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Deterministic assisted cloning of an unknown single-particle four-dimensional quantum state

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Abstract. In this paper, we present a scheme which can produce a perfect copy of an unknown single-particle four-dimensional quantum state with assistance from a state preparer. Two stages were included in this scheme. The first stage requires the usual teleportation, after Alice's (the state sender) generalised Bell state measurement. Bob (the state receiver) can get the original state with unit probability. In the second stage, after having received Victor's (the state preparer) classical message, and using the rest resource of the teleportation process, the perfect copy of an original unknown state can be produced in Alice's place. To realise the scheme, several novel sets of measuring basis were introduced. It must be pointed out that, in the present scheme, the total success probability for assisted cloning of a perfect copy of the unknown state can reach 1.

Keywords. Quantum assisted cloning; single-particle four-dimensional quantum state; projective measurement.

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1. Introduction

Quantum entanglement has generated much interest in quantum information processing such as quantum teleportation [1], quantum dense coding [2], quantum cryptography [3], quantum secret sharing [4], remote state preparation (RSP) [5,6] and so on. Entanglement is also related to quantum cloning. Unlike classical information, an unknown quantum state cannot be cloned exactly because of the no-cloning theorem [7]. However, quantum cloning approximately is necessary in quantum information [8]. Hence, various cloning machines have been proposed. Universal quantum cloning machine was originally addressed by Bužek and Hillery [9]. The probabilistic cloning machine was first presented by Duan and Guo using a general unitary-reduction operation with the post-election of the measurement results [10]. The other category of quantum cloning were developed in [11,12].

In the last decade, Pati [13] proposed a scheme where one can produce copies and orthogonal complement copies of an arbitrary unknown state with minimal assistance from a state preparer. Inspired by Pati's paper, a lot of schemes for assisted cloning have been proposed [14–22]. Zhan [14] generalised Pati's assisted cloning scheme to the case of an unknown two-qubit entangled state. Wang *et al* [23] proposed a scheme for cloning an unknown single-qutrit equatorial state with assistance. Recently, a lot of schemes for assisted cloning of unknown high-dimensional equatorial states have been proposed [24–28]. However, we find that so far no scheme has been reported for assisted cloning of an unknown general high-dimensional state. In this paper, we present a new scheme for quantum cloning of an unknown single-particle four-dimensional (FD) state with assistance. To complete the scheme, several novel sets of measuring basis were introduced. In the present scheme, the total success probability for assisted cloning of a perfect copy of the unknown state can reach 1.

2. Quantum cloning of an unknown single-particle four-dimensional state with assistance

Suppose that there are three participants, the state preparer Victor, the state sender Alice and the state receiver Bob. The sender Alice has an unknown input single-particle FD state

$$|\psi\rangle = (\alpha_0|0\rangle + \alpha_1 e^{i\delta_1}|1\rangle + \alpha_2 e^{i\delta_2}|2\rangle + \alpha_3 e^{i\delta_3}|3\rangle)_A \quad (1)$$

from Victor, where the parameters $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \delta_1, \delta_2$ and δ_3 are real and $\alpha_0^2+\alpha_1^2+\alpha_2^2+\alpha_3^3 = 1$. Alice wishes to teleport the unknown state $|\psi\rangle$ to receiver Bob, and then to create a copy of this state at her place with the assistance of Victor. Assume that Alice and Bob share two-particle FD maximally entangled states, as quantum channel, which is given by

$$|\varphi\rangle_{A_1B} = \frac{1}{2}\left(|00\rangle + |11\rangle + |22\rangle + |33\rangle\right)_{A_1B}, \quad (2)$$

where particle A_1 belongs to Alice while particle B to Bob. The input state $|\psi\rangle$ is unknown to Bob. The initial state of the whole system can be written as

