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Bäcklund transformation and soliton solutions in terms of the Wronskian for the Kadomtsev–Petviashvili-based system in fluid dynamics

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Abstract. In this paper, investigation is made on a Kadomtsev–Petviashvili-based system, which can be seen in fluid dynamics, biology and plasma physics. Based on the Hirota method, bilinear form and Bäcklund transformation (BT) are derived. N-soliton solutions in terms of the Wronskian are constructed, and it can be verified that the N-soliton solutions in terms of the Wronskian satisfy the bilinear form and Bäcklund transformation. Through the N-soliton solutions in terms of the Wronskian, we graphically obtain the kink-dark-like solitons and parallel solitons, which keep their shapes and velocities unchanged during the propagation.

Keywords. Fluid dynamics; Kadomtsev–Petviashvili-based system; Wronskian; Hirota method; soliton solutions; Bäcklund transformation.

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1. Introduction

Nonlinear evolution equations (NLEEs) have been studied in fields such as the fluid dynamics, chemistry, biology, plasma physics, nonlinear optics, and condensed matter physics [1–6]. Efforts have been dedicated to find analytic solutions for the NLEEs, including the soliton, periodic and rational solutions [1–3,7]. Methods such as the inverse scattering method [4], Painlevé analysis method [9], variable separation method [9], Bäcklund transformation (BT) [10], Hirota method [11,12] and Darboux transformation [13] have been developed to derive analytic solutions. Based on the bilinear forms for some NLEEs, e.g., the Korteweg–de Vries, Kadomtsev–Petviashvili (KP) and breaking-soliton equations, the Wronskian technique has been developed to derive the N-soliton solutions in terms of the Wronskian [14–22].

The KP equation [23,24],

$$(-4u_t + 6uu_x + u_{xxx})_x + 3u_{yy} = 0, \tag{1}$$

has been used to describe the evolution of nonlinear, long waves of small amplitude with slow dependence on the transverse coordinate, where u is a real function with respect to two scaled space coordinates x, y and retarded time t, and the subscripts denote the partial derivatives.

Through the inner parameter-dependent symmetry constraints $H_y = u_{xx} - 2uu_x - 2V_x$ and $V_y = -V_{xx} - 2(uV)_x$ on eq. (1), an integrable (2 + 1)-dimensional KP-based system can be obtained [25–28], as follows:

$$H_t = -4[H_{xx} + H^3 - 3HH_x + 3H\partial_y^{-1}V_x + 3\partial_y^{-1}(VH)_x]_x, \tag{2a}$$

$$V_t = -4(V_{xx} + 3HV_x + 3H^2V + 3V\partial_y^{-1}V_x)_x, \tag{2b}$$

where H and V are the functions with respect to x, y and t. System (2) has also been proved to be an extension of the Whitham–Broer–Kaup system for the long waves by incorporating or mimicking the convective, dispersive and viscous effects [27]. Infinitely many Lax pairs for system (2) has been given in [25]. Kink soliton solutions, periodic soliton solutions and lattice soliton solutions for system (2) have been discussed in [26]. V-shaped and A-shaped soliton fusion and Y-shaped soliton fission phenomena for system (2) have been reported in [27]. Soliton-like solutions and periodic solutions for system (2) have been obtained in [28].

However, Bäcklund transformation (BT) in the bilinear form and soliton solutions in terms of the Wronskian for system (2) have not been derived. Motivated by the above, we shall derive the bilinear form, BT for system (2) using the Hirota method in §2. Using the Wronskian technique, N-soliton solutions in terms of the Wronskian for system (2) will be constructed and verified in §3. Graphically, evolution of the one-soliton and parallel solitons will be presented. Finally, we shall work out our conclusions in §4.

