



Bound state solutions to the Schrödinger equation for some diatomic molecules

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Abstract. The bound state solutions to the radial Schrödinger equation are obtained in three-dimensional space using the series expansion method within the framework of a general interaction potential. The energy eigenvalues of the pseudoharmonic and Kratzer potentials are given as special cases. The obtained analytical results are applied to several diatomic molecules, i.e. N₂, CO, NO and CH. In order to check the accuracy of the present method, a comparison is made with similar results obtained in the literature by using other techniques.

Keywords. Schrödinger equation; bound state; eigenvalues; pseudoharmonic potential; Kratzer potential; diatomic molecule.

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1. Introduction

In the realms of quantum physics and quantum chemistry, the most challenging task is to obtain exact analytic solutions to the radial Schrödinger equation (SE) with a given interaction potential [1–3]. The l -state solutions to the SE, in particular, find special interest in chemical physics and molecular spectroscopy. To describe the spectra of diatomic molecules, anharmonic potentials, such as the Morse potential, are generally utilised. The harmonic oscillator potential can be useful only for low-state calculations. The Kratzer and pseudoharmonic potentials are regarded as intermediate ones between anharmonic and harmonic oscillator potentials. Due to the vitality of the anharmonic potentials in molecular physics, molecular spectroscopy, chemical physics and biology, the bound state (i.e. l -state) solutions to the relativistic and non-relativistic SE have been studied by several researchers in the past. A brief account of such studies is as follows.

Dong *et al* [4,5] obtained the eigenvalues and eigenfunctions of the SE with the pseudoharmonic potential in two dimensions (2D) and then established the creation and annihilation operators directly from the eigenfunctions with the factorisation method. Wang *et al* [6] carried out the eigenvalues and eigenfunctions of the SE with the harmonic and pseudoharmonic potentials in an arbitrary D -dimensional space. Qiang *et al* [7]

presented the analytical solutions to the radial SE for the rotating Morse potential. The energy levels and wave functions of all bound states were also obtained for the three diatomic molecules HCl, CO and LiH and the results obtained by this molecular potential were in good agreement with the results of other studies. Analytical expressions of eigenenergies and eigenfunctions were derived by Sun [8] to understand the nature of the bound states of the diatomic molecules using the Morse potential, which is frequently used to describe the vibrational motion of diatomic molecules.

Bayrak *et al* [9] obtained the exact analytical solutions to the radial SE for the Kratzer potential for various values of n and l and calculated the bound state energy eigenvalues and corresponding eigenfunctions for CO, NO, O₂ and I₂ diatomic molecules. The exact solutions to the radial SE for the Mie and pseudoharmonic potentials were obtained by Sever *et al* [10,11] for arbitrary values of l . The calculated eigenfunctions were expressed in terms of Jacobi polynomials and the energy bound states were applied to several diatomic molecules such as N₂, CO, NO and CH. Oyewumi *et al* [12] presented the exact analytical solutions to the SE in N -dimensional space for the pseudoharmonic oscillator potential. The bound state energy eigenvalues, normalised wave functions and expectation values for r^{-2} , r^2 , T , V , H and p^2 were also obtained. From these values, they found that the results obtained in each

case depend on dimensions and potential parameters. Berkdemir *et al* [13] presented the l -state solutions to the SE for diatomic molecules with the Kratzer potential and obtained the bound state energy eigenvalues and the corresponding eigenfunctions by using the Nikiforov–Uvarov method in terms of Jacobi polynomials. The bound states were calculated for various values of l and applied to several diatomic molecules such as N_2 , CO, NO and CH. Arda *et al* [14] studied the exact bound state solutions and normalised eigenfunctions of the radial SE for the Kratzer and pseudoharmonic potentials using the Laplace transform approach and also investigated the numerical results of N_2 and CO molecules. Ikhdaire *et al* [15] obtained the exact solutions to the D -dimensional radial SE for the Kratzer and pseudoharmonic potentials employing the wave function ansatz method and further found the polynomial solutions to the SE for the pseudoharmonic potential of arbitrary l [16]. The energy states of several diatomic molecular systems were also obtained numerically by a proper transformation approach.