$$\begin{aligned}
|\Phi\rangle = |\psi\rangle_A \otimes |\varphi\rangle_{A_1B} = \frac{1}{4}[&|\Psi_{00}\rangle_{AA_1}(\alpha_0|0\rangle \\
&+ \alpha_1 e^{i\delta_1}|1\rangle + \alpha_2 e^{i\delta_2}|2\rangle + \alpha_3 e^{i\delta_3}|3\rangle)_B \\
&+ |\Psi_{10}\rangle_{AA_1}(\alpha_0|0\rangle + i\alpha_1 e^{i\delta_1}|1\rangle - \alpha_2 e^{i\delta_2}|2\rangle \\
&- \alpha_3 e^{i\delta_3}|3\rangle)_B + |\Psi_{20}\rangle_{AA_1}(\alpha_0|0\rangle - \alpha_1 e^{i\delta_1}|1\rangle \\
&+ \alpha_2 e^{i\delta_2}|2\rangle - \alpha_3 e^{i\delta_3}|3\rangle)_B + |\Psi_{30}\rangle_{AA_1}(\alpha_0|0\rangle \\
&- i\alpha_1 e^{i\delta_1}|1\rangle - \alpha_2 e^{i\delta_2}|2\rangle + i\alpha_3 e^{i\delta_3}|3\rangle)_B \\
&+ |\Psi_{01}\rangle_{AA_1}(\alpha_0|1\rangle + \alpha_1 e^{i\delta_1}|2\rangle + \alpha_2 e^{i\delta_2}|3\rangle \\
&+ i\alpha_3 e^{i\delta_3}|0\rangle)_B + |\Psi_{11}\rangle_{AA_1}(\alpha_0|1\rangle + i\alpha_1 e^{i\delta_1}|2\rangle\rangle \\
&- \alpha_2 e^{i\delta_2}|3 - \alpha_3 e^{i\delta_3}|0\rangle)_B + |\Psi_{21}\rangle_{AA_1}(\alpha_0|1\rangle \\
&- \alpha_1 e^{i\delta_1}|2\rangle + \alpha_2 e^{i\delta_2}|3\rangle - \alpha_3 e^{i\delta_3}|0\rangle)_B \\
&+ |\Psi_{31}\rangle_{AA_1}(\alpha_0|1\rangle - i\alpha_1 e^{i\delta_1}|2\rangle - \alpha_2 e^{i\delta_2}|3\rangle \\
&+ i\alpha_3 e^{i\delta_3}|0\rangle)_B + |\Psi_{02}\rangle_{AA_1}(\alpha_0|2\rangle + \alpha_1 e^{i\delta_1}|3\rangle \\
&+ \alpha_2 e^{i\delta_2}|0\rangle + \alpha_3 e^{i\delta_3}|1\rangle)_B + |\Psi_{12}\rangle_{AA_1}(\alpha_0|2\rangle \\
&+ i\alpha_1 e^{i\delta_1}|3\rangle - \alpha_2 e^{i\delta_2}|0\rangle - i\alpha_3 e^{i\delta_3}|1\rangle)_B \\
&+ |\Psi_{22}\rangle_{AA_1}(\alpha_0|2\rangle - \alpha_1 e^{i\delta_1}|3\rangle + \alpha_2 e^{i\delta_2}|0\rangle \\
&- \alpha_3 e^{i\delta_3}|1\rangle)_B + |\Psi_{32}\rangle_{AA_1}(\alpha_0|2\rangle - i\alpha_1 e^{i\delta_1}|3\rangle \\
&- \alpha_2 e^{i\delta_2}|0\rangle + i\alpha_3 e^{i\delta_3}|1\rangle)_B + |\Psi_{03}\rangle_{AA_1}(\alpha_0|3\rangle \\
&+ \alpha_1 e^{i\delta_1}|0\rangle + \alpha_2 e^{i\delta_2}|1\rangle + \alpha_3 e^{i\delta_3}|2\rangle)_B \\
&+ |\Psi_{13}\rangle_{AA_1}(\alpha_0|3\rangle + i\alpha_1 e^{i\delta_1}|0\rangle - \alpha_2 e^{i\delta_2}|1\rangle \\
&- i\alpha_3 e^{i\delta_3}|2\rangle)_B + |\Psi_{23}\rangle_{AA_1}(\alpha_0|3\rangle - \alpha_1 e^{i\delta_1}|0\rangle \\
&+ \alpha_2 e^{i\delta_2}|1\rangle - \alpha_3 e^{i\delta_3}|2\rangle)_B + |\Psi_{33}\rangle_{AA_1}(\alpha_0|3\rangle \\
&- i\alpha_1 e^{i\delta_1}|0\rangle - \alpha_2 e^{i\delta_2}|1\rangle + i\alpha_3 e^{i\delta_3}|2\rangle)_B], \quad (3)
\end{aligned}$$