2. Bilinear form and Bäcklund transformation

Through the transformations $W_y = V_x$, $P_y = (VH)_x$ [26], system (2) can be written as follows:

$$H_t + 4(H_{xx} + H^3 - 3HH_x + 3HW + 3P)_x = 0, \tag{3a}$$

$$V_t + 4(V_{xx} + 3V_x H + 3H^2 V + 3VW)_x = 0, \tag{3b}$$

$$W_y - V_x = 0, \quad P_y - (VH)_x = 0. \tag{3c}$$

In order to obtain the bilinear form for eqs (3), we take the following dependent variable transformations:

$$H = (\log f)_x, \tag{4a}$$

$$V = (\log f)_{xy}, \tag{4b}$$

$$W = (\log f)_{xx} = H_x, \tag{4c}$$

$$P = (\log f)_x (\log f)_{xx} = HH_x, \tag{4d}$$

where f is a function of variables x, y and t. Substituting eqs (4) into eqs (3), we obtain

$$\frac{1}{2f^2}[(D_x D_t + D_y D_t + 4D_x^4 + 4D_x^3 D_y) f \cdot f] + 12\frac{1}{f^2} D_x (f_{xx} + f_{xy}) \cdot f_x = 0, \tag{5}$$

and the bilinear form for eqs (3) is obtained as follows:

$$(D_x D_t + D_y D_t + 4D_x^4 + 4D_x^3 D_y) f \cdot f = 0, \tag{6a}$$

$$(D_x + D_y) f \cdot 1 = 0 \Rightarrow \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) f \cdot f = 0, \tag{6b}$$

where D_x, D_y and D_t are the Hirota bilinear derivative operators [11], defined as

$$D_x^m D_y^n D_t^l (\alpha \cdot \beta) \equiv \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^l \times \alpha(x, y, t) \beta(x', y', t') \Big|_{x'=x, y'=y, t'=t},$$

with x', y' and t' being the formal variables, m, n and l being the non-negative integers, while α is a function with respect to x, y and t, and β is a function with respect to x', y' and t'. According to bilinear form (6), we shall derive the BT for eqs (3) between two different solutions $H = (\log f)_x$, $V = (\log f)_{xy}$, $W = (\log f)_{xx}$ and $P = (\log f)_x (\log f)_{xx}$, and $H' = (\log f')_x$, $V' = (\log f')_{xy}$, $W' = (\log f')_{xx}$ and $P' = (\log f')_x (\log f')_{xx}$ for eqs (3). Setting

$$P_1 = [(D_x D_t + D_y D_t + 4D_x^4 + 4D_x^3 D_y) f' \cdot f'] f^2 - [(D_x D_t + D_y D_t + 4D_x^4 + 4D_x^3 D_y) f \cdot f] f'^2, \tag{7a}$$

$$P_2 = \left[\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) f' \cdot f'\right] f^2 - \left[\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) f \cdot f\right] f'^2, \tag{7b}$$

with the following Hirota bilinear operator identities:

$$(D_x D_t f' \cdot f') f^2 - (D_x D_t f \cdot f) f'^2 = 2D_x (D_t f' \cdot f) \cdot (f' f),$$

$$(D_y D_t f' \cdot f') f^2 - (D_y D_t f \cdot f) f'^2 = 2D_y (D_t f' \cdot f) \cdot (f' f),$$

$$D_x (D_y f' \cdot f) \cdot (f' f) = D_y (D_x f' \cdot f) \cdot (f' f),$$

$$(D_x^4 f' \cdot f') f^2 - (D_x^4 f \cdot f) f'^2 = 2D_x (D_x^3 f' \cdot f) \cdot (f' f) - 6D_x (D_x^2 f' \cdot f) \cdot (D_x f' \cdot f),$$

$$(D_x^3 D_y f' \cdot f') f^2 - (D_x^3 D_y f \cdot f) f'^2 = 2D_y (D_x^3 f' \cdot f) \cdot (f' f) - 6D_x (D_x D_y f' \cdot f) \cdot (D_x f' \cdot f),$$