Oyewumi *et al* [17] developed the bound state energies and eigenfunctions of diatomic molecules such as CO, NO, CH, O_2 , N_2 , H_2 and ScH with the pseudoharmonic potential for different values of n and l by solving the three-dimensional SE using spectrum generating algebra. They also calculated the expectation values of r^2 and p^2 . Falaye *et al* [18] determined the non-relativistic l -state solutions of the ring-shaped Mie-type potential and presented the energy eigenvalues of some diatomic molecules. They also obtained the probability density distribution of electrons. Ikot *et al* [19] determined the thermodynamical properties of some diatomic molecules in the framework of a general molecular potential using asymptotic iteration method.

Keeping in view the voluminous studies on the structure of diatomic molecules and hence to determine the eigenvalue spectra of some important diatomic molecules, in the present work, we consider a general potential of the form

$$V(r) = ar^2 + br - \frac{c}{r} + \frac{d}{r^2} + e, \quad (1)$$

where a, b, c, d and e are arbitrary constant potential parameters which, for a given system, are chosen in such a way that one can obtain the improved results. It is to be noted that the inverse square term, d/r^2 , makes the potential more singular and produces better confinement as compared to Cornell and Coulomb perturbed potentials [20].

The advantage of solving the SE for a general potential lies in the fact that general solutions to such a potential can directly provide results for some physically

relevant potentials as special cases. We also exploit the same idea in §3 for determining energy eigenvalues for the pseudoharmonic and Kratzer potentials from the general results. Recently, the ubiquitous potential (1) was also successfully used by us to investigate the energy eigenvalues of heavy and light mass quarkonia [21].

Ikhdaire *et al* [22] solved the SE for general potential (1) in the presence of magnetic and AB flux fields using the wave function ansatz method and obtained the energy spectra of diatomic molecules such as NO, CO, N_2 and CH for the different values of vibrational and rotational quantum numbers. Therefore, here, we analytically solve the radial SE for potential (1) using the series expansion method [20,23] and apply our general results to investigate the spectra of various molecular potentials such as the Kratzer and pseudoharmonic potentials.

The paper is organised as follows: In §2, the analytical solutions to the 3D radial SE are derived for potential (1). In §3, we apply the analytic expressions to obtain the eigenvalues spectra of N_2 , NO, CO and CH molecules and compare the results with the existing ones in the literature. Finally, §4 contains the concluding remarks.

2. Solutions to the SE for interaction potential (1)

Consider the 3D radial SE [24,25] for the interaction potential (1) as

$$\left[\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2} \left(E - ar^2 - br + \frac{c}{r} - \frac{d}{r^2} - e \right) \right] \Psi(r) = 0, \quad (2)$$

where $l = 0, 1, 2, \dots$ is the angular momentum quantum number, μ is the reduced mass of a diatomic molecule, r is the internuclear separation and E denotes the energy eigenvalues of the system.

The above equation can be written as

$$\left[\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{L(L+1)}{r^2} + (\epsilon - a_1)r^2 - b_1r + \frac{c_1}{r} \right] \Psi(r) = 0, \quad (3)$$

with $L = (-1 + \sqrt{(2l+1)^2 + (8\mu d/\hbar^2)})/2$, $\epsilon = (2\mu/\hbar^2)(E - e)$, $a_1 = 2\mu a/\hbar^2$, $b_1 = 2\mu b/\hbar^2$, $c_1 = 2\mu c/\hbar^2$. Now make an ansatz for the radial wave function [26] as

$$\Psi(r) = e^{-\alpha r^2 - \beta r} F(r), \quad (4)$$

where α and β are positive constants. Using this wave function, eq. (3) becomes

Table 1. Comparison of bound state energy eigenvalues (eV) of N₂ molecule.