where $|\Psi_{nm}\rangle$ are generalised Bell states of the Hilbert space of two FD particles

$$|\Psi_{nm}\rangle = \sum_{j=0}^{3} e^{\pi ijn/2}|j\rangle \otimes |(j+m) \bmod 4\rangle/2, \quad (4)$$

where $n, m, j = 0, 1, 2, 3$. More explicitly,

$$\begin{aligned}
|\Psi_{00}\rangle &= \frac{1}{2}(|00\rangle + |11\rangle + |22\rangle + |33\rangle), \\
|\Psi_{10}\rangle &= \frac{1}{2}(|00\rangle + i|11\rangle - |22\rangle - i|33\rangle), \\
|\Psi_{20}\rangle &= \frac{1}{2}(|00\rangle - |11\rangle + |22\rangle - |33\rangle), \\
|\Psi_{30}\rangle &= \frac{1}{2}(|00\rangle - i|11\rangle - |22\rangle + i|33\rangle), \\
|\Psi_{01}\rangle &= \frac{1}{2}(|01\rangle + |12\rangle + |23\rangle + |30\rangle), \\
|\Psi_{11}\rangle &= \frac{1}{2}(|01\rangle + i|12\rangle - |23\rangle - i|30\rangle), \\
|\Psi_{21}\rangle &= \frac{1}{2}(|01\rangle - |12\rangle + |23\rangle - |30\rangle), \\
|\Psi_{31}\rangle &= \frac{1}{2}(|01\rangle - i|12\rangle - |23\rangle + i|30\rangle), \\
|\Psi_{02}\rangle &= \frac{1}{2}(|02\rangle + |13\rangle + |20\rangle + |31\rangle), \\
|\Psi_{12}\rangle &= \frac{1}{2}(|02\rangle + i|13\rangle - |20\rangle - i|31\rangle), \\
|\Psi_{22}\rangle &= \frac{1}{2}(|02\rangle - |13\rangle + |20\rangle - |31\rangle), \\
|\Psi_{32}\rangle &= \frac{1}{2}(|02\rangle - i|13\rangle - |20\rangle + i|31\rangle), \\
|\Psi_{03}\rangle &= \frac{1}{2}(|03\rangle + |10\rangle + |21\rangle + |32\rangle), \\
|\Psi_{13}\rangle &= \frac{1}{2}(|03\rangle + i|10\rangle - |21\rangle - i|32\rangle), \\
|\Psi_{23}\rangle &= \frac{1}{2}(|03\rangle - |10\rangle + |21\rangle|32\rangle), \\
|\Psi_{33}\rangle &= \frac{1}{2}(|03\rangle - i|10\rangle - |21\rangle + i|32\rangle). \quad (5)
\end{aligned}$$

Through simple calculation, it can be shown that the single-particle operation

$$U_{nm} = \sum_{j=0}^{3} e^{\pi ijn/2}|j\rangle\langle(j+m) \bmod 4|, \quad (6)$$

where $n, m, j = 0, 1, 2, 3$, will transform $|\Psi_{00}\rangle$ into the corresponding states in eq. (5) respectively

$$U_{nm}|\Psi_{00}\rangle = |\Psi_{nm}\rangle. \quad (7)$$

More explicitly, the operations U_{nm} can be described as

$$U_{00}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},\quad U_{10}=\begin{pmatrix}1&0&0&0\\0&i&0&0\\0&0&-1&0\\0&0&0&-i\end{pmatrix},$$

$$U_{20}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix},$$

$$U_{30}=\begin{pmatrix}1&0&0&0\\0&-i&0&0\\0&0&-1&0\\0&0&0&i\end{pmatrix},$$

$$U_{01}=\begin{pmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{pmatrix},\quad U_{11}=\begin{pmatrix}0&0&0&-i\\1&0&0&0\\0&i&0&0\\0&0&-1&0\end{pmatrix},$$

$$U_{21}=\begin{pmatrix}0&0&0&-1\\1&0&0&0\\0&-1&0&0\\0&0&1&0\end{pmatrix},$$

$$U_{31}=\begin{pmatrix}0&0&0&i\\1&0&0&0\\0&-i&0&0\\0&0&-1&0\end{pmatrix},$$

$$U_{02}=\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix},\quad U_{12}=\begin{pmatrix}0&0&-1&0\\0&0&0&-i\\1&0&0&0\\0&i&0&0\end{pmatrix},$$

