Pramana – J. Phys. (2018) 90:51
https://doi.org/10.1007/s12043-018-1541-1

Nucleus-acoustic shock waves in white dwarfs

S JANNAT* and A A MAMUN

Department of Physics, Jahangirnagar University, Savar, Dhaka 1342, Bangladesh
*Corresponding author. E-mail: saudiajannat@gmail.com

MS received 4 August 2017; revised 3 November 2017; accepted 9 November 2017; published online 9 March 2018

Abstract. The nucleus-acoustic shock waves (NASWs) propagating in a white dwarf plasma system, which contain non-relativistically or ultrarelativistically degenerate electrons, non-relativistically degenerate, viscous fluid of light nuclei, and immobile nuclei of heavy elements, have been theoretically investigated. We have used the reductive perturbation method, which is valid for small but finite-amplitude NASWs to derive the Burgers equation. The NASWs are, in fact, associated with the nucleus-acoustic (NA) waves in which the inertia is provided by the light nuclei, and restoring force is provided by the degenerate pressure of electrons. On the other hand, the stationary heavy nuclei participate only in maintaining the background charge neutrality condition at equilibrium. It is found that the viscous force acting in the fluid of light nuclei is a source of dissipation, and is responsible for the formation of NASWs. It is also observed that the basic features (polarity, amplitude, width, etc.) of the NASWs are significantly modified by the presence of heavy nuclei, and that NASWs are formed with either positive or negative potential depending on the values of the charge density of the heavy nuclei. The basic properties are also found to be significantly modified by the effects of ultrarelativistically degenerate electrons. The implications of our results in white dwarfs are briefly discussed.

Keywords. Acoustic waves; degenerate; shock waves; white dwarfs.

PACS Nos **52.35.Fp; 52.35.Tc; 97.20.Rp**

Nowadays, astrophysical compact objects have received a great deal of attention in understanding the electrostatic nonlinear phenomena. A degenerate plasma system is a collection of free and non-interacting particle species with extremely high density and low temperature. The astrophysical compact objects like white dwarfs [1–3] are one of the most suitable examples of degenerate plasma systems. These compact objects have ceased burning thermonuclear fuel, and so they no longer generate thermal pressure. These objects can exist there against the gravitational collapse due to the degenerate pressure of electron. Their basic properties are completely different from that in terrestrial environments. The space-experimental observations and theoretical analysis imply that these compact objects are of two categories in which white dwarfs are of the first category whose interior is close to a dense solid surrounded by degenerate electrons, non-degenerate light nuclei, and immobile heavy nuclei [2,3]. Chandrasekhar [4] assumed that the core of white dwarfs contains pure helium (He: 4 nucleons) nuclei. After that, Koester [3] noticed that the core of white dwarfs contains carbon (C: 12 nucleons) or oxygen (O: 16 nucleons) nuclei (instead of helium) as well as nuclei of heavy elements like rubidium (Rb: 86 nucleons), and that the electron species is relativistically degenerate only within the inner core of the white dwarfs, but is non-relativistically degenerate in their outer mantle [5]. Recent work of Garcia-Berro *et al* [6] implies that the main constituents of compact objects (e.g. white dwarfs) are degenerate electrons and nuclei of light as well as heavy elements. The typical parameters for the white dwarfs [1–3,7,8] (containing non- or ultrarelativistically degenerate electron species and non-relativistically degenerate species of light ($^{12}_{6}$C) and heavy ($^{86}_{37}$Rb) nuclei) are tabulated in table 1.

We also note that the number density (n_{h0}) of the heavy nuclear species is determined by the charge neutrality condition, i.e. by $n_{h0} = (n_{e0} - n_{p0} - Z_l n_{l0})/Z_h$, where Z_l (Z_h) is the number of protons in a light (heavy) nucleus. The degeneracy of electrons and light nuclei arises due to Heisenberg's uncertainty principle. This indicates that the momentum of a highly compressed particle is extremely uncertain, because the particles are located in a very confined space. Therefore, even though the plasma is very cold, the plasma particles must move very fast on an average, and gives rise to very high

Table 1. Typical parameters for white dwarfs [1–3,7].