<i>n</i>	<i>l</i>	<i>E_{nl}</i>	Ref. [22]	Ref. [16]	Ref. [14]	Ref. [11]
0	0	0.10918452	0.10918501	0.1091559	0.109180	0.10915590
1	0	0.32742891	0.32743034	0.3273430	0.327414	0.32734304
	1	0.32792762	0.32792905	0.3278417	0.327913	0.32784167
2	0	0.54567331	0.54567566	0.5455302	0.545648	0.54553018
	1	0.54617203	0.54617437	0.5460288	0.546147	0.54602881
	2	0.54716942	0.54717177	0.5470260	0.547145	0.54702603
3	0	0.76391770	0.76392098	–	0.763883	–
	1	0.76441641	0.76441969	–	0.764382	–
	2	0.76541381	0.76541710	–	0.765380	–
	3	0.76690982	0.76691313	–	0.766877	–
4	0	0.98216208	0.98216631	0.9819045	0.982117	0.98190446
	1	0.98266080	0.98266502	0.9824031	0.982616	0.98240309
	2	0.98365820	0.98366242	0.9834003	0.983614	0.98340031
	3	0.98515421	0.98515845	0.9848961	0.985111	0.98489606
	4	0.98714876	0.98715302	0.9868903	0.987107	0.98689026
5	0	1.20040647	1.20041163	1.2000916	–	1.20009160
	1	1.20090519	1.20091034	1.2005902	–	1.20059020
	2	1.20190258	1.20190774	1.2015875	–	1.20158750
	3	1.20339859	1.20340377	1.2030832	–	1.20308320
	4	1.20539314	1.20539834	1.2050774	–	1.20507740
	5	1.20788609	1.20789131	1.2075699	–	1.20756990

Table 2. Comparison of bound state energy eigenvalues (eV) of CO molecule.

<i>n</i>	<i>l</i>	<i>E_{nl}</i>	Ref. [22]	Ref. [16]	Ref. [14]	Ref. [11]
0	0	0.10194829	0.10195779	0.1019306	0.101953	0.10193061
1	0	0.30572525	0.30575368	0.3056722	0.305738	0.30567217
	1	0.30620386	0.30623238	0.3061508	0.306217	0.30615078
2	0	0.50950221	0.50954957	0.5094137	0.509524	0.50941373
	1	0.50998081	0.51002827	0.5098923	0.510003	0.50989234
	2	0.51093801	0.51098564	0.5108495	0.510961	0.51084953
3	0	0.71327916	0.71334546	–	0.713310	–
	1	0.71375778	0.71382416	–	0.713789	–
	2	0.71471497	0.71478153	–	0.714747	–
	3	0.71615066	0.71621750	–	0.716183	–
4	0	0.91705613	0.91714135	0.9168969	0.917095	0.91689685
	1	0.91753473	0.91762005	0.9173755	0.917574	0.91737546
	2	0.91849193	0.91857742	0.9183327	0.918532	0.91833265
	3	0.91992764	0.92001339	0.9197684	0.919969	0.91976835
	4	0.92184175	0.92192787	0.9216825	0.921885	0.92168247
5	0	1.12083308	1.12093724	1.1206384	–	1.12063840
	1	1.12131170	1.12141594	1.1211170	–	1.12111700
	2	1.12226889	1.12237331	1.1220742	–	1.12207420
	3	1.12370458	1.12380928	1.1235099	–	1.12350990
	4	1.12561870	1.12572376	1.1254240	–	1.12542400
	5	1.12801112	1.12811660	1.1278165	–	1.12781650

$$\begin{aligned}
 &F''(r) + \left[-4\alpha r - 2\beta + \frac{2}{r} \right] F'(r) \\
 &+ \left[(4\alpha^2 - a_1)r^2 + (4\alpha\beta - b_1)r + (-6\alpha + \beta^2 + \epsilon) \right. \\
 &\left. + (c_1 - 2\beta)\frac{1}{r} - \frac{L(L+1)}{r^2} \right] F(r) = 0. \tag{5}
 \end{aligned}$$

The functional $F(r)$ is considered as a series of the form

$$F(r) = \sum_{n=0}^{\infty} a_n r^{2n+L}. \tag{6}$$

Table 3. Comparison of bound state energy eigenvalues (eV) of NO molecule.

