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Strange quark star with Tolman IV background

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Abstract. In the present article, we have studied the behaviour of static, spherically symmetric, anisotropic stellar models within the framework of MIT Bag model in the Tolman IV background. Different physical properties like energy conditions, stability, compactness factor and surface red-shift are investigated through graphical plots and mathematical calculations. The interior solutions found are non-singular in nature, i.e., regular at the centre. The interior spherically symmetric solutions have been matched to an exterior Schwarzschild geometry.

Keywords. Tolman IV potential; MIT Bag constant; quark stars.

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1. Introduction

For the last few decades, researchers are interested in the study of strange stars consisting of quark matters. Hadrons are the collection of more fundamental particles called quarks as suggested independently by Gell-Mann [1] and George Zweig [2] in 1964. The presence of quark stars for static matter in equilibrium was first investigated by Itoh [3]. Bodmer [4] proposed that the matter consisting of u, d and s quarks may be the absolute fundamental state of matter at zero pressure and temperature. After a supernova explosion [5] resulting in the collapse of the core of a massive star, strange stars are supposed to be formed. Alford [6] suggested that hadrons are squeezed into quark matter with colour superconducting phases in the dense cores of neutron stars. The behaviour of quark matter with ultrahigh densities is not properly understood till now. As the quarks are not seen as free particles, QCD Lagrangian provides the quark confinement mechanism.

The transformation of ordinary matter into strange quark matter is possible in the ideal environment as provided by the core of a proto-neutron star or neutron star. There is another possibility of the formation of strange star by the rapidly spinning dense star which can gradually form sufficient mass to experience a phase transition. The general MIT Bag model proposed that the universal pressure B, Bag constant, provides the quark confinement. The value of Bag constant for stable strange quark matter is $B \sim 55$–$75\,\text{MeV fm}^{-3}$. It can be claimed that strong interactions of QCD is responsible to create the microscopic effects in the strange matter and it has been incorporated in the final form of equation of state. Studies of compact astrophysical objects with a quark equation of state have been done in the works of Chodos *et al* [7], Farhi and Jaffe [8], Witten [9], Bombaci [10], Dey *et al* [11–13], Kalam *et al* [14], Komathiraj and Maharaj [15] and Weber [16].

The observational data on gravitational waves to obtain the equation of state for quark matter are given by Sotani *et al* [17]. Harko and Cheng [18] studied the collapse of strange matter are given by in spherically symmetric fields. Yilmaz and Baysal [19] found charged strange matter in rotating fields. The exact analytical solutions for anisotropic stellar models of strange quark star with linear equation of state based on MIT Bag model were investigated by Mak and Harko [20] and Sharma and Maharaj [21]. Mak and Harko [22] proposed a charged strange quark star by assuming spherical symmetry and the existence of conformal killing vector.

In 1939, the static solutions for a fluid sphere have been proposed by Tolman [23]. He obtained eight different internal solutions to the Einstein field equation (EFE) for spherically symmetric, static fluid sphere. Generally, two different classes of solutions for the stellar model are obtained from EFEs. The first one provides neutron stars for which matter is normal and density as well as pressure vanish at the surface. ‘Self-bound’ stars fall into the second class for which density is finite and about 2–3 times higher than the normal matter density at the surface where pressure vanishes. The first class contains Tolman VII solution with vanishing surface density and Tolman IV solution, generated by $e^{\nu} = B_N(1+Cr^2)^N$, represents second class where N is the positive integer and $B_N, C > 0$ are constants which provides physically acceptable solutions. Some remarkable works have been done in the following papers by Thirukkanesh and Ragel [24], Banerjee *et al* [25], Singh *et al* [26], Bhar *et al* [27].

In the context of recent works, we aim to find a strange anisotropic stellar model with Tolman IV type metric function admitting MIT Bag constant. The plan of investigations for this paper is given as follows: In §2, we provide EFEs for a particular choice of metric potential of a static spherically symmetric space–time, which is transformed as suggested by Durgapal and Bannerji [28]. In §3, new exact solutions to the EFEs are obtained. Some physical features are discussed in §4–7. Finally, in §8, some concluding remarks are given.

2. Field equations

The line element of a static spherically symmetric space–time in standard form of Schwarzschild coordinate $x^a = (t, r, \theta, \phi)$, is given by

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \quad (1)$$

where the metric coefficient λ and ν are functions of the radial coordinate r. We consider that the matter distribution inside the compact star is anisotropic in nature. Now we assume the energy–momentum tensor to be of the form

$$T_\nu^\mu = (\rho + p_r)u^\mu u_\nu - p_t g_\nu^\mu + (p_r - p_t)\eta^\mu \eta_\nu, \quad (2)$$

with $u^i u_j = -\eta^i \eta_j = 1$ and $u^i \eta_j = 0$. Here, the vector u_i is the fluid 4-velocity and η^i is the space-like vector which is orthogonal to u^i. Here ρ is the energy density, p_t is the tangential pressure and p_r is the radial pressure. Then the Einstein equations can be written as

$$8\pi\rho(r) = \frac{1}{r^2}(1 - e^{-\lambda}) + \frac{e^{-\lambda}\lambda'}{r}, \quad (3)$$

$$8\pi p_r(r) = -\frac{1}{r^2}(1 - e^{-\lambda}) + \frac{\nu' e^{-\lambda}}{r}, \quad (4)$$

$$8\pi p_t(r) = \frac{e^{-\lambda}}{4}\left(2\nu'' + (\nu')^2 + \frac{2\nu'}{r} - \nu'\lambda' - \frac{2\lambda'}{r}\right), \quad (5)$$

where the prime denotes derivative with respect to r. The mass within the radius r of the sphere is given by

$$m(r) = 4\pi \int_0^r \omega^2 \rho(\omega)\, d\omega. \quad (6)$$

Now, using the transformations according to Durgapal and Bannerji [28], we have

$$x = r^2, \quad Z(x) = e^{-\lambda(r)}, \quad y^2(x) = e^{\nu(r)}, \quad (7)$$

where x is the independent variable. Then Einstein field equations become

$$8\pi\rho(r) = \frac{1}{x}(1 - Z) - 2\dot{Z}, \quad (8)$$

$$8\pi p_r(r) = \frac{-1}{x}(1 - Z) + \frac{4\dot{y}}{y}Z, \quad (9)$$

$$8\pi p_t(r) = 4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} \quad (10)$$

and the mass function turns out to be

$$m(x) = 2\pi \int_0^x \sqrt{\omega}\rho(\omega)d\omega. \quad (11)$$

In this paper, we consider the following equation of state relating density and radial pressure:

$$p_r = \frac{1}{3}(\rho - 4B), \quad (12)$$

where B is the Bag constant.