Parameters	Numerical values
m_l ($^{12}_{6}$C)	$12m_p$
m_h ($^{86}_{37}$Rb)	$86m_p$
n_{e0}	10^{29} cm^{-3}
n_{l0}	1.666×10^{28} cm^{-3}

pressure, known as degenerate pressure, which does not depend on thermal temperature, but on degenerate particle number density. About 85 years ago, Chandrasekhar [9] first recognised that the outward pressure of white dwarfs counterbalances the inward pull of their gravity, and that the outward pressure is provided by the degenerate pressure of plasma particles. The pressure associated with electron and light nuclei can be expressed as $P_j = K_j n_j^{\alpha} = K_j n_j^{\gamma}$, where $j = e$ for electron and $j = l$ for light nuclei; n_j is the number density of electrons or light nuclei; α, γ, and K_j can be defined as $\alpha = \gamma = 5/3$ and $K_j = (3/5)\lambda_c \hbar c$ (with $\lambda_c = \pi\hbar/m_j c$) for the non-relativistic limit; on the other hand $\alpha = 5/3$, $\gamma = 4/3$, and $K_j = (3/4)\hbar c$ for ultrarelativistic limit [5,10–12], where $\Lambda_{cj} = \pi\hbar/m_j c$ and $\hbar$ is the Planck's constant h divided by 2π. Interstellar compact objects like white dwarfs, provide some kind of cosmic laboratories for studying not only the properties of matter in them, but also for investigating the propagation of waves and their instabilities [5,13–16] in such a dense degenerate plasma medium for which the quantum [5] effect is important. Recent discovery [17] of gravitational waves emitted by two merging black holes has opened up a new era of theoretical and observational research in astrophysics [17–19] which leads us to expect that in the near future a similar or different kind of waves (like nucleus-acoustic (NA) waves [20,21]) and associated nonlinear structures, like solitons, shocks, vortices, etc. will be detected from other astrophysical compact objects like white dwarfs. It is important to mention here that in astrophysical compact objects like white dwarfs the degenerate pressure, which provides the restoring force in the NA waves, is the most important pressure to make stable (against gravitational collapse) such astrophysical compact objects [9]. The NA waves can also be termed as 'degenerate pressure-driven NA waves'. This is because in the NA waves, the inertia is provided by the mass density of light nucleus species (which is much greater than that of the heavy nucleus species). This means that the NA waves and their consequences are very important from the view of observing the new waves and associated nonlinear structures in astrophysical compact objects [17–19]. It should be further noted here that our present work is different from the recent works of Mamun *et al* [20,21] from the point of view of both degenerate plasma model considered and the results obtained.

The nonlinear dynamics of our present NA waves in such a white dwarf plasma system is described by

$$\frac{\partial n_l}{\partial t} + \frac{\partial}{\partial x}(n_l u_l) = 0, \tag{1}$$

$$\frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_l}\frac{\partial n_l^{\alpha}}{\partial x} = \eta \frac{\partial^2 u_l}{\partial x^2}, \tag{2}$$

$$\frac{\partial \phi}{\partial x} - \frac{K_2}{n_e}\frac{\partial n_e^{\gamma}}{\partial x} = 0, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1+\mu_h)n_e - n_l - \mu_h, \tag{4}$$

where n_s is the number density of the species s, and is normalised by n_{s0} where n_{s0} is the equilibrium number density of the species s ($s = l$ for degenerate light nuclear species, $s = e$ for degenerate electron species, $s = h$ for stationary heavy nuclear species); u_s is the speed of degenerate light nuclear fluid, and is normalised by $C_l = (m_e c^2/m_l)^{1/2}$ ($m_e(m_l)$ being the electron (light nuclei) rest mass and c being the speed of light in vacuum); ϕ is the electrostatic wave potential, and is normalised by $m_e c^2/e$ (e being the magnitude of the charge of an electron); time variable t is normalised by $\omega_{pl}^{-1} = (m_l/(4\pi n_{l0} e^2))^{1/2}$, and space variable x is normalised by $\lambda_m = (m_e c^2/4\pi e^2 n_{l0})^{1/2}$; $\mu_h = Z_h n_{h0}/Z_l n_{l0}$, $K_1 = n_{l0}^{\alpha-1} K_l/m_e c^2$, and $K_2 = n_{e0}^{\gamma-1} K_e/m_e c^2$. We should mention here that the assumption of stationary heavy nucleus is valid when $\omega \gg \omega_{ph}$, where ω (ω_{ph}) is the frequency of the NA waves (heavy nucleus plasma). We note that in our plasma model, electron species are assumed to be either non-relativistically or ultrarelativistically degenerate, but light nuclei are assumed to be non-relativistically degenerate. This is due to the fact that the mass of an electron is much less than that of a light nucleus (at least 1836 times), and the number density of the electron species is higher than that of the light nucleus species depending on the presence of heavy nucleus.