$$U_{22}=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{pmatrix},$$

$$U_{32}=\begin{pmatrix}0&0&-1&0\\0&0&0&i\\1&0&0&0\\0&-i&0&0\end{pmatrix},$$

$$U_{03}=\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{pmatrix},\quad U_{13}=\begin{pmatrix}0&i&0&0\\0&0&-1&0\\0&0&0&-i\\1&0&0&0\end{pmatrix},$$

$$U_{23}=\begin{pmatrix}0&-1&0&0\\0&0&1&0\\0&0&0&-1\\1&0&0&0\end{pmatrix},$$

$$U_{33}=\begin{pmatrix}0&-i&0&0\\0&0&-1&0\\0&0&0&i\\1&0&0&0\end{pmatrix}. \tag{8}$$

Now let Alice perform a generalised Bell state measurement on her particles A and A_1, and then announce publicly her result of the measurement. According to the outcome of Alice's measurement, Bob can employ a suitable operation U_{nm} (see eqs (7) and (8)) on his particle B, and the original state $|\psi\rangle$ can be recovered. For instance, without loss of generality, assume that the result of Alice's measurement is $|\Psi_{13}\rangle_{AA_1}$, by eq. (3), Bob can perform a unitary operation U_{31} on the particle B, and the original state $|\psi\rangle$ can be obtained. Thus, the teleportation is successfully realised. By eq. (3), if Alice's measurement results are the other 15 cases, Bob can employ the appropriate unitary operations on his particle B and then the original state $|\psi\rangle$ can be always obtained. Here we no longer depict them one by one.

To create a copy of the unknown single-particle FD state $|\psi\rangle$, Alice first performs the unitary operations $(U_{30})_A \otimes (U_{03})_{A_1}$ on particles A and A_1 in $|\Psi_{13}\rangle_{AA_1}$ respectively. The state of particles A and A_1 will become

$$|u\rangle = \frac{1}{2}(|00\rangle + |11\rangle + |22\rangle + |33\rangle)_{AA_1}. \tag{9}$$

Next, Alice can introduce an auxiliary FD particle A_2 with the initial state $|0\rangle_{A_2}$ and the state of particles A, A_1 and A_2 will be described as

$$|v\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle + |33\rangle)_{AA_1} \otimes |0\rangle_{A_2}. \tag{10}$$

Then, Alice applies the higher-dimensional C-NOT gate operation C_{AA_2} on her particles with particle A as the controlled particle and A_2 as the target one. Here, operation C_{AA_2} acts on a pair of particles A and A_2 in the following manner [23]:

$$C_{AA_2}|k,l\rangle_{AA_2} = |k,k+l\rangle_{AA_2}. \tag{11}$$

After that, the state of particles A, A_1 and A_2 will become

$$|v'\rangle = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)_{AA_1A_2}. \tag{12}$$

Then Alice sends FD particles A_1 and A_2 to Victor.

In order to help Alice create the perfect copy of the original state $|\psi\rangle$, Victor needs to perform single-particle FD projective measurement on his own particles A_1 and A_2. As Victor knows the parameters $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \delta_1, \delta_2$ and δ_3 in the original state $|\psi\rangle$, he can choose to measure the particles A_1 and A_2 in any basis. The first measuring basis chosen by Victor is a set of mutually orthogonal basis vectors (MOBVs) $\{|\tau_k\rangle\}$ $(k = 0, 1, 2, 3)$, which is given by

$$\begin{pmatrix}|\tau_0\rangle\\|\tau_1\rangle\\|\tau_2\rangle\\|\tau_3\rangle\end{pmatrix}=G\begin{pmatrix}|0\rangle\\|1\rangle\\|2\rangle\\|3\rangle\end{pmatrix},\tag{13}$$

where

$$G=\begin{pmatrix}\alpha_0&\alpha_1&\alpha_2&\alpha_3\\\alpha_1&-\alpha_0&\alpha_3&-\alpha_2\\\alpha_2&-\alpha_3&-\alpha_0&\alpha_1\\\alpha_3&\alpha_2&-\alpha_1&-\alpha_0\end{pmatrix}.\tag{14}$$