3. Solution for field equations

To solve the above-mentioned system of equations, we consider the ansatz proposed by Tolman [23], as

$$e^{-\lambda} = \frac{(1 + ar^2)(1 - br^2)}{(1 + 2ar^2)}, \quad (13)$$

where a and b are constants. Using eqs (7) and (13), we have

$$Z(x) = \frac{(1 + ax)(1 - bx)}{(1 + 2ax)}. \quad (14)$$

From eqs (7)–(14), we get

$$\frac{\dot{y}}{y} = \frac{3(a + b) + (7ab + 2a^2)x + 6a^2bx^2}{12(1 + ax)(1 - bx)(1 + 2ax)} - \frac{8\pi B(1 + 2ax)}{3(1 + ax)(1 - bx)} + \frac{a + b + abx}{(1 + ax)(1 - bx)}, \quad (15)$$

which is the first-order differential equation with respect to r. Integrating eq. (15) we have

$$y = D(1 + ax)^P(1 - bx)^Q(1 + 2ax)^2, \quad (16)$$

where, for notational convention we use

$$P = \frac{11a - 2b + 96\pi B}{12(a + b)}$$

and

$$Q = -\left[\frac{10ab + 3b^2 + 8a^2}{(a+b)(2a+b)} + \frac{8B\pi(2a+b)}{3b(a-b)} + \frac{(2a+b)}{a+b}\right]$$

and D is the constant of integration.

Then solving the system of eqs (3)–(5), one can generate an exact solution for the EFEs in the form of

$$e^{-\lambda} = \frac{(1+ar^2)(1-br^2)}{(1+2ar^2)}, \tag{17}$$

$$e^{\nu} = D^2(1+ax)^{2P}(1-bx)^{2Q}(1+2ax)^4, \tag{18}$$

$$\rho(r) = \frac{3(a+b) + (7ab+2a^2)x + 6a^2bx^2}{8\pi(1+2ax)^2}, \tag{19}$$

$$p_r(r) = \frac{1}{3}(\rho - 4B), \tag{20}$$

$$p_t(r) = p_r + \Delta, \tag{21}$$

where Δ represents the anisotropic factor, and is defined as

$$\Delta = \frac{1}{8\pi}u(x)v(x) - \frac{a+b+2abx+2a^2bx^2}{8\pi(1+2ax)^2} + \frac{4B}{3} - \frac{4Bx(a+b+2abx+2a^2bx^2)}{3(1+ax)(1-bx)(1+2ax)} - \frac{g(x)}{24\pi(1+2ax)^2} + \frac{x(b^2-a^2+2abx(a+b)+a^2b^2x^2)}{(8\pi)(1+ax)(1-bx)(1+2ax)} + \frac{xd(x)}{24\pi(1+ax)(1-bx)(1+2ax)^3}, \tag{22}$$

with

$$g(x) = 3(a+b) + (7ab+2a^2)x + 6a^2bx^2,$$
$$d(x) = 3b^2 - 7a^2 + ab + 18abx(a+b) - 12a^3x + 33a^2b^2x^2 + 28a^3b^2x^3 + 28a^3bx^2 - 4a^4x^2 + 8a^4bx^3 + 12a^4b^2x^4, \tag{23}$$

$$u(x) = \frac{g(x)}{12(1+ax)(1-bx)(1+2ax)} - \frac{(8\pi)B(1+2ax)}{3(1+ax)(1-bx)} + \frac{a+b+abx}{(1+ax)(1-bx)}, \tag{24}$$

$$v(x) = \frac{xg(x)}{3(1+2ax)^2} - \frac{32\pi Bx}{3} + \frac{x(a+b+abx)}{1+2ax} + \frac{2(2+5ax-3bx-8abx^2+4a^2x^2-6a^2bx^3)}{(1+2ax)^2}$$

4. Physical acceptability conditions

According to Abreu *et al* [29], for a well-behaved natural solution of an anisotropic configuration, the following conditions should be satisfied:

(i) The solution should be free from physical and geometrical singularities within the radius of the stellar configuration. The value of metric potentials at the centre of the star should be non-zero finite, i.e., $e^{\nu}|_{r=0}$ = constant and $e^{\lambda}|_{r=0} = 1$.

(ii) Matter density (ρ), radial pressure (p_r) and tangential pressure (p_t) should be positive inside the stellar configuration.

(iii) The anisotropic factor Δ should vanish at the centre of the stellar configuration and increase towards the surface.

(iv) For an anisotropic fluid sphere, the casuality condition should be satisfied. For instant, the velocity of sound should be less than the speed of light. More specifically, the sound speed along radial and transverse directions must lie between $0< v^2 < 1$.

(v) Both $d\rho/dr$ and $dp_r/dr < 0$, i.e., ρ and p_r are monotonically decreasing functions of r and they have the maximum value at the centre of the star which decreases radially outwards.

(vi) For static object, the adiabatic index

$$\gamma = \frac{\rho + p_r}{p_r}\frac{dp_r}{d\rho}$$

should be greater than 4/3.

