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Theoretical lower limit of mass of phonon and critical mass for matter–dark matter conversion

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Abstract. From Planck's equation for black body radiation and de Broglie's wave–particle duality relation, we can get a relation between the mass of a phonon and frequency of the emitted radiation. From this relation, we get the theoretical lower limit of the mass of a phonon and critical mass for matter–dark matter conversion. The maximum matter density and limit of the string length are also discussed in this respect. It is observed that there is a critical mass of the smallest particle, which is 7.367×10^{-51} kg, above which we get normal matter and below, the dark matter. It is also observed that if phonon obeys the de Broglie's equation, generation of an electromagnetic radiation of frequency less than 56638721410 Hz is not possible by thermal heating.

Keywords. Mass of phonon; matter–dark matter conversion; string length; black body radiation; space energy theory.

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1. Introduction

German physicist Max Planck started to work on black body radiation [1–3] in 1984 which leads to quantum theory. He developed and used the theory of electromagnetic radiation to explain the nature of radiation of a black body. So far, characteristics of black body radiation are well studied and satisfactorily explained by Planck's theory. By 1925, French physicist Louis de Broglie provided the wave–particle duality relation [4] which is an important discovery towards the formation of quantum mechanics. According to the Planck's theory, the black body is a collection of oscillators which are responsible for its absorption or emission properties. As oscillators are in motion, they should have a de Broglie wavelength. Thus, we may get a relation between absorption (or emission) frequency and the de Broglie frequency of the oscillators. As de Broglie frequency is related to the mass of the oscillator, we can have an equation for it in terms of the Planck's constant (h), the Boltzmann constant (k) and its vibration frequency. Using this equation, we get the lower limit of the mass of the resonator, i.e. phonon. It is also possible to explain the interaction between light and matter in a different way with the help of this relation. Considering the present estimated value of mass density at the centre of a black hole, the dimension of a resonator is calculated which is of the order of 10^{-24} m, higher than the present estimated value of Planck length (10^{-35} m). Present theory is also applied to examine the lower limit of the mass of a particle of real matter. The possibility of matter–dark matter conversion is also tested in the light of the present hypothesis.

2. Theory

According to Planck [5], electromagnetic radiation depends on monochromatic vibrations of the resonators of the body which absorb or emit electromagnetic radiation. Now let us consider a body which is emitting electromagnetic radiation has N number of identical resonators at thermal equilibrium of temperature T. If U is the energy of a single vibrating resonator, then total energy of the body will be UN. Entropy S of a monochromatic vibrating resonator is related to its vibrational energy and temperature as

$$\frac{\mathrm{d}S}{\mathrm{d}U} = \frac{1}{T}. \quad (1)$$

According to the equipartition principle, kinetic energy of one mode of vibration at temperature T is $\frac{1}{2}kT$ where k is the Boltzmann constant. As one complete vibration has two modes, kinetic energy of one resonator at temperature T is kT. Thus, we get

$$U = kT$$

or

$$\mathrm{d}U = k\,\mathrm{d}T. \tag{2}$$

From eqs (1) and (2) we get

$$\mathrm{d}S = kd(\ln T)$$

and

$$S = k\ln T + A, \tag{3}$$

where A is an integration constant. As A is a scaling factor, we may consider $A = 0$ for simplicity. Then, we get from eq. (3),

$$S = k\ln T. \tag{4}$$

If p is the momentum and m is the mass of the resonator, we get

$$\frac{1}{2}kT = \frac{p^2}{2m}$$

or

$$kT = \frac{p^2}{m}. \tag{5}$$

Considering de Broglie's wave–particle duality relation, we get

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}, \tag{6}$$

where h is the Planck's constant, λ is the wavelength associated with that resonator due to its momentum, ν is the frequency corresponding to λ and c is the velocity of light in vacuum. From eqs (5) and (6) we get

$$kT = \frac{h^2\nu^2}{mc^2}$$

or

$$T = \frac{h^2\nu^2}{mkc^2}. \tag{7}$$

Replacing T in eq. (4) by eq. (7) we get

$$S = k\ln\left(\frac{h^2\nu^2}{mkc^2}\right). \tag{8}$$

Planck [5] derived an alternative expression for entropy of a resonator as follows:

$$S = k\left\{\left(1+\frac{U}{h\nu}\right)\ln\left(1+\frac{U}{h\nu}\right) - \frac{U}{h\nu}\ln\frac{U}{h\nu}\right\}. \tag{9}$$

