rangle_{abc} + |0\,1\,1\rangle_{abc}) + \frac{\cos\epsilon}{\sqrt{2}}(|1\,0\,1\rangle_{abc} - |1\,1\,0\rangle_{abc}), \quad (11)$$

$$|\chi^3\rangle_{abc} = \frac{\sin\epsilon}{\sqrt{2}}(|0\,0\,0\rangle_{abc} + |0\,1\,1\rangle_{abc}) + \frac{\cos\epsilon}{\sqrt{2}}(|1\,0\,0\rangle_{abc} - |1\,1\,1\rangle_{abc}). \quad (12)$$

The partially entangled states of eqs (10)–(12) are associated with different choices of the Pauli spin operators σ_{xc}, σ_{yc} and σ_{zc}, respectively, in eq. (1). The genuine tripartite entanglement or three-tangle (9) for each of these partially entangled states is found to be $\sin^2(2\epsilon)$. States (2), (7) and (8) can also be shown to be useful in CDC and having three-tangle of the same form as that of state (1).

4. Operational criterion for CDC

Given a pure non-trivial tripartite entangled state, our aim is to find an operational criterion for this state to be used in CDC. Now it is known that for a general non-trivial tripartite state (whose density matrix is denoted as ρ_{abc}), if the local rank of all the reduced density matrices is 1, then the state ρ_{abc} belongs to a product-class state (a–b–c) whereas if the rank of any two reduced systems is 2 and the other one has rank 1, then the tripartite state falls into the class of biseparable states (a–bc or b–ca or c–ab). The interesting scenario is, however, when $\text{rank}(\rho_a) = \text{rank}(\rho_b) = \text{rank}(\rho_c) = 2$, and there are two inequivalent classes of tripartite entangled states which satisfy this condition [6]. It was shown that any non-trivial tripartite pure entangled state can be characterised into these two inequivalent classes known as GHZ-class and W-class of states [6]. The most general form of states belonging to the inequivalent GHZ-class is given by

$$|\psi_{\text{GHZ}}\rangle_{abc} = \sqrt{K}(\cos\delta\,|0\rangle_a|0\rangle_b|0\rangle_c + \sin\delta\,\mathrm{e}^{\mathrm{i}\varphi}|\varphi\rangle_a|\varphi\rangle_b|\varphi\rangle_c), \quad (13)$$

where

$$|\varphi\rangle_a = \cos\alpha|0\rangle + \sin\alpha|1\rangle,$$
$$|\varphi\rangle_b = \cos\beta|0\rangle + \sin\beta|1\rangle,$$
$$|\varphi\rangle_c = \cos\gamma|0\rangle + \sin\gamma|1\rangle \quad (14)$$

and $K = (1 + 2\cos\delta\sin\delta\cos\alpha\cos\beta\cos\gamma\cos\varphi)^{-1} \in (\frac{1}{2}, \infty)$ is a normalisation factor. The ranges of the five parameters of eqs (13) and (14) are $\delta \in (0, \pi/4]$, $\alpha, \beta, \gamma \in (0, \pi/2]$ and $\varphi \in [0, 2\pi)$. On the other hand, the standard form of W-class of sates is given as

$$|\psi_W\rangle_{abc} = \sqrt{a}|0\,0\,1\rangle + \sqrt{b}|0\,1\,0\rangle + \sqrt{c}|1\,0\,0\rangle + \sqrt{d}|0\,0\,0\rangle, \quad (15)$$

where $a, b, c > 0$ and $d \equiv 1 - (a + b + c) \geq 0$.

The non-trivial tripartite entangled states (13) can be converted locally to the standard

$$|\text{GHZ}\rangle_{abc} = \frac{(|0\,0\,0\rangle_{abc} + |1\,1\,1\rangle_{abc})}{\sqrt{2}}$$

state or vice versa by means of SLOCC, whereas the three parties can locally transform state (15) into the state

$$|W\rangle_{abc} = \frac{(|0\,0\,1\rangle_{abc} + |0\,1\,0\rangle_{abc} + |1\,0\,0\rangle_{abc})}{\sqrt{3}}$$

