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On distinguishing different models of a class of emergent Universe solutions

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Abstract. A specific class of singularity-free cosmological model has recently been considered in light of different observational data such as observed Hubble data, BAO data from luminous red galaxy survey by Slowan digital sky survey (SDSS) and CMB data from WMAP. However, it is observed that only 12–14 data points are used to study the viability of the model in late time. In this paper, we discuss the viability of all the models belonging to the same class of EU in light of union compilation data (SNIa) which consists of over a hundred data points, thus getting a more robust test for viability. More importantly, it is crucial that we can distinguish between the various models proposed in the class of solution obtained. We discuss here why with the present observational data it is difficult to distinguish between all of them. We show that the late-time behaviour of the model is typical to any asymptotically de Sitter model.

Keywords. Dark energy; emergent Universe; observational data.

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1. Introduction

The present phase of accelerated expansion of the Universe [1–4] seems to be an undeniable fact today. The origin of such acceleration is more interesting and more challenging. It is non-trivial to address the cause of the late-time acceleration from fundamental physics. Moreover, it is essential to incorporate a phase of inflation in early Universe in the standard Big-Bang cosmology as the present observations favour such an initial phase. The Big-Bang cosmology, despite its success, has always been a concern for many who remained sceptical about its initial singularity. Emergent Universe (EU) models were studied as early as in 1965 by Harrison [5]. Later, Ellis and Maartens [6] studied a similar model of Universe without any initial singularity. An interesting solution was obtained by Mukherjee *et al* [7] where they obtained an eternally inflating solution in flat Universe, which is called 'emergent Universe', using general relativity only and considering a non-linear equation of state as below.

$$p = A\rho - B\rho^{1/2}. \tag{1}$$

It was suggested that the non-linear equation of state could mimic the evolution of a Universe with a mixture of three different matter energy content. The composition of the Universe, they argued, would depend on the choice of the parameter A. Such a non-linear equation of state is a special case of a more general equation $p = A\rho - B\rho^{\alpha}$. Phenomenological representations of such equation of states can be found in string theory. Models based on such equation of state often interpolate between two phases of the Universe [8]. The model was later studied in different frameworks such as brane world [9,10], Gauss–Bonnet gravity [11], Brans–Dicke theory [12] etc. Apart from the two parameters coming from the equation of state, the model involves a third parameter (K) as an integration constant which should be fixed by the suitable choice of initial condition. Recently, attempts were made to constrain the parameters of the original model [13–16]. It is suggested in [15,16] that some of the choices in ref. [7] are permitted by the present observational data. It is critical that we have a clear idea if the present observational data permit us to distinguish between different models belonging to this class. It is straightforward to explain the present scheme. SNIa data tabulate distance modulus $(\mu(z))$ values obtained at different redshifts (z). $\mu(z)$ values are theoretically calculated for different EU models. The relative difference in distance modulus values can also be obtained as $\Delta\mu/\mu$ for these models. The models are distinguishable only if the value of this relative difference function ($\Delta\mu/\mu$) exceeds the uncertainties of SNIa observation. Also, it is interesting to investigate the constraints on the model parameters put by union compilation data which comprises over five hundred data points. Earlier analyses were based mostly on the observed Hubble data (OHD) with twelve data points. OHD is a collection of measured values of Hubble parameters at different red-shift values from different experiments (for details, see refs [16,17]). There is a particular model where the cosmic fluid behaves like a mixture of matter, exotic matter and dark energy (for detailed discussion, see [7]). If the models can be distinguished, this particular one could be an appealing candidate.

In this particular work, the above issues are addressed along with a study of the late-time behaviour of EU models. The plan of the paper is as follows: in §2 relevant field equations for the EU model are introduced. Data analysis based on union2 compilation of SNIa data [18] is presented in §3. The possibilities of distinguishing different EU models from union2 data and study of the late-time behaviour of these models have been discussed in §4. Finally, a brief discussion of the findings is given in §5.