The second measurement bases chosen by Victor are four sets of MOBVs $\{|\mu_q^{(k)}\rangle\}\,(q,k=0,1,2,3)$, which are given by

$$\begin{pmatrix}|\mu_0^{(k)}\rangle\\|\mu_1^{(k)}\rangle\\|\mu_2^{(k)}\rangle\\|\mu_3^{(k)}\rangle\end{pmatrix}=H^{(k)}\begin{pmatrix}|0\rangle\\|1\rangle\\|2\rangle\\|3\rangle\end{pmatrix},\tag{15}$$

where

$$\begin{aligned}H^{(0)}&=\begin{pmatrix}1&\lambda_1&\lambda_2&\lambda_3\\1&-\lambda_1&\lambda_2&-\lambda_3\\1&-\lambda_1&-\lambda_2&\lambda_3\\1&\lambda_1&-\lambda_2&-\lambda_3\end{pmatrix},\\
H^{(1)}&=\begin{pmatrix}\lambda_1&1&\lambda_3&\lambda_2\\\lambda_1&-1&\lambda_3&-\lambda_2\\\lambda_1&-1&-\lambda_3&\lambda_2\\\lambda_1&1&-\lambda_3&-\lambda_2\end{pmatrix},\\
H^{(2)}&=\begin{pmatrix}\lambda_2&\lambda_3&1&\lambda_1\\\lambda_2&-\lambda_3&1&-\lambda_1\\\lambda_2&-\lambda_3&-1&\lambda_1\\\lambda_2&\lambda_3&-1&-\lambda_1\end{pmatrix},\\
H^{(3)}&=\begin{pmatrix}\lambda_3&\lambda_2&\lambda_1&1\\\lambda_3&-\lambda_2&\lambda_1&-1\\\lambda_3&-\lambda_2&-\lambda_1&1\\\lambda_3&\lambda_2&-\lambda_1&-1\end{pmatrix},\end{aligned}\tag{16}$$

where $\lambda_l=\mathrm{e}^{-i\delta_l}\,(l=1,2,3)$.