(vii) The interior solution should satisfy the energy conditions, namely

- strong energy condition (SEC): $\rho - p_r - 2p_t \geq 0$, $\rho - p_r \geq 0$, $\rho - p_t \geq 0$ or
- dominant energy condition (DEC): $\rho \geq p_r$ and $\rho \geq p_t$.

5. Exterior space–time and matching condition

The interior solution of the spherically symmetric metric is matched to the exterior Schwarzschild space–time at the boundary $r = r_b$ where $r_b > 2M$. The exterior space–time is given by the line element

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{25}$$

The coefficient of g_{rr}, g_{tt} are continuous across the boundary surface $r = r_b$. This implies that

$$1-\frac{2M}{r_b}=\mathrm{e}^{\nu(r_b)}, \tag{26}$$

$$\left(1-\frac{2M}{r}\right)^{-1}=\frac{(1+2ar_b^2)}{(1+ar_b^2)(1-br_b^2)}. \tag{27}$$

From eq. (26) we obtain

$$D=\left(1-\frac{2M}{r_b}\right)^{1/2}(1+ar_b^2)^{-P}(1-br_b^2)^{-Q}\times(1+2ar_b^2)^{-2}. \tag{28}$$

6. Construction of physically stellar models

In this section we shall discuss the properties and the internal structure of compact stars based on the obtained solution. We shall discuss hydrostatic stability, energy conditions, checking the speed of sound and adiabatic index of compact objects by analytical expression along with graphical representations.

6.1 *Behaviour of pressure and density*

An important aspect of compact stars within the framework of General Relativity, is that the energy density and pressure at the interior of the star should be non-negative. Here, the central density and central pressure are obtained as

$$\rho_c=\rho(r=0)=\frac{3(a+b)}{8\pi},$$

$$p_r(r=0)=p_t(r=0)=\frac{(a+b)}{8\pi}-\frac{4B}{3}.$$

The differentiation of the pressure and density equations (19)–(21) with respect to the auxiliary variable x, provides the following equations:

$$\frac{\mathrm{d}\rho}{\mathrm{d}x}=\frac{-(5ab+10a^2+2a^2bx+4a^{3x})}{8\pi(1+2ax)^3}, \tag{29}$$

$$\frac{\mathrm{d}p_r}{\mathrm{d}x}=\frac{1}{3}\frac{\mathrm{d}\rho}{\mathrm{d}x}, \tag{30}$$

$$\frac{\mathrm{d}p_t}{\mathrm{d}x}=\frac{\mathrm{d}p_r}{\mathrm{d}x}+\frac{\mathrm{d}\Delta}{\mathrm{d}x}, \tag{31}$$

where the anisotropic factor is given by

$$\begin{aligned}\frac{\mathrm{d}\Delta}{\mathrm{d}x}=&\frac{1}{8\pi}f_1'(x)f_2(x)+\frac{1}{8\pi}f_1(x)f_2'(x)\\&+\frac{a(2a+b)}{4\pi(1+2ax)^3}\\&-\frac{4Br(x)}{3(1+ax)^2(1-bx)^2(1+2ax)^2}\\&+\frac{h(x)}{8\pi(1+ax)^2(1-bx)^2(1+2ax)^2}\\&+\frac{a(5b+10a+2abx+4a^2x)}{24\pi(1+2ax)^3}\\&+\frac{w(x)}{24\pi(1+ax)^2(1-bx)^2(1+2ax)^4}\end{aligned} \tag{32}$$

and for notational convention we use

$$\begin{aligned}r(x)=&a+b+4abx+13a^2bx^2+ab^2x^2-2a^3x^2\\&+16a^3bx^3-2a^3b^2x^4-4a^4bx^4,\end{aligned}$$

with

$$\begin{aligned}f_1'(x)=&\frac{l(x)}{12(1+ax)^2(1-bx)^2(1+2ax)^2},\\&-\frac{8\pi B(a+b+2abx+2a^2bx^2)}{3(1+ax)^2(1-bx)^2},\\&+\frac{[ab+2a^2bx+2ab^2x+a^2b^2x^2-a^2+b^2]}{(1+ax)^2(1-bx)^2},\end{aligned} \tag{33}$$

$$\begin{aligned}f_2'(x)=&\frac{[3a+3b+8abx-2a^2x+18a^2bx^2+12a^3bx^3]}{3(1+2ax)^3},\\&-\frac{32B\pi}{3}+\frac{a+b+2abx+2a^2bx^2}{(1+2ax)^2},\end{aligned} \tag{34}$$

$$\begin{aligned}h(x)=&b^2-a^2+4a^2bx+4ab^2x+2a^2b^2x\\&+2a^2b^2x^2+3a^3bx^2+ab^3x^2+2a^4x^2\\&+3a^3b^2x^2-a^2b^3x^2+4a^2b^3x^3-4a^4bx^3\\&+4a^3b^3x^4+4a^4b^2x^4+2a^4b^3x^4,\end{aligned}$$

$$\begin{aligned}w(x)=&3b^2-7a^2+ab+32a^2bx+24ab^2x+4a^3x\\&+52a^2b^2x^2+23a^3bx^2+3ab^3x^2-1262a^4x^2\\&+592a^3b^2x^3+312a^4bx^3+30a^2b^3x^3\\&-16a^5x^3+60a^4b^2x^3+578a^4b^2x^4\\&+20a^5bx^4+180a^5b^2x^4+57a^3b^3x^4\\&-60a^4b^3x^4+8a^6x^4+88a^4b^3x^5\\&+180a^5b^2x^5-180a^5b^3x^5-16a^6bx^5\\&+120a^6b^2x^5+176a^5b^3x^6-56a^6b^2x^6\\&-120a^6b^3x^6\end{aligned}$$

and

$$\begin{aligned}l(x)=&ab-7a^2+3b^2+18abx(a+b)\\&-12a^3x+33a^2b^2x^2+28a^3bx^2\\&-4a^4x^228a^3b^2x^3+8a^4bx^3+12a^4b^2x^4.\end{aligned}$$