n	l	E_{nl}	Ref. [22]	Ref. [16]	Ref. [11]
0	0	0.08250989	0.08251026	0.0824883	0.08248827
1	0	0.24742405	0.24742511	0.2473592	0.24735916
	1	0.24784667	0.24784775	0.2477817	0.24778171
2	0	0.41233821	0.41233997	0.4122301	0.41223005
	1	0.41276083	0.41276261	0.4126526	0.41265260
	2	0.41360605	0.41360783	0.4134977	0.41349768
3	0	0.57725234	0.57725482	–	–
	1	0.57767499	0.57767747	–	–
	2	0.57852021	0.57852269	–	–
	3	0.57978797	0.57979045	–	–
4	0	0.74216650	0.74216968	0.7419718	0.74197183
	1	0.74258914	0.74259232	0.7423944	0.74239438
	2	0.74343437	0.74343754	0.7432395	0.74323946
	3	0.74470213	0.74470531	0.7445070	0.74450700
	4	0.74639231	0.74639552	0.7461969	0.74619689
5	0	0.90708066	0.90708453	0.9068427	0.90684272
	1	0.90750328	0.90750718	0.9072653	0.90726527
	2	0.90834850	0.90835240	0.9081104	0.90811035
	3	0.90961627	0.90962017	0.9093779	0.90937789
	4	0.91130647	0.91131038	0.9110678	0.91106778
	5	0.91341895	0.91342289	0.9131799	0.91317990

Table 4. Comparison of bound state energy eigenvalues (eV) of CH molecule.

n	l	E_{nl}	Ref. [22]	Ref. [16]	Ref. [11]
0	0	0.16867860	0.16867933	0.1686344	0.16863440
1	0	0.50513964	0.50514181	0.5050072	0.50500718
	1	0.50872344	0.50872564	0.5085903	0.50859034
2	0	0.84160069	0.84160429	0.841380	0.84137996
	1	0.84518448	0.84518812	0.8449631	0.84496312
	2	0.85234720	0.85235089	0.8521246	0.85212458
3	0	1.17806173	1.17806677	–	–
	1	1.18164552	1.18165060	–	–
	2	1.18880823	1.18881337	–	–
	3	1.19954015	1.19954539	–	–
4	0	1.51452277	1.51452925	1.5141255	1.51412550
	1	1.51810656	1.51811307	1.5177087	1.51770870
	2	1.52526928	1.52527585	1.5248701	1.52487010
	3	1.53600120	1.53600786	1.5356002	1.53560020
	4	1.550287849	1.55029463	1.5498843	1.54988430
5	0	1.85098381	1.85099173	1.8504983	1.85049830
	1	1.85456759	1.85457555	1.8540815	1.85408150
	2	1.86173031	1.86173833	1.8612429	1.86124290
	3	1.87246223	1.87247034	1.8719729	1.87197290
	4	1.88674888	1.88675711	1.8862571	1.88625710
	5	1.90457108	1.90457946	1.9040761	1.90407610

Substituting eq. (6) into eq. (5), we obtain

$$\sum_{n=0}^{\infty} a_n \{ (2n+L)(2n+L-1) + 2(2n+L) - L(L+1) \} r^{2n+L-2} + \{ -2\beta(2n+L) + c_1 - 2\beta \} r^{2n+L-1} + \{ -4\alpha(2n+L) + \beta^2 + \epsilon_r - 6\alpha \} r^{2n+L} + \{ 4\alpha\beta - b_1 \} r^{2n+L+1} + \{ 4\alpha^2 - a_1 \} r^{2n+L+2} = 0. \tag{7}$$