By eqs (13)–(16), the state $|v'\rangle$ (see eq. (12)) can be rewritten as

$$\begin{aligned}|v''\rangle=\frac{1}{4}[&|\tau_0\rangle_{A_1}|\mu_0^{(0)}\rangle_{A_2}(\alpha_0|0\rangle+\alpha_1\mathrm{e}^{i\delta_1}|1\rangle\\
&+\alpha_2\mathrm{e}^{i\delta_2}|2\rangle+\alpha_3\mathrm{e}^{i\delta_3}|3\rangle)_A\\
&+|\tau_0\rangle_{A_1}|\mu_1^{(0)}\rangle_{A_2}(\alpha_0|0\rangle-\alpha_1\mathrm{e}^{i\delta_1}|1\rangle\\
&+\alpha_2\mathrm{e}^{i\delta_2}|2\rangle-\alpha_3\mathrm{e}^{i\delta_3}|3\rangle)_A\\
&+|\tau_0\rangle_{A_1}|\mu_2^{(0)}\rangle_{A_2}(\alpha_0|0\rangle-\alpha_1\mathrm{e}^{i\delta_1}|1\rangle\\
&-\alpha_2\mathrm{e}^{i\delta_2}|2\rangle+\alpha_3\mathrm{e}^{i\delta_3}|3\rangle)_A\\
&+|\tau_0\rangle_{A_1}|\mu_3^{(0)}\rangle_{A_2}(\alpha_0|0\rangle+\alpha_1\mathrm{e}^{i\delta_1}|1\rangle\\
&-\alpha_2\mathrm{e}^{i\delta_2}|2\rangle-\alpha_3\mathrm{e}^{i\delta_3}|3\rangle)_A\\
&+|\tau_1\rangle_{A_1}|\mu_0^{(1)}\rangle_{A_2}(\alpha_1\mathrm{e}^{i\delta_1}|0\rangle-\alpha_0|1\rangle\\
&+\alpha_3\mathrm{e}^{i\delta_3}|2\rangle-\alpha_2\mathrm{e}^{i\delta_2}|3\rangle)_A\\
&+|\tau_1\rangle_{A_1}|\mu_1^{(1)}\rangle_{A_2}(\alpha_1\mathrm{e}^{i\delta_1}|0\rangle+\alpha_0|1\rangle\\
&+\alpha_3\mathrm{e}^{i\delta_3}|2\rangle+\alpha_2\mathrm{e}^{i\delta_2}|3\rangle)_A\\
&+|\tau_1\rangle_{A_1}|\mu_2^{(1)}\rangle_{A_2}(\alpha_1\mathrm{e}^{i\delta_1}|0\rangle+\alpha_0|1\rangle\\
&-\alpha_3\mathrm{e}^{i\delta_3}|2\rangle-\alpha_2\mathrm{e}^{i\delta_2}|3\rangle)_A\\
&+|\tau_1\rangle_{A_1}|\mu_3^{(1)}\rangle_{A_2}(\alpha_1\mathrm{e}^{i\delta_1}|0\rangle-\alpha_0|1\rangle\\
&-\alpha_3\mathrm{e}^{i\delta_3}|2\rangle+\alpha_2\mathrm{e}^{i\delta_2}|3\rangle)_A\\
&+|\tau_2\rangle_{A_1}|\mu_0^{(2)}\rangle_{A_2}(\alpha_2\mathrm{e}^{i\delta_2}|0\rangle\\
&-\alpha_3\mathrm{e}^{i\delta_3}|1\rangle-\alpha_0|2\rangle+\alpha_1\mathrm{e}^{i\delta_1}|3\rangle)_A\\
&+|\tau_2\rangle_{A_1}|\mu_1^{(2)}\rangle_{A_2}(\alpha_2\mathrm{e}^{i\delta_2}|0\rangle+\alpha_3\mathrm{e}^{i\delta_3}|1\rangle\\
&-\alpha_0|2\rangle-\alpha_1\mathrm{e}^{i\delta_1}|3\rangle)_A\\
&+|\tau_2\rangle_{A_1}|\mu_2^{(2)}\rangle_{A_2}(\alpha_2\mathrm{e}^{i\delta_2}|0\rangle+\alpha_3\mathrm{e}^{i\delta_3}|1\rangle\\
&+\alpha_0|2\rangle+\alpha_1\mathrm{e}^{i\delta_1}|3\rangle)_A\\
&+|\tau_2\rangle_{A_1}|\mu_3^{(2)}\rangle_{A_2}(\alpha_2\mathrm{e}^{i\delta_2}|0\rangle-\alpha_3\mathrm{e}^{i\delta_3}|1\rangle\\
&+\alpha_0|2\rangle-\alpha_1\mathrm{e}^{i\delta_1}|3\rangle)_A\\
&+|\tau_3\rangle_{A_1}|\mu_0^{(3)}\rangle_{A_2}(\alpha_3\mathrm{e}^{i\delta_3}|0\rangle+\alpha_2\mathrm{e}^{i\delta_2}|1\rangle\\
&-\alpha_1\mathrm{e}^{i\delta_1}|2\rangle-\alpha_0|3\rangle)_A\\
&+|\tau_3\rangle_{A_1}|\mu_1^{(3)}\rangle_{A_2}(\alpha_3\mathrm{e}^{i\delta_3}|0\rangle-\alpha_2\mathrm{e}^{i\delta_2}|1\rangle\\
&-\alpha_1\mathrm{e}^{i\delta_1}|2\rangle+\alpha_0|3\rangle)_A\\
&+|\tau_3\rangle_{A_1}|\mu_2^{(3)}\rangle_{A_2}(\alpha_3\mathrm{e}^{i\delta_3}|0\rangle-\alpha_2\mathrm{e}^{i\delta_2}|1\rangle\\
&+\alpha_1\mathrm{e}^{i\delta_1}|2\rangle-\alpha_2|3\rangle)_A\\
&+|\tau_3\rangle_{A_1}|\mu_3^{(3)}\rangle_{A_2}(\alpha_3\mathrm{e}^{i\delta_3}|0\rangle+\alpha_2\mathrm{e}^{i\delta_2}|1\rangle\\
&+\alpha_1\mathrm{e}^{i\delta_1}|2\rangle+\alpha_0|3\rangle)_A].\end{aligned}\tag{17}$$