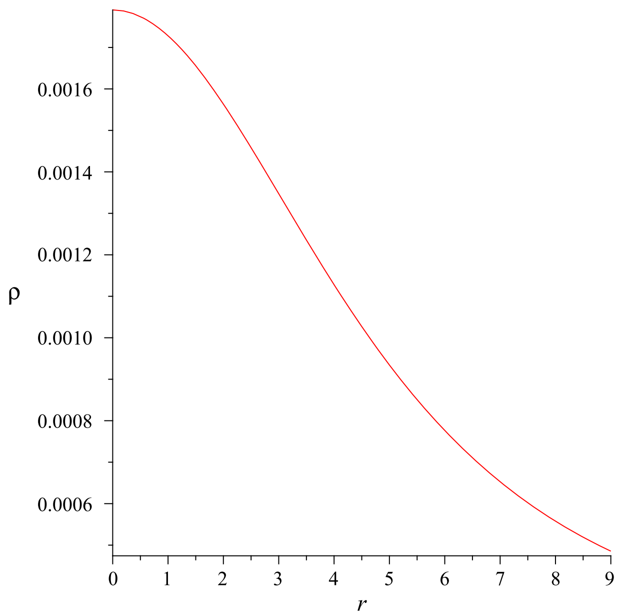

Figure 1. Variation of energy density (ρ) with the radial coordinate inside the star. The parameters are $a = 0.012$, $b = 0.003$ and $B = 0.0001$.

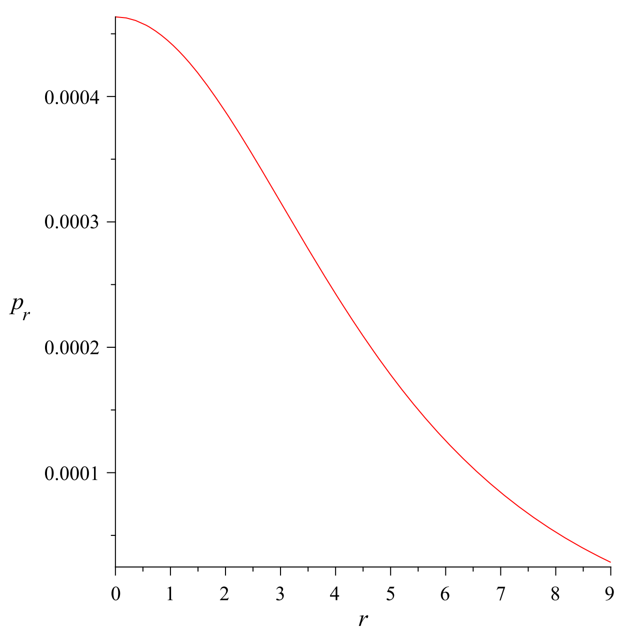

Figure 2. Variation of radial pressure with the radial coordinate inside the star for the same values as mentioned in figure 1.

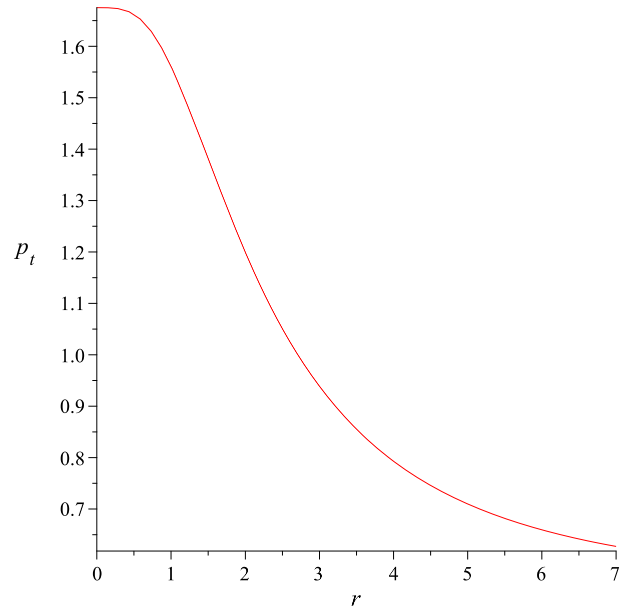

Figure 3. Transverse pressure (p_t) inside the stellar interior is shown against r for the same values mentioned in figure 1.

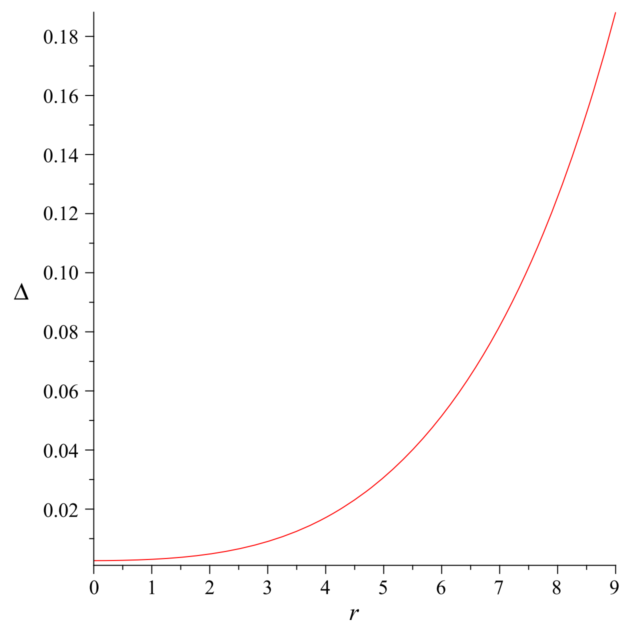

Figure 4. Variations of anisotropic factor (Δ) is plotted with respect to r.