To derive the Burgers equation for the NA waves propagating in a white dwarf plasma system, first we introduce the stretched coordinates [22]:

$$\xi = \epsilon(x - V_p t),$$
$$\tau = \epsilon^2 t, \tag{5}$$

where V_p is the speed of the wave phase (ω/k with k being the wave number of the NA waves) and ϵ is a smallness parameter measuring the weakness of the dissipation ($0 < \epsilon < 1$), then expand n_l, n_e, u_l, and ϕ in power series of ϵ as follows:

$$n_l = 1 + \epsilon n_l^{(1)} + \epsilon^2 n_l^{(2)} + \epsilon^3 n_l^{(3)} + \cdots, \tag{6}$$

$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \epsilon^3 n_e^{(3)} + \cdots, \tag{7}$$

$$u_l = \epsilon u_l^{(1)} + \epsilon^2 u_l^{(2)} + \epsilon^3 u_l^{(3)} + \cdots, \tag{8}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \cdots. \tag{9}$$

Now, expressing (1)–(4) in terms of ξ and τ by using (5), and then substituting (6)–(9), we can easily develop different sets of equations in various powers of ϵ. To the lowest order in ϵ, we have

$$u_l^{(1)} = \frac{V_p \phi^{(1)}}{(V_p^2 - K_1\alpha)}, \tag{10}$$

$$n_l^{(1)} = \frac{\phi^{(1)}}{(V_p^2 - K_1\alpha)}, \tag{11}$$

$$n_e^{(1)} = \frac{\phi^{(1)}}{K_2\gamma}, \tag{12}$$

and

$$V_p = \sqrt{K_1\alpha + \frac{K_2\gamma}{(1+\mu_h)}}. \tag{13}$$

To the next higher order in ϵ, we obtain another set of nonlinear equations:

$$\frac{\partial n_l^{(1)}}{\partial \tau} - V_p \frac{\partial n_l^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi}\left[u_l^{(2)} + n_l^{(1)} u_l^{(1)}\right] = 0, \tag{14}$$

$$\frac{\partial u_l^{(1)}}{\partial \tau} - V_p \frac{\partial u_l^{(2)}}{\partial \xi} + u_l^{(1)} \frac{\partial u_l^{(1)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} + K_1\alpha \frac{\partial}{\partial \xi}\left[n_l^{(2)} + \frac{(\alpha-2)}{2}\left(n_l^{(1)}\right)^2\right] = 0, \tag{15}$$

$$\frac{\partial \phi^{(2)}}{\partial \xi} - K_2\gamma \frac{\partial}{\partial \xi}\left[n_e^{(2)} + \frac{(\gamma-2)}{2}\left(n_e^{(1)}\right)^2\right] = 0, \tag{16}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - (1+\mu_h) n_e^{(2)} + n_l^{(2)} = 0. \tag{17}$$

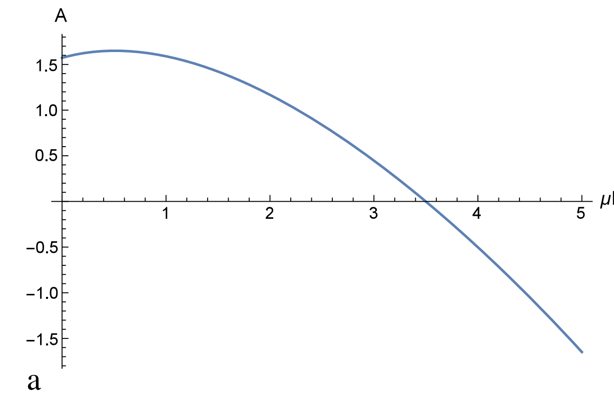
We finally eliminate n_l^2, u_l^2, n_e^2, and ϕ^2 to obtain the Burgers equation:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \tag{18}$$