we may rewrite P_1 and P_2 as

$$\begin{aligned} P_1 &= 2D_x (D_t f' \cdot f) \cdot (f' f) + 2D_y (D_t f' \cdot f) \cdot (f' f) + 8D_x (D_x^3 f' \cdot f) \cdot (f' f) - 24\, D_x (D_x^2 f' \cdot f) \cdot (D_x f' \cdot f) \\ &\quad + 8D_y (D_x^3 f' \cdot f) \cdot (f' f) - 24 D_x (D_x D_y f' \cdot f) \cdot (D_x f' \cdot f) \\ &= 2D_x [(D_t + 4D_x^3 + 12\lambda D_x) f' \cdot f] \cdot (f' f) - 24 D_x [(D_x^2 - \lambda) f' \cdot f] \cdot (D_x f' \cdot f) \\ &\quad + 2D_y [(D_t + 4D_x^3 + 12\lambda D_x) f' \cdot f] \cdot (f' f) - 24 D_x [(D_x D_y + \lambda) f' \cdot f] \cdot (D_x f' \cdot f), \end{aligned} \tag{8a}$$

$$P_2 = 2f' f (f'_x f - f' f_x + f'_y f - f' f_y) = 2[(D_x + D_y) f' \cdot f] \cdot (f' f). \tag{8b}$$

The BT for eqs (3) can be displayed as

$$(D_x + D_y) f' \cdot f = 0, \tag{9a}$$

$$(D_x D_y - \mu D_x + \lambda) f' \cdot f = 0, \tag{9b}$$

$$(D_x^2 - \mu D_x - \lambda) f' \cdot f = 0, \tag{9c}$$

$$(D_t + 4D_x^3 + 12\lambda D_x) f' \cdot f = 0, \tag{9d}$$

where both μ and λ are the constants.

In order to get the one-soliton solutions for eqs (3) from BT (9), a seed solution for eqs (3) can be taken as $f = 1$, i.e. $H = 0$, $V = 0$, $W = 0$ and $P = 0$. Letting $\mu = 0$ and $\lambda = k^2$, and substituting the seed solution into BT (9), we obtain

$$f = e^{\xi} + e^{-\xi}, \tag{10}$$

where $\xi = kx - ky - 16k^3 t + \xi^0$, k and ξ^0 are all arbitrary constants. Substituting eq. (10) into eqs (4), we obtain one-soliton solutions for eqs (3), which can be expressed as

$$H = k \tanh \xi, \tag{11a}$$

$$V = -k^2 \mathrm{sech}^2\, \xi, \tag{11b}$$

$$W = k^2 \mathrm{sech}^2 \xi, \tag{11c}$$

$$P = k^3 \mathrm{sech}^2\, \xi \tanh \xi. \tag{11d}$$

3. *N*-Soliton solutions in terms of Wronskian

Suppose that N-soliton solutions in terms of Wronskian for eqs (3) can be defined by

$$F = W(\phi_1, \phi_2, \ldots, \phi_N) = \begin{vmatrix} \phi_1 & \phi_1^{(1)} & \phi_1^{(2)} & \cdots & \phi_1^{(N-1)} \\ \phi_2 & \phi_2^{(1)} & \phi_2^{(2)} & \cdots & \phi_2^{(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_N & \phi_N^{(1)} & \phi_N^{(2)} & \cdots & \phi_N^{(N-1)} \end{vmatrix} = |\widehat{N-1}|, \tag{12}$$

where $\phi_j^i = \partial^i \phi_j / \partial x^i$ $(i = 1, 2, \ldots, N-1)$, $\phi_j = e^{\xi_j} + (-1)^{j+1} e^{-\xi_j}$ and $\xi_j = k_j x - k_j y - 16k_j^3 t + \xi_j^0$ with ξ_j^0 and k_j $(j = 0, 1, \ldots, N)$ being the constants. It can be seen that ϕ_j satisfies the following relations:

$$\phi_{j,y} = -\phi_{j,x}, \tag{13a}$$

$$\phi_{j,xx} = k_j^2 \phi_j, \tag{13b}$$

$$\phi_{j,t} = -16\phi_{j,xxx}. \tag{13c}$$

According to the properties of the derivative of Wronskian, the derivatives of F with respect to the variables x, y and t can be obtained as follows:

