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Geometry of magnetic rotational (MR) band-crossing phenomenon in MR bands

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Abstract. A semiclassical (SC) approach is proposed to calculate the $B(M1)$ transition rates in the band-crossing region of two magnetic rotational (MR) bands. In the present work, a geometry is suggested for the shear blades to govern its behaviour during the band-crossing. In the crossing region, gradual alignment of two nucleons is responsible for the crossing behaviour and it must give a quantised resultant angular momentum. As an example, it is successfully implemented for the MR bands in the mass $A = 110$ and $A = 200$ regions. A good agreement of the present semiclassical calculations with the experimental values is presented and furthermore, it is seen that the present proposal is also helpful to see the core contribution in the MR phenomenon.

Keywords. Magnetic rotational band; band-crossing; semiclassical; $B(M1)$; alignment plot.

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1. Introduction

In nuclear landscape, many dipole ($\Delta I = 1$) bands have been reported which are known as magnetic rotational (MR) bands and Frauendörf [1] already established that these bands occur due to the highly asymmetric nuclear current distribution. Most of these bands have a regular sequence of state of fixed parity I^π, $I^\pi + 1$, $I^\pi + 2$, …, connected by strong $\Delta I = 1$, $M1$ transitions and relatively weak crossover $E2$ transitions. The $B(M1)$ transition rate is large and decreases with increasing angular momentum [2–4]. An excellent data compilation on these bands is available in [5] and more than 190 such bands have been found in about 80 nuclides. In some of the nuclei, their magnetic rotational character is also retained after the band-crossing. In such situations, the MR character is found in both the bands and this phenomenon is known as MR band-crossing. At the band head, the proton and the neutron angular momentum blades (J_π and J_ν) are nearly perpendicular to each other and the total angular momentum, J, lies between these with some tilted angle, υ, with respect to the nuclear symmetry axis. The higher angular momentum states are generated by gradual closing of the two blades step by step towards J. The MR band-crossing occurs due to the alignment of a pair of valence nucleons and the shear blades reopen to build a new shear band [6–8]. The observed transition rates $B(M1)$ of these bands have been well described in the framework of the tilted axis cranking (TAC) model by Frauendörf [1,6] and semiclassical geometrical model by Clark and Macchiavelli [7,9,10]. However, these models could not reproduce the $B(M1)$ value of the states in the crossing region. Semiclassical models [11] have also been successfully used to study the signature splitting phenomenon in normal deformed bands [12], and to study the properties of superdeformed bands [13,14].

In the present paper, an extension of the semiclassical (SC) approach of Macchiavelli [10] has been proposed to explain the MR band-crossing. This model is used to calculate the $B(M1)$ value in the crossing region (§2). The results and discussion are given in §3 with examples in mass $A = 110$ and $A = 200$ regions. Finally, we conclude the present work in §4.

2. Semiclassical model for MR band-crossing

The present geometrical approach is that, in the MR band-crossing, a nucleon pair which is responsible for the band-crossing is coupled to zero angular momentum before the crossing and the alignment of the nucleon pair changes the structure of the band after the crossing. In the crossing region, there will be some states in which the two aligning valence nucleons are only partially aligned. The resultant angular momentum should be quantised. Therefore, at these intermediate states, only those angles that quantise the resultant angular momentum, are possible between the aligning pair. Using the above interpretation of MR band-crossing, the $B(M1)$ value can be calculated when the band changes its structure during the crossing. From the geometry of the MR band given in ref. [10], the $B(M1)$ value is proportional to the square of the perpendicular component of the magnetic moment ($\mu_\perp$) and it shows decreasing behaviour with the spin. $B(M1)$ is given in terms of proton angle θ_π and $g_{\text{eff}} = g_\nu - g_\pi$ as

$$B(M1) = \frac{3}{4\pi}\frac{1}{2}\vec{\mu}_\perp^2 = \frac{3}{4\pi}g_{\text{eff}}^2 j_\pi^2 \frac{1}{2}\sin^2\theta_\pi \quad [\mu_N^2]. \tag{1}$$