Equation (9) may be written as

$$S = k\ln\left\{\left(1+\frac{U}{h\nu}\right)\left(1+\frac{h\nu}{U}\right)^{U/h\nu}\right\}. \tag{10}$$

Comparing eqs (8) and (10) we get

$$\left(\frac{h^2\nu^2}{mkc^2}\right) = \left\{\left(1+\frac{U}{h\nu}\right)\left(1+\frac{h\nu}{U}\right)^{U/h\nu}\right\}. \tag{11}$$

From eqs (2) and (5) we get

$$U = \frac{h^2\nu^2}{mc^2}. \tag{12}$$

Thus,

$$\frac{U}{h\nu} = \frac{h\nu}{mc^2}. \tag{13}$$

Putting this value of $U/h\nu$ in eq. (11), we get

$$\left(\frac{h^2\nu^2}{mkc^2}\right) = \left(1+\frac{h\nu}{mc^2}\right)\left(1+\frac{mc^2}{h\nu}\right)^{h\nu/mc^2}. \tag{14}$$

In eq. (14), m is the mass of the resonator (which is considered as phonon in modern physics). Thus, mc^2 is the energy (E) equivalent to the mass of the resonator. Following Einstein's energy quantisation relation, we can write

$$mc^2 = E = h\nu_0$$

or

$$\frac{mc^2}{h} = \nu_0. \tag{15}$$

Here, it should be mentioned that ν and ν_0 are not the same. ν is the vibrational frequency of the resonator and ν_0 is the frequency of mass equivalent of the resonator. Replacing mc^2/h by ν_0 in eq. (14), we get

$$\frac{h\nu^2}{k\nu_0} = \left(1+\frac{\nu}{\nu_0}\right)\left(1+\frac{\nu_0}{\nu}\right)^{\nu/\nu_0}. \tag{16}$$

3. Numerical analysis and discussions

As h and k are constants in eq. (16), we have a very interesting relation between ν and ν_0. That is, the lowest energy vibration (i.e. the lowest value of frequency, ν) of a resonator (phonon) depends on its mass (defined by ν_0) only. Planck [2] considered that a resonator emits electromagnetic radiation with a frequency which is the same as that of the resonator. Thus, it is possible that any object which emits electromagnetic radiations, should have a critical temperature below which all vibrating resonators would be at its lowest energy level. Thus, the emission spectrum of an object below the critical

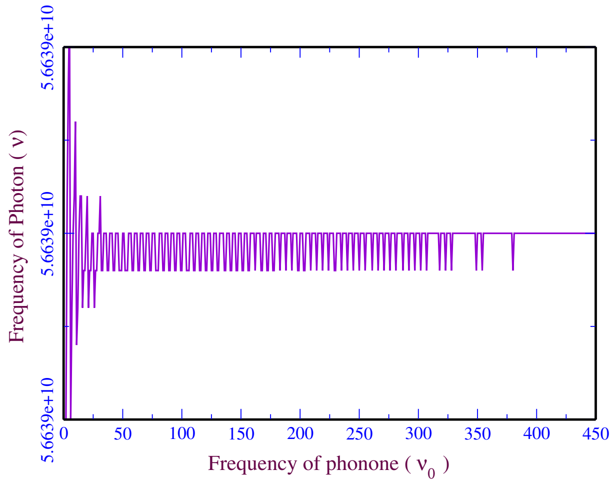

Figure 1. Calculation of minimum frequency of the resonator using numerical variation method.

temperature should give information about the number of resonators with different masses. Not only that, as different frequencies correspond to the resonators of different masses, we should be able to calculate the mass of every resonator using eq. (16).