where the nonlinear coefficient A and dissipative coefficient C are given by

$$A = \frac{1}{2V_p}\left[(1+\mu_h)(\gamma-2) + \frac{3V_p^2}{V_p^2 - K_1\alpha} + \frac{K_1\alpha(\alpha-2)}{V_p^2 - K_1\alpha}\right],$$

$$C = \frac{\eta}{2}.$$



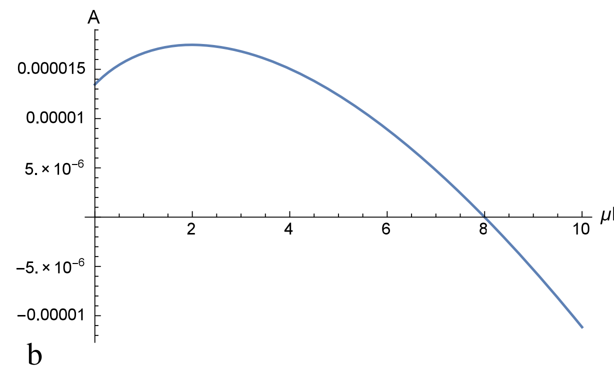


Figure 1. Variation of A with μ_h (**a**) for non-relativistically degenerate electron species and (**b**) for ultrarelativistically degenerate electron species for $\alpha = 5/3$.

Now we look for stationary shock wave solution of this Burgers equation by considering a moving frame $\zeta = \xi - U_0\tau$, where ζ (U_0) is normalised by λ_D (C_l), and apply the appropriate boundary conditions, viz. $\phi \to 0$, $\mathrm{d}\phi/\mathrm{d}\zeta \to 0$, $\mathrm{d}^2\phi/\mathrm{d}\zeta^2 \to 0$ at $\zeta \to \pm\infty$. These allow us to write the stationary shock wave solution as

$$\phi = \phi_m\left[1 - \tanh\left(\frac{\zeta}{\Delta}\right)\right], \tag{19}$$

where ϕ_m (height normalised by $m_e c^2/e$) and Δ (thickness normalised by λ_D) are given by

$$\phi_m = \frac{U_0}{A} \quad \text{and} \quad \Delta = \frac{2C}{U_0}. \tag{20}$$

It is clear from (19) and (20) that NASWs exist, which are formed due to the balance between nonlinearity and dissipation, because $C > 0$ and that the NASWs with $\phi > 0$ ($\phi < 0$) exist if $A > 0$ ($A < 0$) because $U_0 > 0$. Thus, we have numerically shown the parametric regimes corresponding to $A > 0$ ($A < 0$). These are displayed in figure 1 which implies that the NASWs exist with $\phi < 0$ when $\mu > 8$ ($\mu > 3.501$) for non-relativistically (ultrarelativistically) DES.

The NASWs with $\phi > 0$ and $\phi < 0$ for ultrarelativistically and non-relativistically degenerate electron

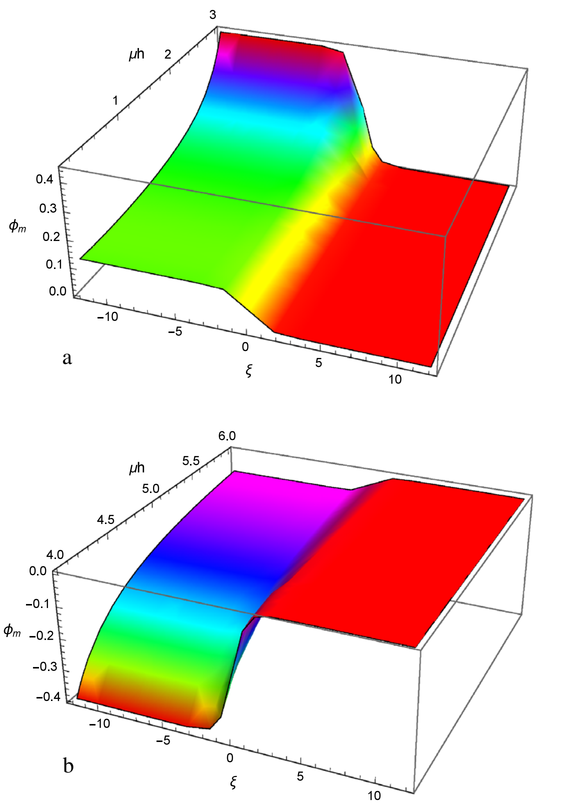


Figure 2. Variation of the effect of μ_h on ξ and ϕ of NASWs (**a**) for $\mu_h < \mu_c$ and (**b**) for $\mu_h > \mu_c$ when electrons are in the ultrarelativistic state.