$$F = |\widehat{N-1}|, \quad F_x = |\widehat{N-2}, N|,$$

$$F_y = -|\widehat{N-2}, N|,$$

$$F_t = -16(|\widehat{N-4}, N-2, N-1, N| - |\widehat{N-3}, N-1, N+1| + |\widehat{N-2}, N+2|),$$

$$F_{xx} = |\widehat{N-3}, N-1, N| + |\widehat{N-2}, N+1|,$$

$$F_{xy} = -F_{xx} = -(|\widehat{N-3}, N-1, N| + |\widehat{N-2}, N+1|),$$

$$F_{xxx} = |\widehat{N-4}, N-2, N-1, N| + 2|\widehat{N-3}, N-1, N+1| + |\widehat{N-2}, N+2|,$$

$$F_{xxy} = -F_{xxx} = -(|\widehat{N-4}, N-2, N-1, N| + 2|\widehat{N-3}, N-1, N+1| + |\widehat{N-2}, N+2|),$$

$$F_{xt} = -16(|\widehat{N-5}, N-3, N-2, N-1, N| - |\widehat{N-3}, N+1| + |\widehat{N-2}, N+3|),$$

$$F_{yt} = -F_{xt} = 16(|\widehat{N-5}, N-3, N-2, N-1, N| - |\widehat{N-3}, N, N+1| + |\widehat{N-2}, N+3|),$$

$$F_{xxxx} = |\widehat{N-5}, N-3, N-2, N-1, N| + 3|\widehat{N-4}, N-2, N-1, N+1| + 2|\widehat{N-3}, N, N+1| + 3|\widehat{N-3}, N-1, N+2| + |\widehat{N-2}, N+3|,$$

$$F_{xxxy} = -F_{xxxx} = -(|\widehat{N-5}, N-3, N-2, N-1, N| + 3|\widehat{N-4}, N-2, N-1, N+1| + 2|\widehat{N-3}, N, N+1| + 3|\widehat{N-3}, N-1, N+2| + |\widehat{N-2}, N+3|),$$

where $\widehat{N-K}$ indicates that there are consecutive derivatives up to the order of $N-K+1$. Substituting all the derivatives above F into bilinear form (6), we can obtain

$$\begin{aligned}
&(D_x D_t + D_y D_t + 4D_x^4 + 4D_x^3 D_y) F \cdot F \\
&= 2\{-F_t(F_x + F_y) - 4(-3F_{xy}F_{xx} - 3F_{xx}^2 \\
&\quad + 3F_x F_{xxy} + F_y F_{xxx} + 4F_x F_{xxx}) \\
&\quad + F[F_{yt} + F_{xt} + 4(F_{xxxy} + F_{xxxx})]\} \\
&= 2\{-F_t(F_x - F_x) - 4[-3F_{xx}(-F_{xx} + F_{xx}) \\
&\quad - 3F_x F_{xxx} - F_x F_{xxx} + 4F_x F_{xxx}] \\
&\quad + F[-F_{xt} + F_{xt} + 4(-F_{xxxx} + F_{xxxx})]\} \\
&= 0,
\end{aligned} \tag{14a}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) F \cdot F = 2F(F_x + F_y) = 0. \tag{14b}$$

Thus, we have verified that solutions (12) are solutions for bilinear form (6). Taking $\phi_j = e^{\xi_j} + (-1)^{j+1}e^{-\xi_j}$ and $\xi_j = k_j x - k_j y - 16k_j^3 t + \xi_j^0$ with ξ_j^0 and k_j as some constants, we find that ϕ_j satisfies relations (13). Hence, we can obtain the N-soliton solutions for system (2) in terms of the Wronskian as follows:

$$H = (\log|\widehat{N-1}|)_x, \tag{15a}$$

$$V = (\log|\widehat{N-1}|)_{xy}. \tag{15b}$$

In the following, we shall verify that solutions (12) satisfy BT (9).