In eq. (16), ν_0 is the frequency related to the mass of the resonator. ν_0 is a non-zero positive quantity. To get an idea about the mass of a resonator, let us consider a small value of ν_0 as $\nu_0 = 1$ Hz. Then mass of this resonator would be 7.36×10^{-51} kg, which is 10^{21} times lighter than that of the lightest quark (the down quark, mass $= 7.297 \times 10^{-30}$ kg). Putting $\nu_0 = 1$ Hz in eq. (16) we get $\nu = 56638721410$ Hz. Thus, if the lowest value of ν_0 is 1 Hz and both Planck's theory for black body radiation and de Broglie's wave–particle duality relation are true, we can conclude that, it is impossible to generate an electromagnetic radiation with frequency less than 56638721410 Hz by thermal excitation only. Thus, 56638721410 Hz or 56.63872141 GHz frequency may be considered as cut-off frequency of thermal emission. This explains why we never get any significant emission at very low frequency even at very low temperature. In Planck's equation, there is no cut-off frequency except an exponential decay, which implies that every object, even at very low temperature would emit electromagnetic radiations spontaneously, though the energy density may be very small, which is not true.

The lowest value of ν may be calculated from eq. (16) by numerical method. A plot of ν against ν_0 is presented in figure 1. It is observed that for very small values of ν_0, the lowest value of ν varies between 5.663872×10^{10} Hz and 5.663895×10^{10} Hz. After $\nu_0 = 380$ Hz, ν does not change with ν_0. We get a fixed value of $\nu = 5.663895 \times 10^{10}$ Hz. This value of ν is marginally higher than that obtained by considering $\nu_0 = 1$ Hz. If we consider $\nu = \nu_0$ then we get $\nu = 83345207372$ Hz, i.e. 83.34 GHz, which is significantly higher than the values obtained from the other two methods. But, in all the three methods, we get ν values in the GHz region. Thus, the lower limit of thermal emission is in the GHz region considering the present values of Planck's constant ($h = 6.62607 \times 10^{-34}$ JS) and Boltzmann constant ($k = 1.3806 \times 10^{-23}$ J K^{-1}).

To find ν_0 for high-energy emission, another set of numerical analysis is done using fixed values of ν taken at different energy regions like X-ray, γ-ray, ultraviolate, visible, infrared, etc. Calculated results are presented in table 1. Mass of the corresponding resonator is also calculated and presented in the same table. Previously, we have found that for 83.34 GHz frequency, $\nu = \nu_0$. Above this value, ν_0 is always greater than ν and below this value, ν_0 is less than ν. Thus, 83.34 GHz frequency may be considered as 'inversion frequency'. Above inversion frequency, ν_0 increases more rapidly than ν. The trend is just opposite below the inversion frequency. The change of ν and ν_0 with an increase of emission energy is presented in figure 2. Near cut-off frequency, ν_0 exponentially drops to 1 Hz. At very high-energy region, ν_0 exponentially increases. This implies that the probability of emission of very high-energy electromagnetic radiation on thermal heating is very less. These two limits (upper limit and lower limit) explain physically why we always get a peak instead of any exponential increase or decrease in energy density plot against temperature of a black body. The existence of cut-off frequency is very important, else we should have lower energy emission from every object even at very low temperature.

From table 1, we get that the mass of the resonator for emission in the UV–visible reason is nearly equal to the mass of an electron (9.11×10^{-31} kg). It is well known that electromagnetic radiation in this region is related to the electronic transition. This fact supports the work presented in this paper. Mass of the resonator in the X-ray and γ-ray regions is almost equal to the mass of proton and neutron (1.67×10^{-27} kg). This also supports the conclusion made in this work as we know that γ-ray radiation from an atom is related to neutron and proton.