Now let Victor perform single-particle FD projective measurements on his own particles A_1 and A_2 under the bases $\{|\tau_k\rangle\}\,(k=0,1,2,3)$ and $\{|\mu_q^{(k)}\rangle\}\,(q,k=0,1,2,3)$ respectively, and publicly announce the results of his measurement. In accord with the outcomes of Victor's measurement, Alice can recover the original state $|\psi\rangle$ by a suitable unitary operation. For example, without loss of generality, we assume that Victor's first measurement result is $|\tau_2\rangle_{A_1}$, and he should employ the measuring bases $\{|\mu_q^{(2)}\rangle\}\,(q=0,1,2,3)$ to measure the particle A_2. If his second measurement outcome is $|\mu_1^{(2)}\rangle_{A_2}$, by eq. (17), the particle A will be collapsed into the state

$$\begin{aligned}|p\rangle=\frac{1}{4}(&\alpha_2\mathrm{e}^{i\delta_2}|0\rangle+\alpha_3\mathrm{e}^{i\delta_3}|1\rangle-\alpha_0|2\rangle\\
&-\alpha_1\mathrm{e}^{i\delta_1}|3\rangle)_A.\end{aligned}\tag{18}$$

Table 1. The relation between two single-particle measurement results $|\tau_k\rangle_{A_1}(k=0,1,2,3)$ and $|\mu_q^{(k)}\rangle_{A_2}(k,q=0,1,2,3)$ performed by Victor and the unitary operations u_A performed by Alice. MR denotes the result of measurement on the particles A_1 and A_2 performed by Victor; I is the 2×2 identity matrix and $\sigma_x,\sigma_y,\sigma_z$ are the Pauli matrices.

MR	u_A	MR	u_A
$\|\tau_0\rangle_{A_1}\|\mu_0^{(0)}\rangle_{A_2}$	$\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$	$\|\tau_2\rangle_{A_1}\|\mu_0^{(2)}\rangle_{A_2}$	$\begin{pmatrix} 0 & -\sigma_z \\ \sigma_z & 0 \end{pmatrix}$
$\|\tau_0\rangle_{A_1}\|\mu_1^{(0)}\rangle_{A_2}$	$\begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}$	$\|\tau_2\rangle_{A_1}\|\mu_1^{(2)}\rangle_{A_2}$	$\begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$
$\|\tau_0\rangle_{A_1}\|\mu_2^{(0)}\rangle_{A_2}$	$\begin{pmatrix} \sigma_z & 0 \\ 0 & -\sigma_z \end{pmatrix}$	$\|\tau_2\rangle_{A_1}\|\mu_2^{(2)}\rangle_{A_2}$	$\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
$\|\tau_0\rangle_{A_1}\|\mu_3^{(0)}\rangle_{A_2}$	$\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$	$\|\tau_2\rangle_{A_1}\|\mu_3^{(2)}\rangle_{A_2}$	$\begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix}$
$\|\tau_1\rangle_{A_1}\|\mu_0^{(1)}\rangle_{A_2}$	$\begin{pmatrix} -i\sigma_y & 0 \\ 0 & -i\sigma_y \end{pmatrix}$	$\|\tau_3\rangle_{A_1}\|\mu_0^{(3)}\rangle_{A_2}$	$\begin{pmatrix} 0 & -\sigma_x \\ \sigma_x & 0 \end{pmatrix}$
$\|\tau_1\rangle_{A_1}\|\mu_1^{(1)}\rangle_{A_2}$	$\begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_x \end{pmatrix}$	$\|\tau_3\rangle_{A_1}\|\mu_1^{(3)}\rangle_{A_2}$	$\begin{pmatrix} 0 & i\sigma_y \\ -i\sigma_y & 0 \end{pmatrix}$
$\|\tau_1\rangle_{A_1}\|\mu_2^{(1)}\rangle_{A_2}$	$\begin{pmatrix} \sigma_x & 0 \\ 0 & -\sigma_x \end{pmatrix}$	$\|\tau_3\rangle_{A_1}\|\mu_2^{(3)}\rangle_{A_2}$	$\begin{pmatrix} 0 & -i\sigma_y \\ -i\sigma_y & 0 \end{pmatrix}$
$\|\tau_1\rangle_{A_1}\|\mu_3^{(1)}\rangle_{A_2}$	$\begin{pmatrix} -i\sigma_y & 0 \\ 0 & i\sigma_y \end{pmatrix}$	$\|\tau_3\rangle_{A_1}\|\mu_3^{(3)}\rangle_{A_2}$	$\begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}$

Alice can perform a unitary operation

$$u_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} \tag{19}$$

on her particle A, where I is the 2×2 identity matrix, and then the copy of the unknown state $|\psi\rangle$ can be obtained at her side. That is, the quantum-assisted cloning has been completed successfully. The relation of two single-particle measurement results, $\{|\tau_k\rangle_{A_1}\}$ and $\{|\mu_q^{(k)}\rangle_{A_2}\}$ $(k,q=0,1,2,3)$, performed by Victor on the particles A_1 and A_2 and the unitary operations u_A performed by Alice on the particle A is given in table 1. It is easily found that, in the present scheme, the total success probability for assisted cloning of a perfect copy of the unknown state can reach 1, that is to say, the scheme is deterministic.