2. Relevant field equations for the EU model

Friedmann equation for a flat Universe is

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}, \tag{2}$$

where H is the Hubble parameter and a is the scale factor of the Universe. The conservation equation is given by

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + 3H\,(p+\rho) = 0. \tag{3}$$

Using the EOS given by eq. (1) in eq. (2), and eq. (3)

$$\rho\,(z) = \left(\frac{B}{A+1}\right)^2 + \frac{2BK}{(A+1)^2}\,(1+z)^{3(A+1)/2} + \left(\frac{K}{A+1}\right)^2 (1+z)^{3(A+1)}, \tag{4}$$

were z is the cosmological red-shift. Scale factor $a(t)$ is related to cosmological red-shift (z): $a(t) = 1/(1+z)$. The first term in the right-hand side of eq. (4) is a constant which can be interpreted as the cosmological constant (describes dark energy). Equation (4) can be written as

Table 1. Best-fit values of B and K from union2 data.

Model	B	K	$\chi^2_{\min}$ (/d.o.f.)
$A=0$	0.867	1.133	0.974
$A=1$	1.491	0.528	0.985
$A=1/3$	1.121	0.879	0.974

$$\rho(z) = \rho_1 + \rho_2(1+z)^{3(A+1)/2} + \rho_3(1+z)^{3(A+1)}, \tag{5}$$

were $\rho_1 = (B/(A+1))^2$, $\rho_2 = 2BK/(A+1)^2$ and $\rho_3 = (K/(A+1))^2$ are densities at the present epoch. The Friedmann equation (eq. (2)) can be written in terms of red-shift and density parameter:

$$H^2(z) = H_0^2(\Omega_1 + \Omega_2(1+z)^{3(A+1)/2} + \Omega_3(1+z)^{3(A+1)}), \tag{6}$$

where the density parameter is defined as $\Omega = (8\pi G\rho/3H_0^2) = \Omega(A, B, K)$. Different compositions of cosmic fluids are obtained for different values of A. For example, the case $A = 0$ was considered in [14] and the model included dark energy, dark matter and dust in the Universe (for details, see [13]). With $A = A_0$, eq. (6) can be written as

$$H^2(H_0, B, K, z) = H_0^2 E^2(B, K, z), \tag{7}$$

$$E^2(B, K, z) = \Omega_\Lambda + \Omega_2\,(1+z)^{3(A+1)/2} + \Omega_3\,(1+z)^{3(A+1)}, \tag{8}$$

where the constant part of the DP (Ω_1) has been replaced by a new notation Ω_Λ.

3. Analysis of the EU model with SNIa data

In a flat Universe, the Hubble-free luminosity distance ($D_L \equiv H_0 d_L$) is defined as

$$D_L(z) = (1+z)\int_0^z \frac{H_0}{H(z'; a_1, a_2, ..., a_n)}\mathrm{d}z', \tag{9}$$

where $a_1, a_2, ..., a_n$ are theoretical model parameters. The distance modulus is defined as in ref. [19]:

$$\mu_{\mathrm{th}} = 5\log_{10}(D_L(z)) + \mu_0, \tag{10}$$

where $\mu_0 = 42.38 - 5\log_{10} h$. h is the dimensionless Hubble parameter at the present epoch. Consequently, a χ^2 function can be defined as

$$\chi^2_{\mathrm{SNIa}}(B, K) = \sum_1^N \frac{(\mu_{\mathrm{obs}}(z_i) - \mu_{\mathrm{th}}(z_i))^2}{\sigma_i^2}, \tag{11}$$

where $\mu_{\mathrm{obs}}(z_i)$ is the observed distance modulus value at a red-shift z_i and σ_i is the associated uncertainty in