The $(N-1)$-soliton solutions for bilinear form (6) have the form of

$$\begin{aligned} F' &= W(\phi_1, \phi_2, \ldots, \phi_{N-1}, \tau) \\ &= \begin{vmatrix} \phi_1 & \phi_1^{(1)} & \phi_1^{(2)} & \cdots & \phi_1^{(N-2)} & 0 \\ \phi_2 & \phi_2^{(1)} & \phi_2^{(2)} & \cdots & \phi_2^{(N-2)} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \phi_{N-1} & \phi_{N-1}^{(1)} & \phi_{N-1}^{(2)} & \cdots & \phi_{N-1}^{(N-2)} & 0 \\ \phi_N & \phi_N^{(1)} & \phi_N^{(2)} & \cdots & \phi_N^{(N-2)} & 1 \end{vmatrix} \\ &= |\widehat{N-2}, \tau|, \end{aligned} \tag{16}$$

where $\tau = (0, 0, \ldots, 0, 1)^T$ and T represents the transpose of a matrix. Then, the derivatives of F' are computed as follows:

$$\begin{aligned}
F' &= |\widehat{N-2}, \tau|, \quad F'_x = |\widehat{N-3}, N-1, \tau|, \\
F'_y &= -F'_x = -|\widehat{N-3}, N-1, \tau|, \\
F'_{xy} &= -|\widehat{N-4}, N-2, N-1, \tau| - |\widehat{N-3}, N, \tau|, \\
F'_{xx} &= |\widehat{N-4}, N-2, N-1, \tau| + |\widehat{N-3}, N, \tau|, \\
F'_{xxx} &= |\widehat{N-5}, N-3, N-2, N-1, \tau| \\
&\quad + 2|\widehat{N-4}, N-2, N, \tau| \\
&\quad + |\widehat{N-3}, N+1, \tau|, \\
F'_t &= -16(|\widehat{N-5}, N-3, N-2, N-1, \tau| \\
&\quad - |\widehat{N-4}, N-2, N, \tau| \\
&\quad + |\widehat{N-3}, N+1, \tau|), \\
k_N^2 F' F &= \left[\left(\sum_{j=1}^{N} k_j^2\right) F\right] F' - \left[\left(\sum_{j=1}^{N-1} k_j^2\right) F'\right] F \\
&= |\widehat{N-2}, \tau|(-|\widehat{N-3}, N-1, N| \\
&\quad + |\widehat{N-2}, N+1|) \\
&\quad + |\widehat{N-1}|(|\widehat{N-4}, N-2, N-1, \tau| \\
&\quad - |\widehat{N-3}, N, \tau|), \\
k_N^2 F'_x F &= \left[\left(\sum_{j=1}^{N} k_j^2\right) F\right] F'_x - \left[\left(\sum_{j=1}^{N-1} k_j^2\right) F'_x\right] F \\
&= |\widehat{N-3}, N-1, \tau|(|\widehat{N-2}, N+1| \\
&\quad - |\widehat{N-3}, N-1, N|) \\
&\quad + |\widehat{N-1}|(|\widehat{N-5}, N-3, N-2, N-1, \tau| \\
&\quad - |\widehat{N-3}, N+1, \tau|), \\
k_N^2 F' F_x &= \left[\left(\sum_{j=1}^{N} k_j^2\right) F_x\right] F' - \left[\left(\sum_{j=1}^{N-1} k_j^2\right) F'\right] F_x \\
&= |\widehat{N-2}, \tau|(|\widehat{N-2}, N+2| \\
&\quad - |\widehat{N-4}, N-2, N-1, N|) \\
&\quad - |\widehat{N-2}, N|(|\widehat{N-3}, N, \tau| \\
&\quad - |\widehat{N-4}, N-2, N-1, \tau|).
\end{aligned}$$

Besides, the Wronskian identities are listed as [20]

$$\sum_{j=1}^{N} |\alpha_1, \ldots, \alpha_{j-1}, b\alpha_j, \alpha_{j+1}, \ldots, \alpha_N| = \left(\sum_{j=1}^{N} b_j\right) |\alpha_1, \ldots, \alpha_N|,$$

where α_j's are N-dimensional column vectors and $b\alpha_j$ denotes $(b_1\alpha_{j,1}, b_2\alpha_{j,2}, \ldots, b_N\alpha_{j,N})^T$.

$$|Mab||Mcd| - |Mac||Mbd| + |Mad||Mbc| = \frac{1}{2}\begin{vmatrix} M & 0 & a & b & c & d \\ 0 & M & a & b & c & d \end{vmatrix} = 0,$$

where M is an $N \times (N-2)$ matrix and a, b, c, d represent the N-dimensional column vectors.