The shear angle $\theta_{\nu\pi}$ is given as

$$\cos\theta_{\nu\pi} = \frac{J(J+1) - J_\nu(J_\nu+1) - J_\pi(J_\pi+1)}{2\sqrt{J_\nu(J_\nu+1)J_\pi(J_\pi+1)}}. \tag{2}$$

This expression has been extended in terms of the shear angle in ref. [15] as

$$B(M1) = \frac{3}{4\pi}\frac{(2J_\nu+1)^2(2J_\pi+1)^2}{16J(2J+1)} \times (g_\nu - g_\pi)^2 \sin^2\theta_{\nu\pi} \quad [\mu_N^2]. \tag{3}$$

Assume that a shear band is built by recoupling two long angular momenta J_π and J_ν, which are formed by coupling one or more protons and neutrons, respectively. This band is crossed by another band that arises due to the alignment of a pair of neutron (J_ν^1 and J_ν^2) along J_ν, coupled to form J_ν'. The shears will re-open with the two shear blades formed by J_π and J_ν'. In the crossing region, J_ν^1 and J_ν^2 are partially aligned with an angle ϕ between them and the effective neutron angular momentum, J_ν^{eff}, is formed by the coupling of J_ν and J_ν^{12} where

$$J_\nu^{12} = \sqrt{(J_\nu^1)^2 + (J_\nu^2)^2 + 2(J_\nu^1)(J_\nu^2)\cos\phi}, \tag{4}$$

In band-crossing, the band changes its structure by the gradual alignment of J_ν^1 and J_ν^2. Figure 1 shows the classical picture of the nucleon alignment in the MR band-crossing. If g_ν is the neutron g-factor before crossing, then, the effective neutron g-factor in the crossing region is

$$g_\nu^{\text{eff}} = g_\nu + g_\nu^{12}, \tag{5}$$

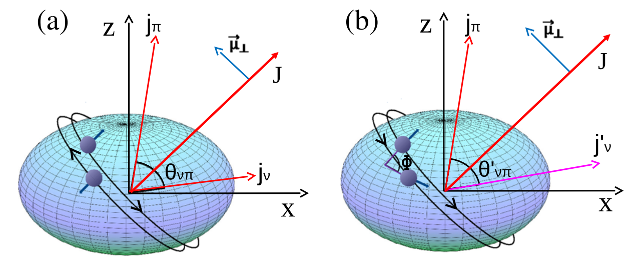

Figure 1. Classical picture of nucleon alignment and angular momentum coupling of the MR band-crossing. (**a**) The aligning nucleon pair is antiparallel before the band-crossing and are coupled to zero angular momentum. (**b**) In band-crossing, the nucleon pair starts aligning with an angle ϕ between them and their resultant angular momentum vector contributes in forming the shears blade.

where g_ν^{12} can be calculated from the g-factors of the aligning pair g_ν^1 and g_ν^2 as [16]

$$g_\nu^{12} = \frac{1}{2}(g_\nu^1 + g_\nu^2) + \frac{J_\nu^1(J_\nu^1+1) - J_\nu^2(J_\nu^2+1)}{2J_\nu^{12}(J_\nu^{12}+1)}(g_\nu^1 - g_\nu^2). \tag{6}$$

From the semiclassical expression, eq. (3), $B(M1)$ and the shear angle $\theta_{\nu\pi}$ of the states in the crossing region can be expressed as

$$B(M1) = \frac{3}{4\pi}\frac{(2J_\nu^{\text{eff}}+1)^2(2J_\pi+1)^2}{16J(2J+1)} \times (g_\nu^{\text{eff}} - g_\pi)^2 \sin^2\theta_{\nu\pi}[\mu_N^2], \tag{7}$$

$$\cos\theta_{\nu\pi} = \frac{J(J+1) - J_\nu^{\text{eff}}(J_\nu^{\text{eff}}+1) - J_\pi(J_\pi+1)}{2\sqrt{J_\nu^{\text{eff}}(J_\nu^{\text{eff}}+1)J_\pi(J_\pi+1)}}. \tag{8}$$

Same approach can be used if the aligning pair is a proton pair.