We have found that if the lower limit of ν_0 is 1 Hz, then the lower limit of mass of a resonator (phonon) is 7.367×10^{-51} kg. In fact, according to $E = mc^2$ relation, we can conclude that this is the lower limit of the mass of an object which can exist as a particle if the frequency of electromagnetic radiation is an integer. Though frequency of electromagnetic radiation is not an integer, we may fix $\nu_0 = 1$ Hz as cut-off frequency as we found that near this value, we observe changes of properties. Cut-off frequency may be found in a

Table 1. Frequency and mass of the resonator for higher energy emission.

Region of emission	ν (Hz)	ν_0 (Hz)	Mass of the resonator (kg)
γ-ray	3×10^{21}	4.32×10^{32}	3.18×10^{-18}
X-ray	3×10^{19}	4.32×10^{28}	3.18×10^{-22}
X-ray	3×10^{17}	5.00×10^{24}	3.68×10^{-26}
UV	7.5×10^{14}	2.99×10^{19}	2.20×10^{-31}
Visible	4.3×10^{14}	8.99×10^{18}	6.62×10^{-32}
IR	3×10^{12}	5.00×10^{14}	3.68×10^{-36}
IR	3×10^{11}	3.12×10^{12}	2.30×10^{-38}
Microwave	2×10^{11}	1.18×10^{12}	8.69×10^{-39}
Microwave	1×10^{11}	1.66×10^{11}	1.22×10^{-39}
Microwave	9×10^{10}	1.14×10^{11}	8.40×10^{-40}
Microwave	8.33×10^{10}	8.33×10^{10}	6.14×10^{-40}
Microwave	8.00×10^{10}	6.96×10^{10}	5.13×10^{-40}
Microwave	7.00×10^{10}	3.41×10^{11}	2.51×10^{-40}
Microwave	6.00×10^{10}	7.19×10^{9}	5.30×10^{-41}
Microwave	5.90×10^{10}	4.95×10^{9}	3.65×10^{-41}
Microwave	5.80×10^{10}	2.80×10^{9}	2.08×10^{-41}
Microwave	5.70×10^{10}	7.28×10^{8}	5.36×10^{-42}
Microwave	5.67×10^{10}	1.22×10^{8}	8.99×10^{-43}
Microwave	5.665×10^{10}	2.21×10^{7}	1.63×10^{-43}
Microwave	5.664×10^{10}	2.12×10^{6}	1.56×10^{-44}
Microwave	5.6639×10^{10}	1.2×10^{5}	8.84×10^{-45}
Microwave	5.66389×10^{10}	2.0	1.47×10^{-50}
Microwave	$5.663872141 \times 10^{10}$	1.0	7.37×10^{-51}

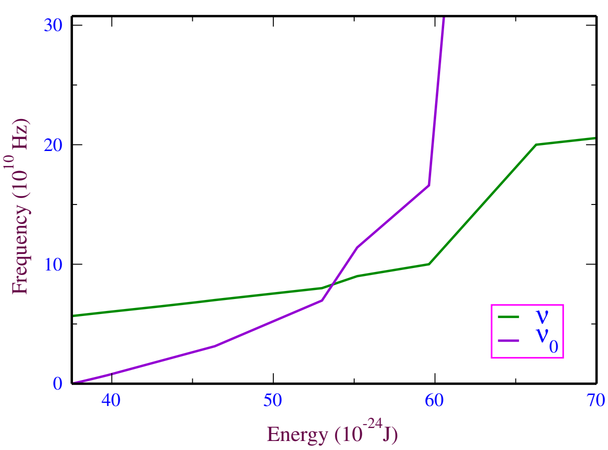

Figure 2. Variation of ν and ν_0 at different energy regions.