Taking $\mu = 0$ and $\lambda = k_N^2$, and substituting the derivatives of F and F' into BT (9) with the Wronskian identities can yield

$$\begin{aligned} &(D_x + D_y)F' \cdot F \\ &= (F'_x F - F' F_x) + (F'_y F - F' F_y) = 0, \end{aligned} \tag{17a}$$

$$\begin{aligned} &(D_x D_y - \mu D_x + \lambda)F' \cdot F \\ &= -F'_y F_x - F_y F'_x + F' F_{xy} + F F'_{xy} + k_N^2 F' \cdot F \\ &= -\begin{vmatrix} \widehat{N-3} & 0 & N-2 & N-1 & N & \tau \\ 0 & \widehat{N-3} & N-2 & N-1 & N & \tau \end{vmatrix} = 0, \end{aligned} \tag{17b}$$

$$\begin{aligned} &(D_x^2 - \lambda)F' \cdot F \\ &= (D_x^2 - k_N^2)F' \cdot F \\ &= -2F_x F'_x + F' F_{xx} + F F'_{xx} - k_N^2 F' \cdot F \end{aligned}$$

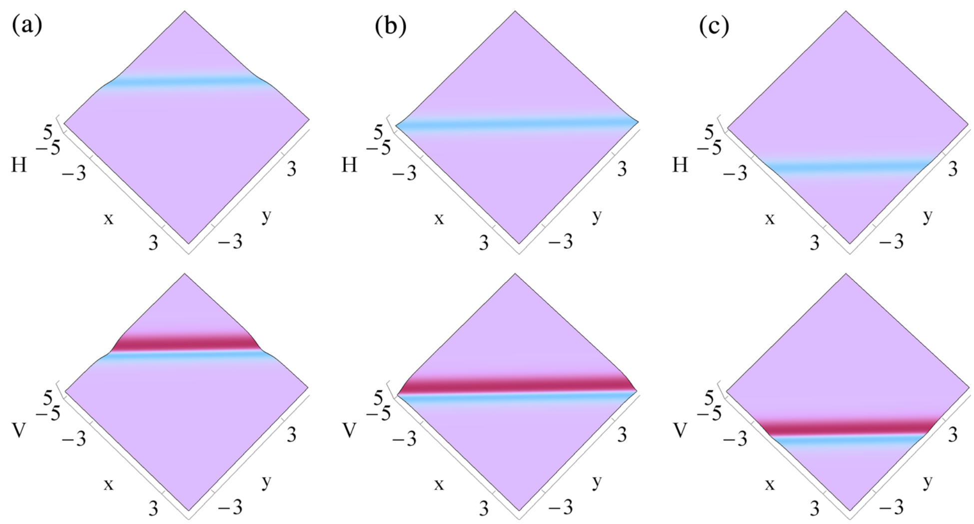

Figure 1. One soliton via solutions (15) with $N = 1$. The parameters adopted here are $k_1 = 1.5$, $\xi_1^0 = 0$ for (**a**) $t = -0.1$; (**b**) $t = 0$; (**c**) $t = 0.1$.