3. Results and discussion

It has been observed in shear bands that the $B(M1)$ value decreases with increasing spin and after band-crossing, the $B(M1)$ value jumps to a higher value and then again follows the same decreasing trend. A deeper understanding of MR band-crossing can be done by calculating J^{eff} and g^{eff} for possible angle ϕ between the aligning pair and observing the $B(M1)$ behaviour. In the crossing region, the $B(M1)$ value should be of some intermediate value so that it changes from a lower value before crossing to a higher value above crossing by a step by step increment. To test the validity of the proposed geometrical picture, $B(M1)$ values of the states

Table 1. Calculated $J_\nu^{\rm eff}$, $g_\nu^{\rm eff}$ and $B(M1)$ values of the states in the crossing region for the possible angle ϕ between the aligning pair. The bands are numbered according to refs [5,18].

Bands	J ($\hbar$)	$\phi°$	$J_\nu^{\rm eff}$ ($\hbar$)	$g_\nu^{\rm eff}$	$B(M1)$ [μ_N^2]
^{108}Cd-Band 1	16.5	164	12.5	−0.203	0.54
BC – $\pi[g_{9/2}^{-3}g_{7/2}] \otimes \nu[h_{11/2}(g_{7/2}/d_{5/2})^1]$	17.0	146	13.5	−0.255	1.23
AC – $\pi[g_{9/2}^{-3}g_{7/2}] \otimes \nu[h_{11/2}^3(g_{7/2}/d_{5/2})^1]$	17.5	130	14.5	−0.271	1.82
^{196}Pb-Band 3	21.0	159	11.0	−0.192	3.02
BC – $\pi[h_{9/2}i_{13/2}]_{K=11^-} \otimes \nu[i_{13/2}^{-1}(p_{3/2}/f_{5/2})^1]$	22.0	134	13.0	−0.219	5.36
AC – $\pi[h_{9/2}i_{13/2}]_{K=11^-} \otimes \nu[i_{13/2}^{-3}(p_{3/2}/f_{5/2})^1]$	23.0	107	15.0	−0.226	7.85
	24.0	74	17.0	−0.228	10.41
^{197}Pb-Band 1	39/2	155	8.5	−0.413	1.15
BC – $\pi[h_{9/2}i_{13/2}]_{K=11^-} \otimes \nu[i_{13/2}^{-1}]$	41/2	130	10.0	−0.448	2.71
AC – $\pi[h_{9/2}i_{13/2}]_{K=11^-} \otimes \nu[i_{13/2}^{-3}]$	43/2	104	11.5	−0.458	4.35
	45/2	72	13.0	−0.462	6.17
^{199}Pb-Band 1	39/2	160	8.5	−0.366	1.09
BC – $\pi[h_{9/2}i_{13/2}]_{K=11^-} \otimes \nu[i_{13/2}^{-1}]$	41/2	134	10.5	−0.396	3.89
AC – $\pi[h_{9/2}i_{13/2}]_{K=11^-} \otimes \nu[i_{13/2}^{-3}]$	43/2	107	12.5	−0.402	6.93
	45/2	74	14.5	−0.405	10.20