or vice versa under SLOCC [6]. The inequivalence between states (13) and (15) means that if two arbitrary states $|\Phi\rangle$ and $|\Psi\rangle$ can, respectively, be converted either into the GHZ-class or W-class then it will not be possible to transform $|\Phi\rangle$ into $|\Psi\rangle$ or vice versa under SLOCC. We already know that the standard $|\text{GHZ}\rangle_{abc}$ states are suitable for CDC while the the standard $|W\rangle_{abc}$ states are not [16]. Going further, we claim in this paper that if one can characterise a non-trivial tripartite state belonging to GHZ-class (13) then the state is useful for CDC under suitable choice of the state parameters, whereas if the state belongs to the W-class of states, defined in eq. (15), then they are not useful for CDC. This claim will have practical significance in characterising a non-trivial tripartite entangled state belonging to any of these two inequivalent classes of states. The question is: given a state, how to confirm that it belongs to either of these two classes! Here the three-tangle comes to rescue. The genuine tripartite entanglement suggested by Coffman *et al* [19] has the ability to distinguish between the two inequivalent classes of states. It was shown in [6,20,21] that the three-tangle (a measure of genuine tripartite entanglement) is non-vanishing for any state in the GHZ-class (13), while the three-tangle vanishes for any state in the W-class (15).

For example, let us analyse the entanglement properties of the class of states $|\chi\rangle_{abc}$ of eq. (1), whose density matrix is represented by $\rho_{abc} = |\chi\rangle_{abc}\langle\chi|$, and the reduced density matrices are of the following form:

$$\rho_a = \begin{pmatrix} 2k_1^2 & 0 \\ 0 & 2k_1^2 \end{pmatrix}, \quad \rho_b = \begin{pmatrix} k_1^2 + k_2^2 & 0 \\ 0 & k_1^2 + k_2^2 \end{pmatrix},$$
$$\rho_c = \begin{pmatrix} k_1^2 + k_2^2 & 0 \\ 0 & k_1^2 + k_2^2 \end{pmatrix}, \tag{16}$$

where $k_1 = \sin\theta/\sqrt{2}, k_2 = \cos\theta/\sqrt{2}$ and consequently $\text{rank}(\rho_a) = \text{rank}(\rho_b) = \text{rank}(\rho_c) = 2$. So the class of states may belong to either of the two inequivalent classes (GHZ-class or W-class). It has been shown in the previous sections that state (1) has non-vanishing three-tangle and hence belongs to the GHZ-class of state (13). State (1) is also useful for controlled dense coding as shown in §2. Similar analysis can reveal that the entangled states (2), (7) and (8) belong to the GHZ-class. Thus, we see that the states defined in eqs (1), (2), (7) and (8) actually belong to the GHZ-class of state (13) and hence are useful in CDC.

5. Some other non-trivial tripartite entangled states and their utilities in CDC

To support our conjecture that all non-trivial tripartite entangled states falling under GHZ-class with non-vanishing tangle are actually useful in CDC for appropriate choices of the state parameters, we cite some more examples below.

5.1 *Maximal sliced states in CDC*

Here we discuss another class of partially entangled states known as 'maximal slice states' [13,18] from the perspective of CDC. The three-qubit partially entangled set of maximal slice (MS) states is defined as

$$|\text{MS}\rangle_{abc} = \frac{1}{\sqrt{2}}(|0\,0\,0\rangle_{abc} + \cos\alpha|1\,1\,0\rangle_{abc} + \sin\alpha|1\,1\,1\rangle_{abc}). \tag{17}$$

If ρ_{abc}^{MS} denotes the density matrix corresponding to the state $|\text{MS}\rangle_{abc}$, then it can be easily verified that $\text{rank}(\rho_a^{\text{MS}}) = \text{rank}(\rho_b^{\text{MS}}) = \text{rank}(\rho_c^{\text{MS}}) = 2$. Also, we see that the three-tangle for MS state (17) is found to be $\sin^2(\alpha)$ which is non-vanishing for $\alpha \neq n\pi$, where n is any integer. Hence, we conclude that state (17) belongs to the GHZ-class of state (13) and are useful for CDC. Now, the effectiveness of state (17) in CDC has already been investigated in [13] which endorses our conjecture.

5.2 *Symmetric state in CDC*

Next, we consider three-qubit symmetric states [11] defined as

$$|S\rangle_{abc} = p|0\,0\,0\rangle_{abc} + q|1\,1\,1\rangle_{abc} + r|0\,0\,1\rangle_{abc} + s|1\,1\,0\rangle_{abc}, \tag{18}$$

where $p^2+q^2+r^2+s^2 = 1$. Without loss of generality, the coefficients p, q, r, s are considered to be real along with the condition $p \geq q \geq r \geq s$. Writing the three-qubit state (18) in the density matrix form as ρ_{abc}^S, one can verify that the rank of reduced density matrices are $\text{rank}(\rho_a^S) = \text{rank}(\rho_b^S) = \text{rank}(\rho_c^S) = 2$. Using eq. (9), we also find that the three-tangle of the symmetric state is $4(p^2+r^2)(q^2+s^2)$ which is non-vanishing (since all of $p,\ q,\ r,\ s$ are not zero). Hence, state (18) must fall under the GHZ-class of state (13). With state (18), CDC can be successfully performed as shown in [11] which validates our claim.