different way. Let us now consider a particle which has a mass equal to the lower limit of the mass. If we consider it as spherical and having mass density the same as the highest mass density of a black hole [6–8] which is 1.5×10^{20} kg m^{-3}, we get the radius of that particle as 2.272×10^{-24} m. This radius is well above the Planck length [9,10] which is 1.6162×10^{-35} m. Planck length is an important parameter in string theory [11–14]. The characteristic length scale of strings is assumed to be of the order of the Planck length, the scale at which the effects of quantum gravity are believed to become significant [12]. If we consider the present scale of the Planck length to be true, then using the same mass density, we get the mass of the smallest resonator as 2.654×10^{-84} kg, which is equivalent to $\nu_0 = 3.6 \times 10^{-34}$ Hz. Obviously, this value of ν_0 is very very less than our previous assumption, i.e. $\nu_0 = 1$ Hz. On the other hand, if we consider both the present scale of Planck length and the lower limit of mass is true, then we get the mass density of strings as presented in table 2. From the calculated values we found that for three-dimensional string mass, density is of the order of 10^{53} kg m^{-3} (4.166×10^{53} kg m^{-3} for a sphere) which is 10^{33} times higher than the highest mass density of a black hole [6]. This implies that three-dimensional string is not possible or mass density in the centre of a black hole is 10^{33} times higher than the present value.

The quantum tunnelling probability of a particle is inversely proportional to its mass. A particle with tiny mass has quantum tunnelling probability near unity. Thus, there should be a critical mass below which quantum tunnelling probability is unity. That means, such a particle could not be confined in any quantum boundary state. Violation of quantum boundary condition by such a tiny particle implies that it should not interact with any kind of electromagnetic radiation, because interaction of matter with electromagnetic radiation occurs only when at least one of its states, arises due to any kind

Table 2. Mass density of different strings.

Dimension of the string	Shape of the string	Mass density of the string
One-dimensional	Linear	4.558×10^{-16} kg m^{-1}
Two-dimensional	Circular	8.978×10^{18} kg m^{-2}
Three-dimensional	Sphere	4.166×10^{53} kg m^{-3}

of boundary condition, changes. Also, such a particle should expand spontaneously as it is not bound by any boundary condition. Hence, its pressure would be negative. These properties are similar to dark matter [15–19]. Thus, we can consider the lower limit of mass as termed earlier, which is 7.367×10^{-51} kg, is in fact the 'critical mass' of a fundamental particle above which we get normal matter and below, the dark matter.

From the above discussions, we can conclude that normal matter and dark matter are related and partitioned by a critical mass barrier. Till date, it is not undoubtedly proved that dark matter and matter are related and convertible. The other possibility, i.e. matter and dark matter are two different things, is also not proved. Present hypothesis supports matter–dark matter inter-relation. Recent research proves that Einstein's familiar formula ($E = mc^2$) should be scaled [20–23], though different researchers proposed different values of the scaling factor. Thus, critical mass calculated here should be scaled according to the scaling factor for $E = mc^2$ relation. It is also possible that $E = mc^2$ equation is correct. There is no need for inclusion of scaling factor as, according to the present hypothesis, matter is divided into two parts; matter and dark matter.

In space energy theory [24], it is considered that the frequency of any electromagnetic radiation would decrease with time even if it travels through vacuum. Change of wavelength (λ) of an electromagnetic radiation with time (t) is given as

$$\lambda = \lambda_0(1 + \theta t^{1/2}), \tag{17}$$

where θ is a constant. According to eq. (17), every electromagnetic radiation after a certain time should reach a frequency less than 1 Hz and would behave as dark matter. In this process, energy in our Universe is spontaneously converted to dark matter which creates space and hence we observe an accelerating Universe at present [25–28]. Thus, the present hypothesis and space energy theory [24] are complementary to each other.