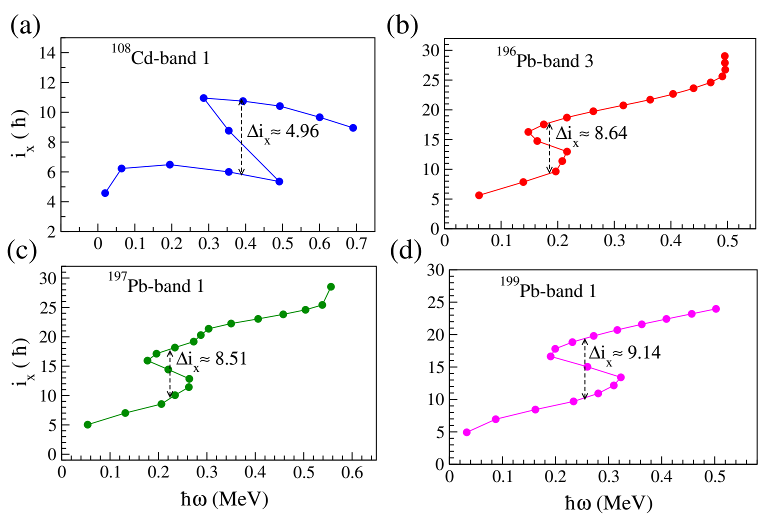

Figure 2. Plot of alignment, i_x, as a function of angular frequency, $\hbar\omega$, along with the gain in angular momentum, Δi_x, after the band-crossing. The solid lines are to guide the experimental points.

in crossing region have been calculated for some of the MR bands in $A \sim 110$ and $A \sim 200$ mass regions. The possible angle ϕ between the aligning pair was set to get quantised $J_\nu^{\rm eff}$ that can couple with J_π to give total angular momentum J of the state. The single-particle g-factors were calculated from the Nilsson orbitals of the bands using the relations $g_\pi = (\Lambda_\pi + g_{s\pi}\Sigma_\pi)/|\Omega_\pi|$ and $g_\nu = g_{s\nu}\Sigma_\nu/|\Omega_\nu|$ [17]. A quenching factor of 0.6 was considered in the calculations, i.e. $g_{s\sigma} = 0.6 \times g_{s\sigma}^{\rm free}$ ($\sigma = \pi$ or ν), $g_{s\pi}^{\rm free} = 5.587$ and $g_{s\nu}^{\rm free} = -3.826$. The total g-factor is given by

$$g_\sigma = \frac{\sum_i g_{s\sigma}^i \Sigma_\sigma^i}{\sum_i |\Omega_\sigma^i|}.$$

The results of the calculations using the suggested configuration for before crossing (BC) and after crossing (AC) of the bands are listed in table 1 for ^{108}Cd-band 1 [18,19], ^{196}Pb-band 3 [20,21], ^{197}Pb-band 1 [22,23] and ^{199}Pb-band 1 [24,25]. The experimental alignment and gain in alignment Δi_x, after the band-crossing were also calculated and is shown in figure 2. The observed values of Δi_x were in agreement with the values of j_ν