Now on equating the coefficients of various powers of r to zero in eq. (7), one finds the following relations:

$$(2n + L)(2n + L + 1) - L - L^2 = 0, \quad (8)$$

$$\epsilon_r = 2\alpha(4n + 2L + 3) - \beta^2, \quad (9)$$

$$\alpha = \frac{\sqrt{a_1}}{2}, \quad (10)$$

$$\beta = \frac{b_1}{4\alpha}, \quad (11)$$

$$c_1 = 2\beta(2n + L + 1). \quad (12)$$

From eqs (9)–(12), the energy eigenvalue expression becomes

$$E_{nl} = e + \sqrt{\frac{\hbar^2 a}{2\mu}} \left(4n + 2 + \sqrt{(2l + 1)^2 + \frac{8\mu d}{\hbar^2}} \right) - \frac{2\mu c^2}{\hbar^2} \left(4n + 1 + \sqrt{(2l + 1)^2 + \frac{8\mu d}{\hbar^2}} \right) \quad (13)$$

and the potential parameters a , b and c satisfy the following restriction:

$$c = \frac{b}{2} \sqrt{\frac{\hbar^2}{2\mu a}} \left(4n + 1 + \sqrt{(2l + 1)^2 + \frac{8\mu d}{\hbar^2}} \right). \quad (14)$$

3. Applications

The results of the previous section of the general potential (1) are now being utilised to develop solutions to some special types of interaction potentials, which have direct bearing in practical problems solving. The pseudoharmonic and Kratzer potentials are successfully employed in the past [27] to study the eigenvalues spectra of a class of diatomic molecules. Keeping this fact in mind, in what follows, we apply the above general results to obtain the energy eigenvalues spectra of four diatomic molecules namely N₂, CO, NO and CH.

3.1 The pseudoharmonic potential

The pseudoharmonic potential can easily be retrieved from the general potential (1) by assigning the potential parameters $a = D_e/r_e^2$, $b = c = 0$, $d = D_e r_e^2$ and $e = -2D_e$

$$V(r) = \frac{D_e}{r_e^2} r^2 + \frac{D_e r_e^2}{r^2} - 2D_e. \quad (15)$$

This potential is an important one not only in classical physics but also in quantum physics and is useful to describe the interaction of some diatomic molecules [28–30]. Now, using eq. (13), the energy eigenvalues spectra of the pseudoharmonic potential become

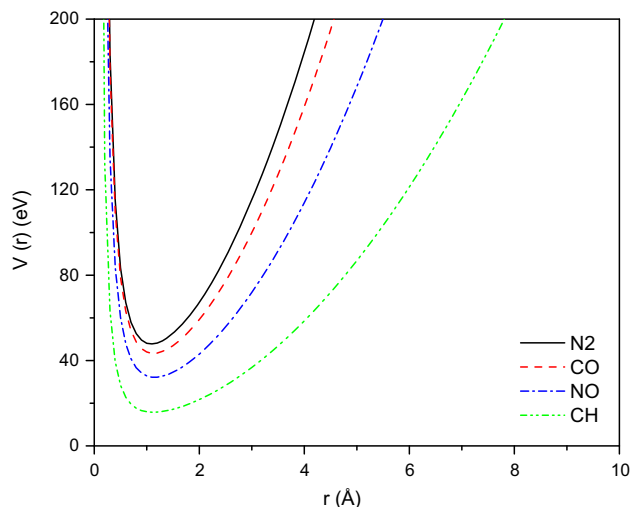


Figure 1. Shape of pseudoharmonic potential for various diatomic molecules.

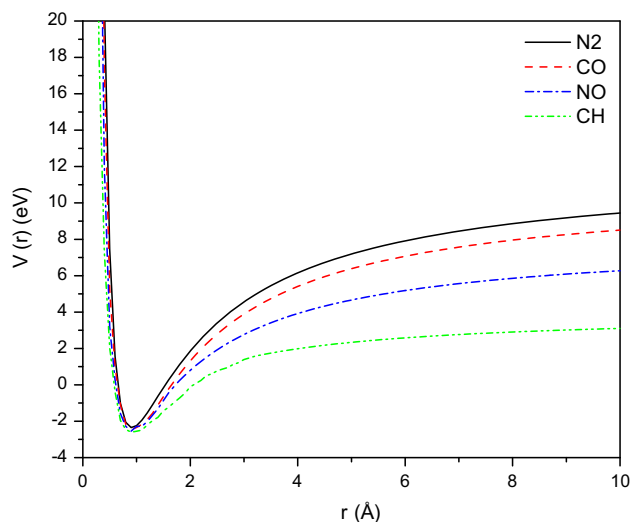


Figure 2. Shape of the Kratzer potential for various diatomic molecules.