For our model $e^{\lambda}|_{r=0} = 1$ and $e^{\nu}|_{r=0} = D^2 = A$, a positive constant. From figures 1–3 it is clear that matter density, radial and transverse pressures are positive and monotonic decreasing function of r. The radial pressure p_r vanishes at the boundary of the star whereas the matter density and transverse pressure are positive at the boundary of the star. The profile of the anisotropic factor Δ is shown in figure 4. Figure shows that $\Delta > 0$ for our model, i.e., anisotropic force is repulsive in nature (figure 5). Moreover, at the centre of the star, Δ vanishes if $a = -b$. According to Bondi [30] the energy tensor should be $\rho - p_r - 2p_t \geq 0$ which is shown graphically in figure 6.

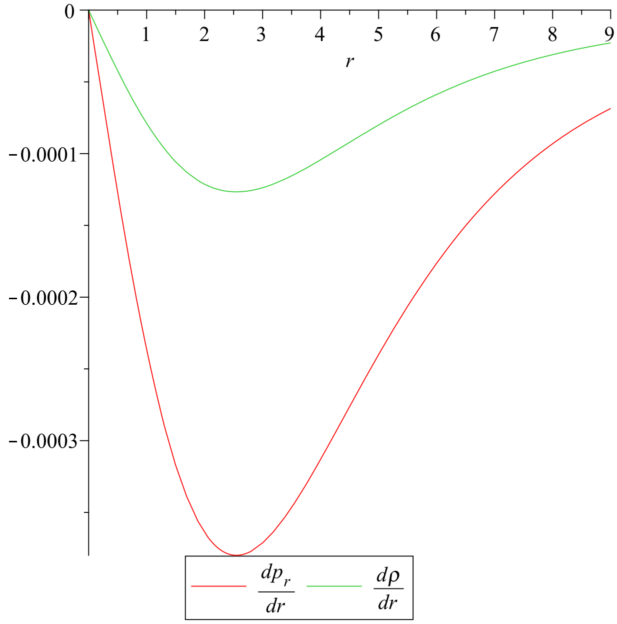

Figure 5. Variation of $\mathrm{d}p_r/\mathrm{d}r$, $\mathrm{d}\rho/\mathrm{d}r$ are shown against r.

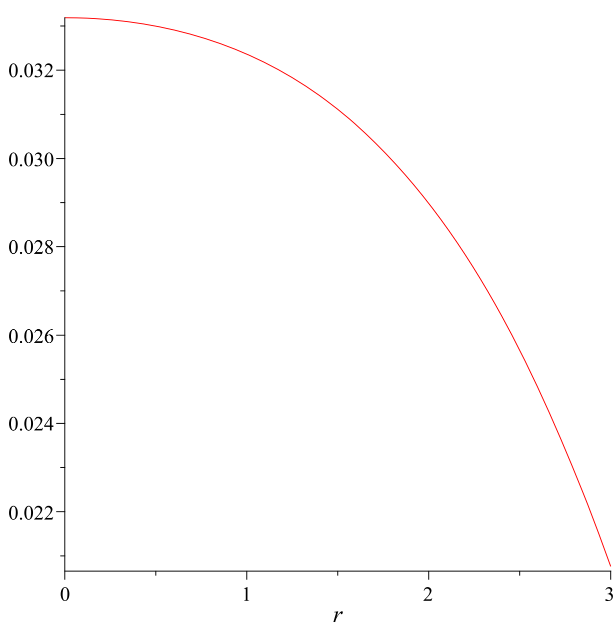

Figure 6. The SEC $\rho - p_r - 2p_t$ is plotted against r for the compact star by taking the same values mentioned in figure 1.

6.2 *Stability condition*

To check the stability of our model, we first discuss the adiabatic index, which can be calculated by the following formula:

$$\gamma = \frac{\rho + p_r}{p_r}\frac{\mathrm{d}p_r}{\mathrm{d}\rho}. \tag{35}$$

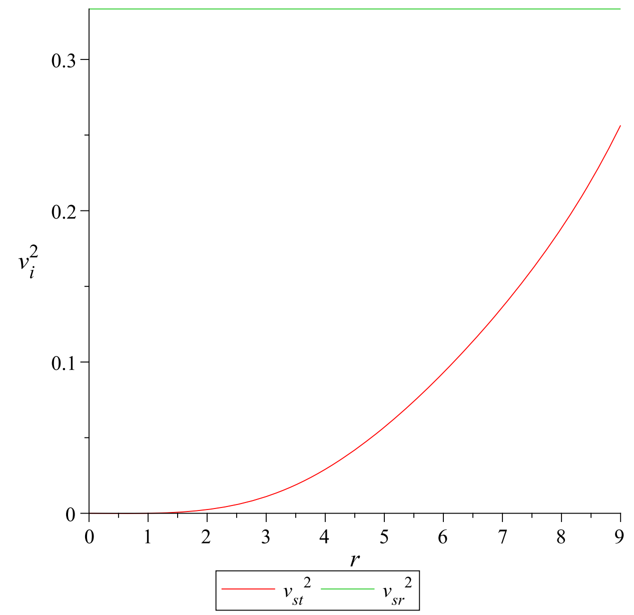

Figure 7. Variation of radial and transverse sound speed are shown against r.

According to Bondi [31] the collapsing condition for an isotropic fluid sphere is valid for $\gamma < \frac{4}{3}$, and for an anisotropic fluid sphere it is given by

$$\gamma < \frac{4}{3} + \left[\frac{4}{3}\frac{p_{ti} - p_{ri}}{r|p'_{ri}|} + \frac{8\pi r}{3}\frac{\rho_i p_{ri}}{|p'_{ri}|}\right]_{\max}, \tag{36}$$

where p_{ri}, p_{ti} and ρ_i are initial values of radial pressure, transverse pressure and density, respectively. The limit of adiabatic index for a stable stellar configuration depends on the types of anisotropy, i.e., either $p_{ti} > p_{ri}$ or $p_{ti} < p_{ri}$. For our proposed model, γ is always greater than $4/3$ and hence stable.