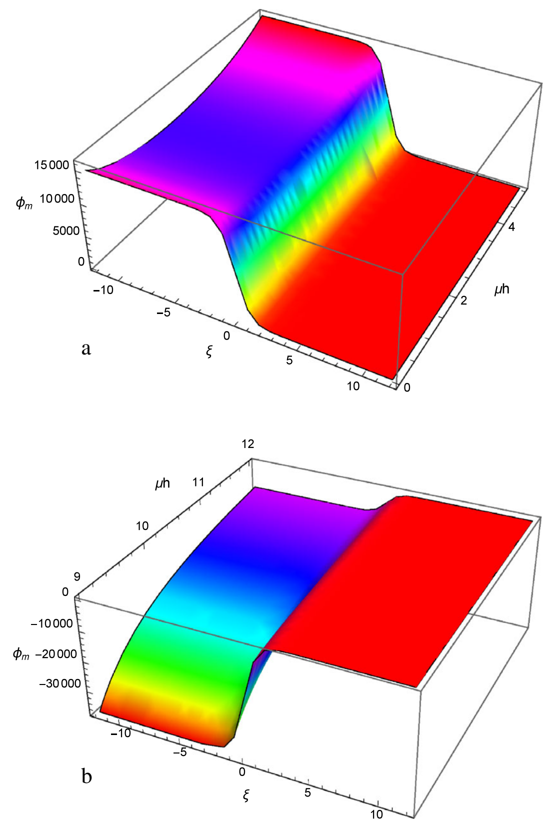


Figure 3. Variation of the effect of μ_h on ξ and ϕ of NASWs (**a**) for $\mu_h < \mu_c$ and (**b**) for $\mu_h > \mu_c$ when electrons are in the non-relativistic state.

species are shown in figures 2 and 3, respectively. It is obvious from figure 2a that the magnitude of the amplitude (width) of the NASWs associated with $\phi > 0$ increases (decreases) with the increase in μ_h. On the other hand, from figure 2b it is clear that the magnitude of the amplitude (width) of the NASWs associated with $\phi < 0$ increases (increases) with the increase in μ_h for ultrarelativistically degenerate electron species. Similarly, it is obvious from figure 3a that the magnitude of the amplitude (width) of the NASWs associated with $\phi > 0$ first decreases (increases) and after that increases (decreases) with the increase in μ_h and from figure 3b, the magnitude of the amplitude (width) of the NASWs associated with $\phi < 0$ increases (increases) with the increase in μ_h for non-relativistically degenerate electron species. We have considered $U_0 = 0.01$ and $\eta = 0.1$ for our numerical analysis. To sum up, we have considered a new white dwarf plasma system containing non/ultrarelativistically degenerate electrons, degenerate light nuclei, and immobile nuclei of heavy elements and have studied the formation of the NASWs, and their basic properties. The results that we have found from this investigation can be summarised as follows:

1. The white dwarf plasma system under consideration supports small but finite-amplitude NASWs with either positive or negative potentials depending on the values of μ_h.
2. The basic features (polarity, amplitude, width, etc.) of the NASWs are significantly modified by the presence of nuclei of heavy elements.
3. The magnitude of the amplitude (width) of the NASWs associated with $\phi > 0$ increases (decreases) with the increase in μ_h while the amplitude (width) of the NASWs associated with $\phi < 0$ increases (increases) with the increase in μ_h for ultrarelativistically degenerate electron species.
4. The magnitude of the amplitude (width) of the NASWs associated with $\phi > 0$ first decreases (decreases) and after that increases (decreases) with the increase in μ_h.
5. The magnitude of the amplitude (width) of the NASWs associated with $\phi < 0$ increases (increases) with the increase in μ_h for non-relativistically degenerate electron species.
6. The NASWs potential is always higher when the electrons are in the ultrarelativistic state than in the non-relativistic state.