3. Conclusion

In conclusion, we have proposed a scheme which can produce perfect copy of unknown single-particle FD quantum state with assistance. In the scheme, the sender Alice wishes to teleport an unknown single-particle FD state, from the state preparer Victor, to the receiver Bob, and then create a perfect copy of the unknown state at her side. To help Alice realise the quantum cloning, Victor will perform single-particle projective measurements on his own particles. According to the public announcements of the results of measurement from Victor, the sender Alice can acquire a perfect copy of the unknown state with unit success probability. We hope that our schemes will be helpful in the deeper understanding of the process of quantum-assisted cloning.

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References

[1] C H Bennett, G Brassard, C Crépeau, R Jozsa, A Peres and W K Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993)
[2] C H Bennett and S J Wiesner, *Phys. Rev. Lett.* **69**, 2881 (1992)
[3] A K Ekert, *Phys. Rev. Lett.* **67**, 661 (1991)
[4] M Hillery and V Bužek, *Phys. Rev. A* **56**, 1212 (1997)
[5] H B Chen, H Fu, X W Li, P C Ma and Y B Zhan, *Pramana – J. Phys.* **86**, 783 (2016)
[6] H Fu, P C Ma, G B Chen, X W Li and Y B Zhan, *Pramana – J. Phys.* **88**, 92 (2017)
[7] W K Wootters and W H Zurek, *Nature (London)* **299**, 802 (1982)
[8] M A Nielsen and I J Chuang, *Quantum computation and quantum information* (Cambridge University Press, Cambridge, 2000)
[9] V Bužek and M Hillery, *Phys. Rev. A* **54**, 1844 (1996)

[10] L M Duan and G C Guo, *Phys. Rev. Lett.* **80**, 4999 (1998)
[11] V Scarani, S Iblisdir, N Gisin and A Acin, *Rev. Mod. Phys.* **77**, 1225 (2005)
[12] H Fan *et al*, *Phys. Rep.* **544**, 241 (2014)
[13] A K Pati, *Phys. Rev. A* **61**, 022308 (2000)
[14] Y B Zhan, *Phys. Lett. A* **336**, 317 (2005)
[15] M Fang, Y M Liu, J Liu, S H Shi and Z J Zhang, *Commun. Theor. Phys.* **46**, 849 (2006)
[16] Y B Zhan, *Int. J. Theor. Phys.* **44**, 1541 (2005)
[17] J Shi and Y B Zhan, *Int. J. Theor. Phys.* **46**, 3331 (2007)
[18] P C Ma and Y B Zhan, *Int. J. Quantum Inf.* **6**, 159 (2008)
[19] K Hou and S H Shi, *Int. J. Theor. Phys.* **48**, 167 (2009)
[20] X F Xiu, *Int. J. Theor. Phys.* **48**, 2559 (2009)
[21] X Q Xiao and J M Liu, *Quantum Inf. Process*. **10**, 567 (2011)
[22] S Y Ma, M X Luo and Y Su, *Chin. Phys. B* **22**, 110304 (2013)
[23] Z Y Wang, H Yuan and S H Shi, *Physica A* **374**, 103 (2007)
[24] Y B Zhan and P C Ma, *Mod. Phys. Lett. B* **21**, 1847 (2007)
[25] P C Ma, Y B Zhan and L L Zhang, *Int. J. Quantum Inf.* **6**, 535 (2008)
[26] P C Ma and Y B Zhan, *Commun. Theor. Phys.* **51**, 57 (2009)
[27] P C Ma and Y B Zhan, *Int. J. Quantum Inf.* **7**, 653 (2009)
[28] X B Chen *et al*, *Commun. Theor. Phys.* **53**, 1067 (2010)