Now, for a physically acceptable model, it is expected that the radial velocity and transverse velocity of sound should be less than unity, i.e., $0 < v_r^2 \leq 1$ and $0 < v_t^2 \leq 1$, and the expressions are given by

$$v_{sr}^2 = \frac{\mathrm{d}p_r}{\mathrm{d}\rho} = \frac{1}{3} < 1, \tag{37}$$

$$v_{st}^2 = \frac{\mathrm{d}p_t}{\mathrm{d}\rho} = \frac{1}{3} + \frac{\mathrm{d}\Delta}{\mathrm{d}\rho}. \tag{38}$$

Here, we represent the radial velocity (v_{sr}^2) and transverse velocity (v_{st}^2) for our anisotropic model with the help of graphical representation due to the complexity of expression of p_t. From figure 7, we can conclude that $0 \leq v_{sr}^2 \leq 1$ and $0 \leq v_{st}^2 \leq 1$ everywhere within the sphere. Now, we apply Herrera's [32] cracking (or overturning) theorem to investigate the stability of local anisotropic matter distribution. Figure 8 indicates that our proposed model is potentially stable. As proposed by Andréasson in [33], we obtain $|v_{st}^2 - v_{sr}^2| \leq 1$, because

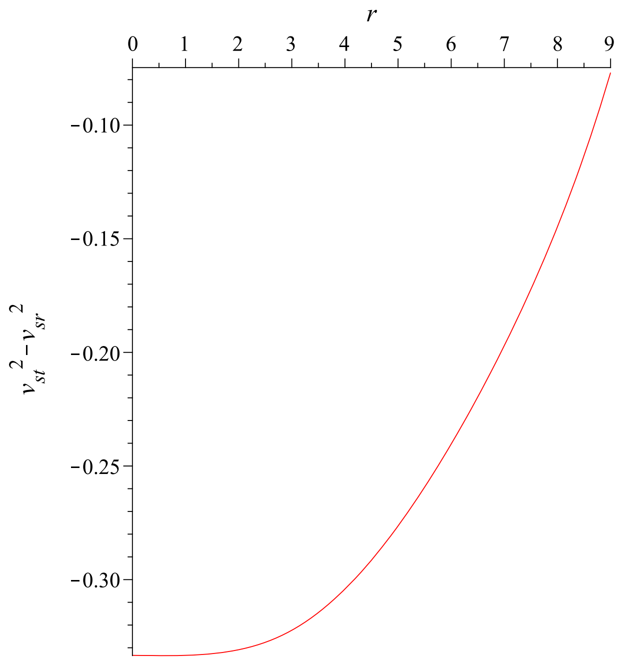

Figure 8. Variation of the square of radial and transverse velocity of sound are plotted against *r*, by employing the same values of the constants as mentioned in figure 1.

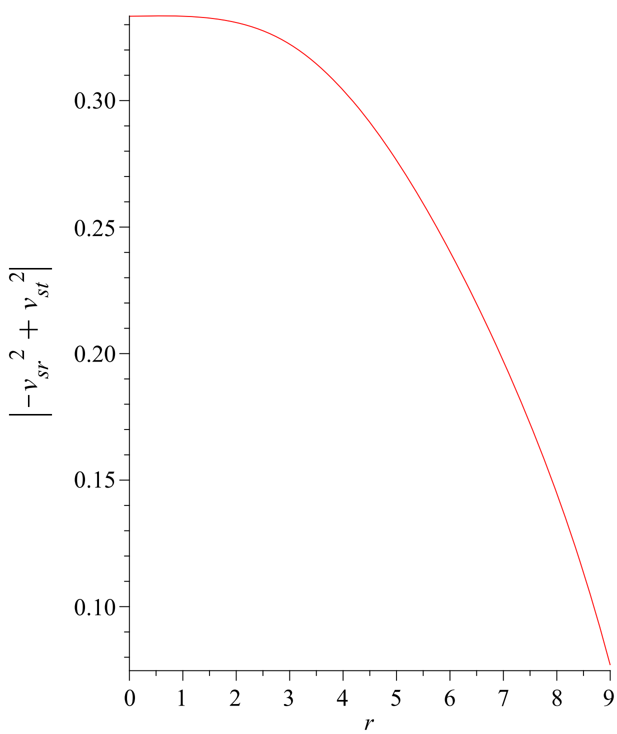

Figure 9. Variation of $|-v_{sr}^2 + v_{st}^2|$ is shown against the radial coordinate for the compact star model.

$0 \leq v_{sr}^2 \leq 1$, $0 \leq v_{st}^2 \leq 1$ (see figure 9). Also, according to Abreu *et al* [29], the region of the anisotropic fluid where $-1 \leq v_{st}^2 - v_{sr}^2 \leq 0$ is potentially stable but the region where $0 \leq v_{st}^2 - v_{sr}^2 \leq 1$ is potentially unstable.

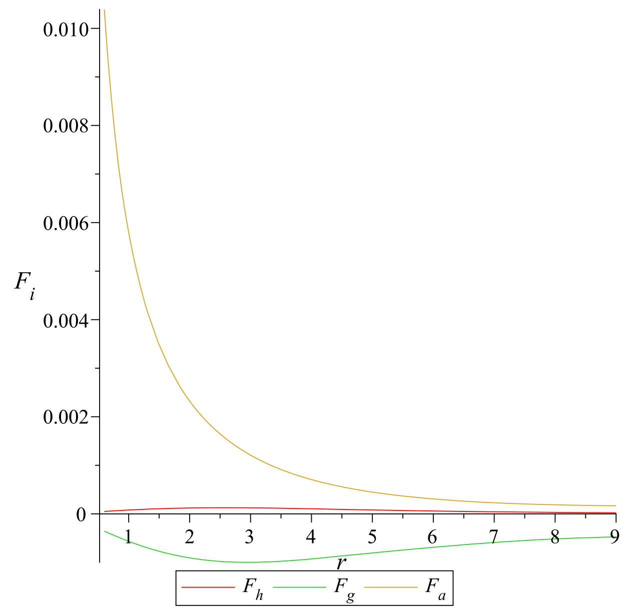

Figure 10. Variation of three different forces, hydrostatic force (F_h), gravitational force (F_g) and anisotropic force (F_a), are shown against *r*.