Following space energy theory, formation of dark matter from electromagnetic radiation is explained in the previous section. But, how does matter to dark matter conversion take place? This is not clear from the present assumptions. We know that if the mass of an individual resonator is less than the critical mass, then it would behave as a dark matter. Still now, we do not know whether the mass of a resonator is a constant quantity or not. If the mass of a resonator is a fixed quantity, then matter to dark matter conversion would not take place. According to the present assumption, matter to dark matter conversion would take place if mass of the resonator changes with temperature. A relation between the rest mass of the resonator and the temperature of the object is presented in eq. (7). In this equation, vibrational frequency of the resonator is also a variable of temperature. Thus, we cannot draw any conclusion unless we have an experimental proof. But, there are only two possibilities here: mass of the resonator may increase or decrease with temperature. Increase of mass with increase of temperature means that association of resonator takes place due to increase in temperature. This implies that at a very very high temperature, for example, at the stage of the first few seconds of our Universe as considered in Big Bang Theory, all resonators were associated as one resonator in a very compact form with extremely high mass density. On cooling, from one resonator of high mass, numbers of the resonator with low mass generates and the process continues until mass of a resonator crosses the critical mass and form dark matter. This supports the Big Bang Theory. On the other hand, if mass of a resonator decreases with increase of temperature, we may conclude that matter is created from dark matter, i.e. at the initial stage of our Universe there was only dark matter; on cooling, dark matter condensed to form matter. But, in that condition total mass of the dark matter of the Universe would decrease with time. Thus, a measurement of the change of mass of dark matter of a confined space would prove which process is the actual process for matter–dark matter conversion. If no change is observed, we can say that there is no interchange between matter and dark matter.

At present, there are no experimental data to prove the assumptions made here. But, two different experiments may be performed which would justify whether these assumptions are true or false. If we do the experiment to measure the wavelength of a monochromatic radiation after a finite time interval and found that its wavelength increases due to time travel then we can conclude that both the space energy theory and assumptions made here

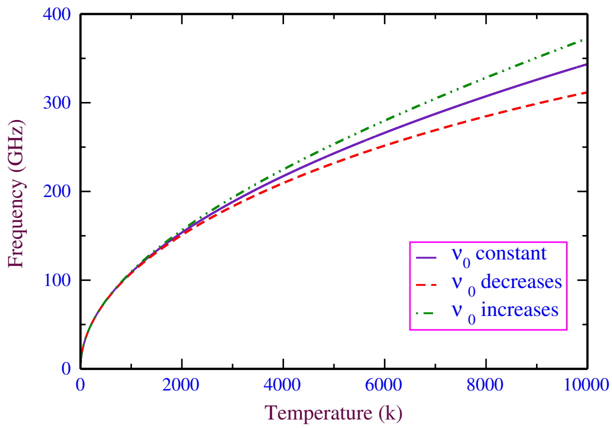

Figure 3. Variation of ν with temperature for static and variable ν_0.

are true. In another experiment, we can measure the frequency of the emitted radiation of a monochromatic source at different temperatures and plot ν vs. T. If the experimental values of ν at different temperatures are less (red line in figure 3) than the corresponding theoretical values considering ν_0 as constant (violet line in figure 3), we can conclude that the mass of a resonator decreases with increase in temperature. If experimental values are higher than the corresponding theoretical values (green line in figure 3), then we should say that the mass of the resonator increases with the increase in temperature. In either case, i.e. any deviation from the ideal plot, we can conclude that assumptions proposed in this article are legal. But only from experiment, we can conclude whether matter is created from dark matter or vice versa.

4. Conclusions

In the present work, by incorporating de Broglie's equation in Planck's equation of black body radiation we reached eq. (16) which opens up a few questions and possibilities. Equation (16) may be considered as the universal equation of state which correlate electromagnetic radiation with string theory, dark energy and dark matter, space energy theory and quantum gravity. Using this equation, we can calculate the mass of a resonator, i.e. phonon. From the value of critical mass, we can calculate the limit of length of a string using maximum mass density and vice versa. From density constrained we can say both maximum mass density and string length may be justified only if the strings are dark matters. Then, different types of strings would be the fundamental particles we are searching for.

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