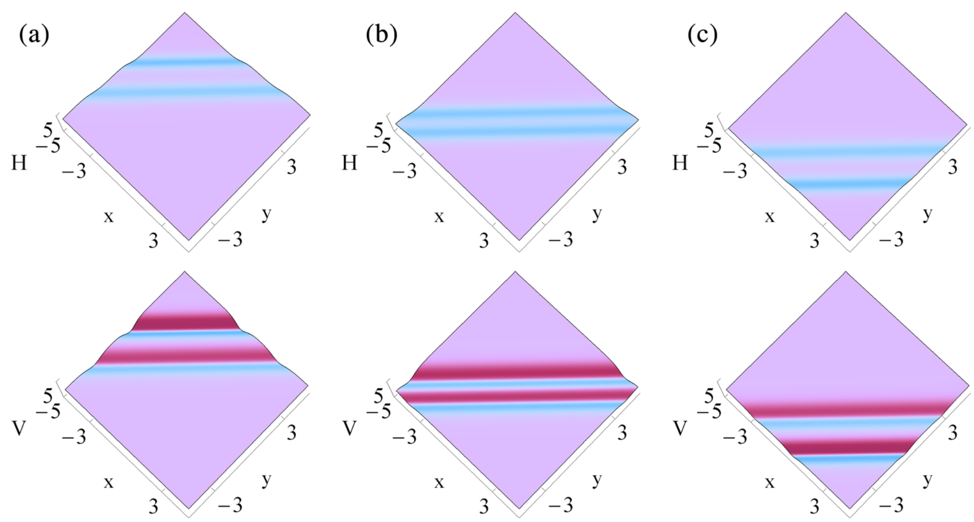

Figure 2. Parallel solitons via solutions (15) with $N = 2$. The parameters adopted here are $\xi_1^0 = 0$, $\xi_2^0 = 0$. $k_1 = -1.4$, $k_2 = 1.7$ for (**a**) $t = -0.1$; (**b**) $t = 0$; (**c**) $t = 0.1$.

$$= \begin{vmatrix} \widehat{N-3} & 0 & N-2 & N-1 & N & \tau \\ 0 & \widehat{N-3} & N-2 & N-1 & N & \tau \end{vmatrix} = 0, \qquad (17c)$$

$$(D_t + 4D_x^3 + 12\lambda D_x)F' \cdot F$$
$$= 12(F'_x F_{xx} - F_x F'_{xx}) - F'[F_t + 4(3k_N^2 F_x + F_{xxx})]$$
$$+ F(F'_t + 12k_N^2 F'_x + 4F'_{xxx})$$
$$= -12 \begin{vmatrix} \widehat{N-3} & 0 & N-2 & N-1 & N+1 & \tau \\ 0 & \widehat{N-3} & N-2 & N-1 & N+1 & \tau \end{vmatrix}$$
$$- 12 \begin{vmatrix} \widehat{N-4} & N-2 & 0 & 0 & N-3 & N-1 & N & \tau \\ 0 & 0 & \widehat{N-4} & N-2 & N-3 & N-1 & N & \tau \end{vmatrix}$$
$$= 0. \qquad (17d)$$

Thus, we have verified that solutions (12) satisfy BT (9).

Here, we present some figures to show the shapes and motion of the one soliton and two solitons for system (2). Figures 1a–1c show the motion of a kink-dark-like soliton: H component shows the kink-shaped soliton while V component displays the soliton under the background which is like the dark soliton. It can be seen that the kink-dark-like soliton propagates stably, and keeps its width and amplitude invariant when propagating.

Figures 2a–2c display the parallel propagation of two-kink-dark-like solitons on the x–y plane at different t's. We can see that the two-kink-dark-like solitons maintain their amplitudes and velocities. During the propagation, there is no energy exchange between the two solitons. In addition to the parallel solitons, the KPII equation has a variety of the soliton interactions, e.g. elastic interaction and inelastic interaction [30–32]. Numbers of the incoming and outgoing solitons of the elastic interaction are the same, when compared with the inelastic interaction between two solitons for eq. (1) [30,31].

4. Conclusions

Investigation has been made on KP-based system (2) in fluid dynamics. Using the Hirota method and symbolic computation, we have deduced bilinear form (6) and BT (9) for system (2), and constructed N-soliton solutions (15) in terms of the Wronskian for system (2) from BT (9). Using the Wronskian technique, it has been verified that solutions (15) satisfy bilinear form (6) and BT (9). Figures 1a–1c have displayed the kink-dark-like soliton for system (2), and the soliton keeps its shape unchanged during the propagation. Figures 2a–2c have illustrated the parallel propagation of two-kink-dark-like solitons on the x–y plane at different t's, during which the two solitons maintain their amplitudes and velocities unchanged.

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