6.3 *TOV equation*

The generalised Tolman–Oppenheimer–Volkov (TOV) equation for an anisotropic fluid distribution is given by the formula

$$\frac{M_{\mathrm{G}}(\rho + p_r)}{r^2} \mathrm{e}^{(\lambda-\nu)/2} + \frac{\mathrm{d}p_r}{\mathrm{d}r} + \frac{2}{r}(p_t - p_r) = 0, \quad (39)$$

where $M_{\mathrm{G}} = M_{\mathrm{G}}(r)$ is the effective gravitational mass inside the fluid sphere of radius r and is given by

$$M_{\mathrm{G}}(r) = \frac{1}{2} r^2 \mathrm{e}^{(\nu-\lambda)/2} \nu'. \quad (40)$$

$M_{\mathrm{G}}(r)$ is obtained from Tolman–Whittaker mass formula. Using eqs (39) and (40) we get the modified TOV equation as

$$F_g + F_h + F_a = 0, \quad (41)$$

where

$$F_g = -\frac{\nu'}{2}(\rho + p_r), \quad (42)$$

$$F_h = -\frac{\mathrm{d}p_r}{\mathrm{d}r}, \quad (43)$$

$$F_a = \frac{2}{r}(p_t - p_r). \quad (44)$$

F_g, F_h, F_a are known as gravitational, hydrostatic and anisotropic forces of the system respectively. The profile of three forces for our proposed model is shown in figure 10. Figure shows that static equilibrium has been attained by F_g, F_h, F_a.

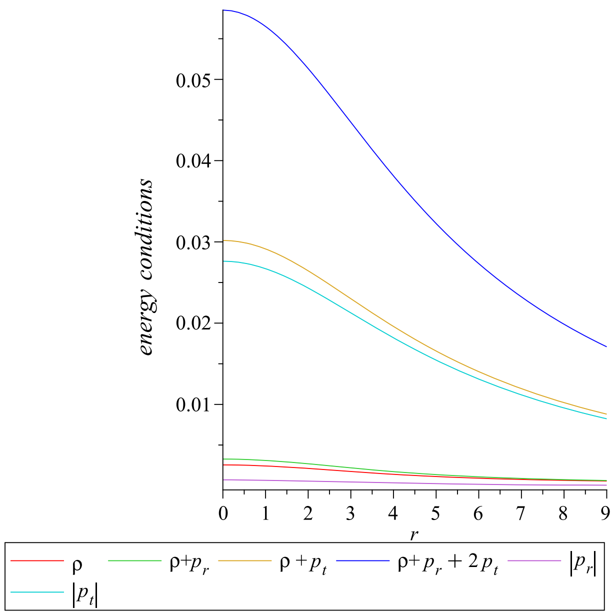

Figure 11. The NEC, WEC, SEC and DEC are plotted against r for the compact star by taking the same values mentioned in figure 1.

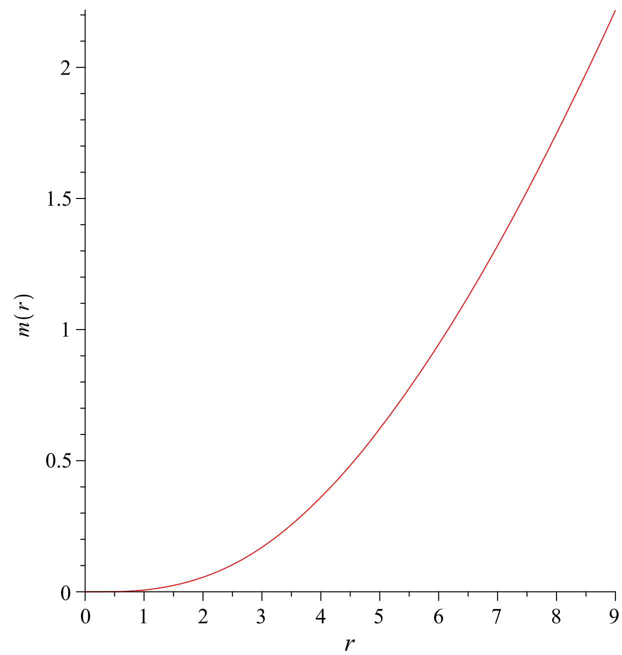

Figure 12. Mass vs. radius curve for the compact star for the same values mentioned in figure 1.

6.4 *Energy condition*

In this section, we now verify different energy conditions of our proposed model. As we know, if and only if the following inequalities hold everywhere inside the stellar model, then we can conclude that the energy conditions, namely weak energy condition (WEC), null energy condition (NEC), strong energy condition (SEC) and dominant energy condition (DEC) are satisfied.

(i) NEC: $\rho + p_r \geqslant 0,$ (45)

(ii) WEC: $\rho + p_r \geqslant 0, \quad \rho > 0,$ (46)

(iii) SEC: $\rho + p_r \geqslant 0, \quad \rho + p_r + 2p_t \geqslant 0,$ (47)

(iv) DEC: $\rho > |p_r|, \quad \rho > |p_t|.$ (48)

Due to the complexity of the expression of p_t, the above inequalities can be proved by graphical representation. The left-hand sides of the above expressions are shown in figure 11. From the figure it is observed that all energy conditions, i.e., null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and dominant energy condition (DEC) are satisfied by our proposed model.

7. Some features

7.1 *Mass of the compact star*

The total mass of the anisotropic compact star within the radius r is given by

$$m(r) = 4\pi \int_0^r r^2 \rho(r) \mathrm{d}r = \frac{br^3}{4} + \frac{r}{4} + \frac{br}{8a} - \frac{(2a+b)r}{8a(1+2ar^2)}. \quad (49)$$

The mass function is given in figure 12. As $r \to 0$, $m(r) \to 0$, the mass function is regular at the centre of the star. Also mass function is a monotonic increasing function of r and is positive inside the stellar configuration.