$$E_{nl} = -2D_e + \sqrt{\frac{\hbar^2 D_e}{2\mu r_e^2}} \left(4n + 2 + \sqrt{(2l + 1)^2 + \frac{8\mu D_e r_e^2}{\hbar^2}} \right), \quad (16)$$

where D_e represents the dissociation energy and r_e is the equilibrium internuclear separation.

By inserting appropriate values of parameters D_e and r_e , the energy spectra of any diatomic molecule can be calculated by using eq. (16). Tables 1–4 show the binding energy levels with different values of vibrational n and rotational l quantum numbers, for N₂

Table 5. The bound state energy eigenvalues (eV) of N₂ and CO molecules.

<i>n</i>	<i>l</i>	N ₂			CO		
		<i>E_{nl}</i>	Ref. [13]	Ref. [14]	<i>E_{nl}</i>	Ref. [13]	Ref. [14]
0	0	0.05443655	0.054430	0.054434	0.05082472	0.050823	0.050827
1	0	0.16207644	0.162057	0.162068	0.15129100	0.151287	0.151296
	1	0.16256502	0.162546	0.162557	0.15175962	0.151755	0.151765
2	0	0.26826045	0.268229	0.268245	0.25036117	0.250354	0.250369
	1	0.26874244	0.268711	0.268728	0.25082329	0.250816	0.250831
	2	0.26970629	0.269675	0.269692	0.25174741	0.251744	0.251756
3	0	0.37301474	0.372972	0.372992	0.34806097	0.348051	0.348070
	1	0.37349025	0.373447	0.373468	0.34851671	0.348507	0.348526
	2	0.37444116	0.374398	0.374419	0.34942807	0.349418	0.349438
	3	0.37586721	0.375823	0.375846	0.35079481	0.350785	0.350806
4	0	0.47636485	0.476313	0.476334	0.44441555	0.444403	0.444425
	1	0.47683401	0.476779	0.476803	0.44486503	0.444852	0.444871
	2	0.47777219	0.477717	0.477742	0.44576388	0.445751	0.445774
	3	0.47917917	0.479124	0.479150	0.44711184	0.447099	0.447123
	4	0.48105461	0.480999	0.481026	0.44890860	0.448895	0.448921
5	0	0.57833578	0.578269	–	0.53944950	0.539434	–
	1	0.57879869	0.578732	–	0.53989284	0.539877	–
	2	0.57972438	0.579658	–	0.54077939	0.540764	–
	3	0.58111263	0.581046	–	0.54210893	0.542093	–
	4	0.58296310	0.582896	–	0.54388112	0.543865	–
	5	0.58527534	0.585208	–	0.54609548	0.546082	–

Table 6. The bound state energy eigenvalues (eV) of NO and CH molecules.

<i>n</i>	<i>l</i>	N ₂		CO	
		<i>E_{nl}</i>	Ref. [13]	<i>E_{nl}</i>	Ref. [13]
0	0	0.04112300	0.041118	0.08322383	0.083214
1	0	0.12232531	0.122311	0.24115051	0.241123
	1	0.12273833	0.122724	0.24440882	0.244381
2	0	0.20229793	0.202274	0.38958988	0.389547
	1	0.20270470	0.202681	0.39265445	0.392611
	2	0.20351812	0.203494	0.39876758	0.398722
3	0	0.28106555	0.281033	0.52928690	0.529229
	1	0.28146621	0.281434	0.53217281	0.532115
	2	0.28226740	0.282235	0.53792974	0.537870
	3	0.28346888	0.283436	0.54652818	0.546467
4	0	0.35865226	0.358611	0.66091486	0.660844
	1	0.35904693	0.359006	0.66363571	0.663565
	2	0.35983614	0.359795	0.66906360	0.668992
	3	0.36101966	0.360978	0.67717107	0.677098
	4	0.36259710	0.362555	0.68791738	0.687842
5	0	0.43508157	0.435032	0.78508342	0.785001
	1	0.43547036	0.435421	0.78765158	0.787569
	2	0.43624783	0.436198	0.79277502	0.792692
	3	0.43741373	0.437364	0.80042821	0.800343
	4	0.43896771	0.438917	0.81057320	0.810487
	5	0.44090928	0.440858	0.82316019	0.823071