5.3 *Non-prototypical type-I GHZ-class in CDC*

The non-prototypical GHZ-type of state (named here as type-I GHZ-class) is of the following form:

$$|N_{\text{I}}\rangle_{abc} = L\{|0\,0\,0\rangle_{abc} + l|1\,1\,1\rangle_{abc}\}, \tag{19}$$

with $L = 1/(\sqrt{1+l^2})$ and $l > 0$ is real. Such a state was proposed by Pati *et al* [22]. CDC was successfully shown with this state (obviously depending on the choice of the state parameter) in [16]. Again using (9), we see that state (19) has tangle of the form $\tau = 4L^4 l^2$, which is obviously non-zero. It is clear from this expression that when $l = 1$, the state has tangle $\tau = 1$. So, as expected, by our conjecture, this state is from the $|\text{GHZ}\rangle$ class.

5.4 *Non-prototypical type-II GHZ-class in CDC*

We have named a special class of states as type-II GHZ-class in this paper. These states are basically transformed from the GHZ-class of states. The type-II classes of states are suitable for perfect teleportation and superdense coding and they can be converted from state $|\psi_{\mathrm{GHZ}}\rangle_{abc}$ by a proper unitary operation. The unitary operation, however, is the tensor product of a two-qubit unitary operation and a one-qubit unitary operation as defined in [23]. We denote this transformed version of GHZ-states by $|N_{\mathrm{II}}\rangle_{abc}$ which is defined as follows [23]:

$$|N_{\mathrm{II}}\rangle_{abc} = \frac{1}{\sqrt{2}}[|\phi\rangle_{ab}\,|0\rangle_c + \mathrm{e}^{\mathrm{i}\epsilon}|0\,0\rangle_{ab}\,|1\rangle_c], \tag{20}$$

where

$$|\phi\rangle = \frac{1}{\sqrt{n+1}}[|1\,0\rangle + \sqrt{n}\mathrm{e}^{\mathrm{i}\alpha}|0\,1\rangle]. \tag{21}$$

The state $|N_{\mathrm{II}}\rangle_{abc}$ can be converted from $|\psi_{\mathrm{GHZ}}\rangle_{abc}$ by

$$|N_{\mathrm{II}}\rangle_{abc} = (U_{ab} \otimes I_c)|\psi_{\mathrm{GHZ}}\rangle_{abc}. \tag{22}$$

Here U_{ab} is a unitary operator acting on particles a and b and is given as

$$U_{ab} = |\phi\rangle\langle 0\,0| + |1\,1\rangle\langle 0\,1| + |\phi^{\perp}\rangle\langle 1\,0| + \mathrm{e}^{\mathrm{i}\epsilon}|0\,0\rangle\langle 1\,1|,$$
$$|\phi^{\perp}\rangle = \frac{1}{\sqrt{n+1}}(\sqrt{n}\mathrm{e}^{-\mathrm{i}\alpha}|1\,0\rangle - |0\,1\rangle). \tag{23}$$

We have shown in our earlier work [16] that the states of the form (20) are useful in CDC. Following the same rank analysis of the reduced systems and evaluating the non-vanishing three-tangle of the state (20), one can confirm that they also belong to the GHZ-class of states (13). This in turn gives us another example in justifying our conjecture.

Disclaimer: The names 'non-prototypical GHZ-states of type-I and II' used in this paper have been chosen just to distinguish between the following two forms of states. The identification of the states by these names are not universal.

6. Conclusion

To conclude, we have shown that the class of non-trivial pure tripartite entangled states defined in eqs (1), (2), (7) and (8) are useful for CDC, which explores the experimental significance of these states. We have shown that if a non-trivial class of three-qubit entangled states fall into the category of GHZ-class of states defined in eq. (13), then those states under suitable choices of state parameters can be used in CDC. This operational criterion is mainly determined by the genuine tripartite entanglement (three-tangle) of the state. To justify our claim further, we have considered different types of three-qubit entangled states which are useful for CDC. In future, one can generalise our result to other multipartite systems in higher dimensions. It was shown that tripartite qutrit states of GHZ-class are useful in CDC [16]. So, investigating the operational criteria for CDC in higher dimensional systems will also be interesting. To generalise our present work for higher dimensions, one may consider 'squashed entanglement' instead of three-tangle [24].

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