7.2 *Compactness*

In this section, we shall briefly address that the mass–radius ratio of the compact star, called compactification factor, is not arbitrarily large. Buchdahl [34] proposed that for (3 + 1)-dimensional anisotropic fluid sphere, it should be $\frac{2M}{r_b} < \frac{8}{9}$. The compactness of the star has been calculated to find the maximum allowable ratio of mass to the radius for our model. The compactification factor is given by

$$u(r) = \frac{m(r)}{r} = \frac{br^2}{4} + \frac{1}{4} + \frac{b}{8a} - \frac{1}{4(1+2ar^2)} - \frac{b}{8a(1+2ar^2)}. \quad (50)$$

The graph of $u(r)$ is plotted in figure 13. The figure shows that $u(r)$ is an increasing function.

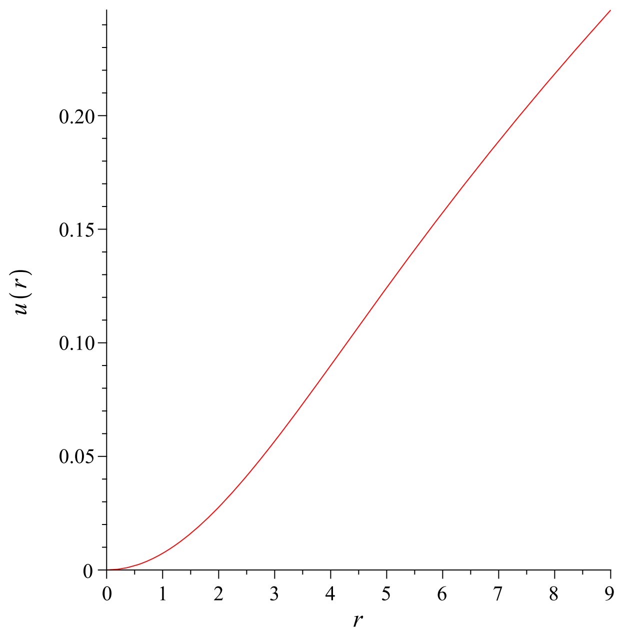

Figure 13. The compactification factor $u(r)$ is shown against r.

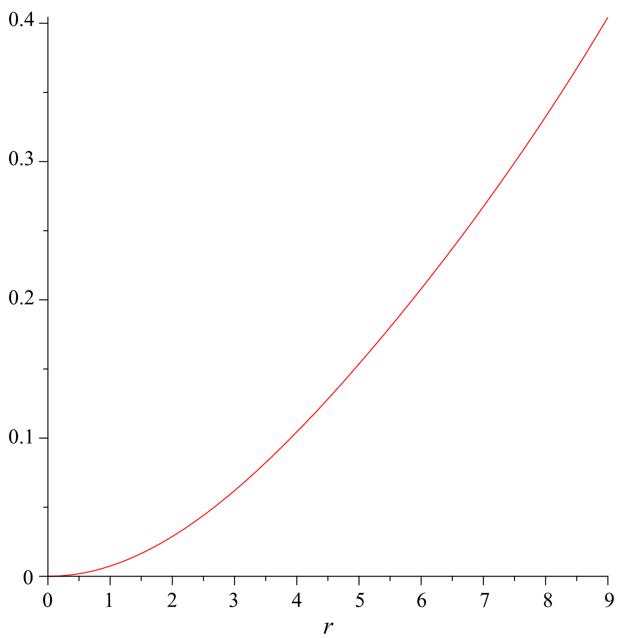

Figure 14. The surface red-shift function Z_s is shown against r.

7.3 *Surface red-shift*

Here, the surface red-shift function of the compact star is defined as

$$1 + Zs = 1 + Z_s. \tag{51}$$

Therefore, the surface red-shift function of the compact star is given by

$$Z_s = (1 - 2u)^{-1/2} - 1. \tag{52}$$

The graph of the surface red-shift is plotted in figure 14.

8. Conclusion

In this work, we proposed a model for relativistic compact objects by considering Tolman IV as a metric potential. The interior solutions of the matter distribution is anisotropic in nature that admits a linear equation of state, namely MIT Bag model. In the present study, we have performed some investigations after considering the following inputs:

(i) From eqs (19), (20) and (21) we observe that all the solutions are non-singular at the stellar interior, i.e., at $r = 0$.

(ii) The nature of the matter density (ρ), radial pressure (p_r) and transverse pressure (p_t) of the proposed model are all positive and monotonic decreasing function of r, as is clear from figures 1–3.

(iii) Taking $a = 0.012, b = 0.003, B = 0.0001$ MeV fm^{-3} and using the conditions $p_r(r = b) = 0$, we obtain the radius of the star as 10.7 km and mass of the star as 2.12 $M_{\odot}$.

(iv) Plugging G and c in the expression of ρ and p_r, we obtained the values of central density and central pressure as 1356.672 MeV fm^{-3} and 973.27 MeV fm^{-3}, respectively.

(v) From graphical representation, it is found that the anisotropic factor $\Delta > 0$, i.e., anisotropic force is repulsive in nature which implies that more compact object will be formed as discussed by Gokhroo and Mehra [35].

(vi) From figure 6, it is found that the trace of energy tensor for the proposed anisotropic stellar model is non-negative.

(vii) The value of compactness factor, 0.1984, and surface red-shift $z_s = 0.288$, for our proposed model are obtained by using the value of parameters.

(viii) Figure 10 shows that three forces acting on our model are counterbalancing to keep the system in static equilibrium and the condition obtained by Andréasson [33] is well satisfied.

(ix) The adiabatic index $\Gamma > 4/3$ and the causality conditions are satisfied for our model ensuring that the model is potentially stable.

Finally, we can say that all these results obtained from our present investigation of compact star is theoretically as well as physically admissible.

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