7. The NASWs with positive (negative) potential exist for $\mu_h < \mu_c$ ($\mu_h > \mu_c$).

To conclude, we consider the NASWs in degenerate plasma system containing non-relativistically or ultra-relativistically degenerate electrons, non-relativistically degenerate light nuclei, and static nuclei of heavy elements. The results of the present investigation will be helpful for understanding the nonlinear features of electrostatic disturbances in astrophysical compact objects like white dwarf plasmas, where the effects of dissipation, higher-order nonlinearity, degenerate pressure, and positively charged immobile heavy nuclei plays an important role. It should be mentioned here that the equilibrium state of the NASWs is not defined by our basic equations. If we want to explain the equilibrium state of the NASWs, then we have to rewrite the basic equations which should be valid only at equilibrium state. At equilibrium, the force associated with electrostatic and degenerate pressures is balanced by the self-gravitational force. However, if we neglect net surface charge density of plasma species, at equilibrium the degenerate pressure force is balanced by the gravitational force. This type of work gives us the opportunity to explain the signature of the existence of shock signals observed by space experiments (in the near future) in astrophysical compact objects like white dwarfs.

References

[1] S L Shapiro and S A Teukolsky, *Black holes, white dwarfs, and neutron stars: The physics of compact objects* (Wiley-VCH Verlag, Weinheim, 2004)
[2] D Koester and G Chanmugam, *Rep. Prog. Phys.* **53**, 837 (1990)
[3] D Koester, *Astron. Astrophys. Rev.* **11**, 33 (2002)
[4] S Chandrasekhar, *Philos. Mag.* **11**, 592 (1931)
[5] P K Shukla and B Eliasson, *Phys. Usp.* **53**, 51 (2010); *Rev. Mod. Phys.* **83**, 885 (2011)
[6] Garcia-Berro, S Torres and L G Althaus, *Nature* **465**, 194 (2010)
[7] A Vanderburg, J A Johnson, S Rappaport, A Bieryla, J Irwin, J A Lewis, D Kipping, W R Brown, P Dufour, D R Ciardi, R Angus, L Schaefer, D W Latham, D Charbonneau, C Beichman, J Eastman, N McCrady, R A Wittenmyer and J T Wright, *Nature* **526**, 546 (2015)
[8] M Amina, S A Ema and A A Mamun, *Pramana–J. Phys.* **81**, 88 (2017)
[9] S Chandrasekhar, *Astrophys. J.* **74**, 81 (1931)
[10] A A Mamun and P K Shukla, *Phys. Plasmas* **17**, 104504 (2010); *Phys. Lett.* **374**, 4238 (2010)
[11] P K Shukla, A A Mamun and D A Mendis, *Phys. Rev. E* **84**, 026405 (2011)
[12] A A Mamun, P K Shukla and D A Mendis, *J. Plasma Phys.* **78**, 143 (2012)
[13] P K Shukla and B Eliasson, *Phys. Rev. Lett.* **96**, 245001 (2006)
[14] G Brodin and M Marklund, *Phys. Plasmas* **14**, 112107 (2007)
[15] M Marklund, B Eiasson and P K Shukla, *Phys. Rev. E* **76**, 067401 (2007)
[16] P K Shukla, *Nat. Phys.* **5**, 92 (2009)
[17] B P Abbott, R Abbott, T D Abbott, M R Abernathy, F Acernese, K Ackley, C Adams, T Adams, P Addesso, R X Adhikari, V B Adya, C Affeldt, M Agathos, K Agatsuma, N Aggarwal, O D Aguiar *et al*, *Phys. Rev. Lett.* **116**, 061102 (2016)
[18] A Kurkela and A Vuorinen, *Phys. Rev. Lett.* **117**, 042501 (2016)
[19] W C G Ho, *Mon. Not. R. Astron. Soc.* **463**, 489 (2016)
[20] A A Mamun, M Amina and R Schlickeiser, *Phys. Plasmas* **23**, 094503 (2016)
[21] A A Mamun, M Amina and R Schlickeiser, *Phys. Plasmas* **24**, 042307 (2017)
[22] H Washimi and T Taniuti, *Phys. Rev. Lett.* **17**, 996 (1966)