(*D_e* = 11.938193820 eV, *r_e* = 1.0940 Å, *μ* = 7.00335 amu), CO (*D_e* = 10.845073641 eV, *r_e* = 1.1283 Å, *μ* = 6.860586 amu), NO (*D_e* = 8.043729 eV, *r_e* = 1.1508 Å, *μ* = 7.468441 amu) and

CH (*D_e* = 31838.08149 eV, *r_e* = 1.1198 Å, *μ* = 0.929931 amu) molecules within the framework of the pseudoharmonic potential. The values of the potential parameters are taken from [17]. From tables 1–4, it

should be noted that the present results are in good agreement with the results of other studies [14,16,21,22]. Figure 1 depicts the shape of potential (15) for various molecules.

3.2 The Kratzer potential

Now adjusting the values of potential parameters $a = b = 0$, $c = 2D_e r_e$, $d = D_e r_e^2$ and $e = D_e$ in eq. (1), the Kratzer potential [9,31–33] is written as

$$V(r) = D_e + \frac{D_e r_e^2}{r^2} - \frac{2D_e r_e}{r}. \quad (17)$$

Figure 2 presents the shape of the Kratzer potential to understand its behaviour, which consists of repulsive and long-range attractive parts. This potential has extensively been used to describe molecular structures and interactions. The energy eigenvalue expression corresponding to this potential is obtained using eq. (13) as

$$E_{nl} = D_e - \frac{2\mu D_e^2 r_e^2}{\hbar^2} \left(n + \beta + \frac{1}{2} \right)^{-2}, \quad (18)$$

where $\beta = \sqrt{\left(l + \frac{1}{2}\right)^2 + (2\mu D_e r_e^2 / \hbar^2)}$. It is worth mentioning that if we take $e = 0$, $\mu = \hbar = 1$ and $l + (1/2) = m$, then energy eigenvalue eq. (18) becomes

$$E_{nm} = \frac{-2D_e^2 r_e^2}{\left(n + \frac{1}{2} + \sqrt{m^2 + 2D_e r_e^2}\right)^2}, \quad (19)$$

which is similar to eq. (16) of [34] and provides solutions to the 2D radial SE for the Kratzer potential.

The energy eigenvalues calculated using eq. (18) for various diatomic molecules within the framework of the Kratzer potential are listed in tables 5 and 6. Again, to check the veracity of the results of the present work, a comparison is made with other similar works [13,14]. It is apparent from tables 5 and 6 that the present results are in close agreement with others.

4. Conclusions

In the present work, we analytically solved the 3D radial SE with a general anharmonic potential by using the series expansion method. The bound state energy eigenvalues and restriction on the potential parameters have been worked out for different values of principal quantum number n and angular quantum number l . To explore the practical utility of the potential (1), from the general eigenvalues expression (13), by suitably adjusting potential parameters, the eigenvalues spectra of the pseudoharmonic and Kratzer potentials are derived (see eqs (16) and (18)). Finally, within the framework of

these interaction potentials, the energy eigenvalues of four diatomic molecules namely N_2 , CO, NO and CH were computed (see tables 1–6). To check the validity of the employed method, the obtained results were compared with other such studies. A close perusal of the data of tables 1–6 reveals a close agreement between the present results and results of other similar works. It is interesting to note that potential (1) can also be a suitable model potential to study the bound state solutions of heavy and light mass quarkonia and quantum dot systems.

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