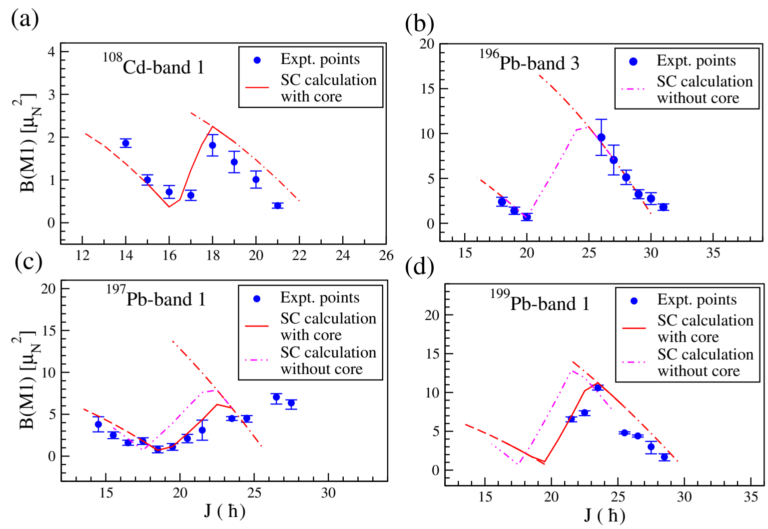

Figure 3. Plot of $B(M1)\,[\mu_N^2]$ as a function of spin J ($\hbar$) for (**a**) ^{108}Cd-band 1, (**b**) ^{196}Pb-band 3, (**c**) ^{197}Pb-band 1 and (**d**) ^{199}Pb-band 1. Solid circles are experimental points taken from refs [5,18]. The dashed lines and dash–dotted lines are the theoretical curves calculated using Macchiavelli formalism [10] of MR bands for the given configuration of BC and AC, respectively. Solid lines are SC calculations of the proposed model for the crossing region joining BC and AC theoretical curves. In (**c**) and (**d**), the dash–double dotted lines are for the case when the core contribution is not considered.

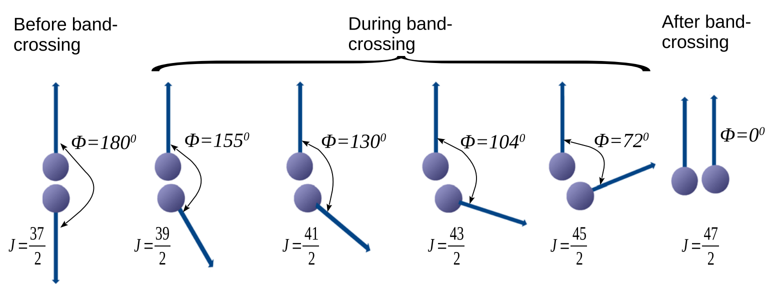

Figure 4. Classical picture of neutron alignment in the MR band-crossing of ^{197}Pb-band 1. The neutron pair is antialigned with $\phi = 180°$ before the band-crossing. In band-crossing, the neutron pair starts aligning in steps, their resultant angular momentum vector contributes in the angular momentum blades, and becomes fully aligned with $\phi = 0°$ after the band-crossing.

and j_π considered in the present calculations. In the case of ^{108}Cd, a core contribution to the total spin is considered in the calculations for BC and AC, following ref. [18]. For ^{197,199}Pb, calculations for both the cases of with and without core contribution were performed and are shown in figures 3c and 3d, respectively. With core contribution, the total spin of the state is given by [9,18]

$$I_{\text{total}} = I_{\text{shear}} + I_{\text{core}}, \quad I_{\text{core}} = I - \frac{\Delta R}{\Delta I}(I - I_b), \quad (9)$$

where I_b is the band head spin of the band.

The possible angle ϕ between the aligning pair was considered in such a manner that j_ν^{eff} (or j_π^{eff}, if the aligning pair is a proton pair) is quantised and the resultant angular momentum of the aligning pair gives regular increment in the angular momentum blade in each step to achieve total Δi_x in the band-crossing. For example, in the case of ^{197}Pb, the band-crossing is brought by the alignment of a neutron pair from $i_{13/2}$ and the neutron blade changes from $j_\nu = 6.5\hbar$ (BC) to $j_\nu = 14.5\hbar$ (AC) in the band-crossing. The neutron pair starts aligning and made a gain of $2.0\hbar$ in the first step, with an angle $\phi = 155°$, giving $j_\nu^{\text{eff}} = 8.5\hbar$. Subsequently, it made a gain of $1.5\hbar$ in each step to achieve a total alignment gain of $8\hbar$ in the band-crossing. A pictorial representation of such step by step alignment of the neutron pair is shown in figure 4.

4. Conclusion

We present a geometrical picture of MR band-crossing and developed a semiclassical approach to calculate the $B(M1)$ values in the crossing region. The present SC calculation shows good agreement with the experimental observations in ^{108}Cd and ^{196,197,199}Pb. In ^{197,199}Pb, consideration of the core contribution in the total angular momentum gives better agreement with the experimental values. However, lack of enough experimental $B(M1)$ values for the crossing regions is a draw-back to test its implementation in general for the band-crossing phenomenon in MR bands. Such a geometrical picture will help in a better perception of shear bands and their crossing. It will propel the theoretical efforts in understanding the structure of these bands.

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