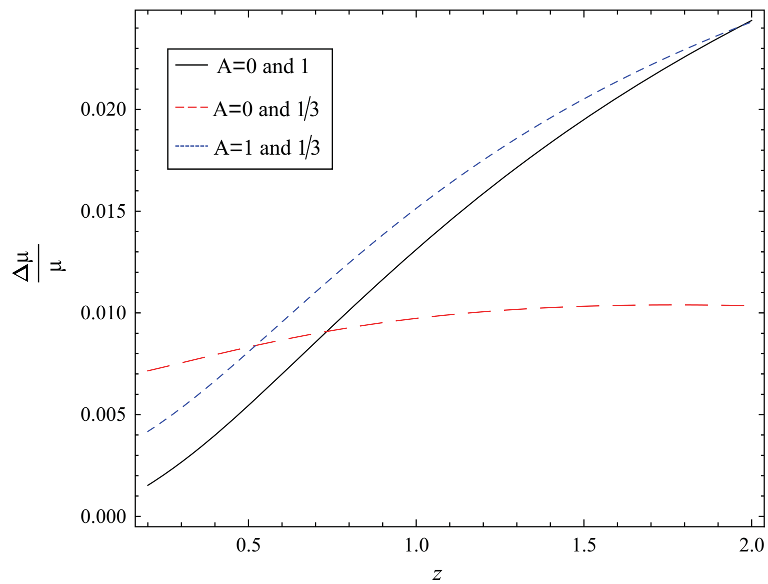

Figure 1. Evolution of difference between distance moduli for various EU models.

measurement. The union2 data set compiled in ref. [18] has been considered here. The above χ^2 function, and the method discussed in ref. [19] can be used to find a χ^2-fit. Findings are given in table 1.

4. Possibilities of distinguishing different EU models with SNIa data

As noted in eq. (10), theoretically it is possible to obtain the distance modulus for various EU models. Theoretical difference between the distance modulus values for two different EU models ($\Delta\mu(z)$) are calculated at different red-shift points. Different EU models cannot be distinguished from SNIa observations if $(\Delta/\mu)(z)$ remains within the associated uncertainties in the measurement of $\mu(z)$. The graphs obtained are shown in figure 1. It is seen that the $(\Delta\mu/\mu)(z)$ values for the models with $A = 0, 1, 0$ and $1/3$ reach around 2% or more only for $z \geq 1$. The associated uncertainties in the measurement of $\mu(z)$ is around 2–4% and for red-shifts $z \sim 1.5$ and higher, the uncertainty is even greater. Thus, it is not possible to distinguish between different EU models from SNIa observation. Distinguishing between EU with $A = 1$ and $1/3$ is even more unlikely.

4.1 *Late-time behaviour of EU models*

It is seen from figure 2 that the difference between different EU models fades when the present epoch is approached. This is not unexpected as these models are asymptotically de Sitter models. Thus, at late time their behaviour should be indistinguishable from one another as well as from a de Sitter Universe. This can also be inferred from figure 1. The $\Delta\mu$ values for any two EU models fall within the uncertainty in the measured μ value (which is around 2–4%) for lower red-shifts. There is a strong possibility that at the present epoch, only late-time behaviour of EU models is observed. All the EU models, belonging to the class under discussion, are asymptotically de Sitter. As given in eq. (14) in ref. [7], the Hubble parameter for EU is

$$H = \frac{\omega\alpha e^{\alpha t}}{\beta + e^{\alpha t}}, \tag{12}$$

where β is a constant, $\alpha = (\sqrt{3}/2)B$ and $\omega = 2/3(A+1)$. In late-time approximation $H \approx \omega\alpha$. The μ vs. z curve for different EU models along with the original union2 data are presented in figure 3. Late-time behaviour of these EU models are almost the same and typical to a de Sitter model. These late-time approximations fit union2 data reasonably well. However, as noted previously, the behaviour is typical for any de Sitter Universe. At late time the models no longer depend on the parameter K. Once A is specified, there is only one free parameter, i.e., B.

5. Conclusion

A class of EU models, presented in ref. [7], is studied and best-fit values of the model parameters are determined from union2 compilation of SNIa data [18]. More importantly, the possibilities of distinguishing different EU models from SNIa observations have also been considered. It is seen that the model with $A = 1$ and

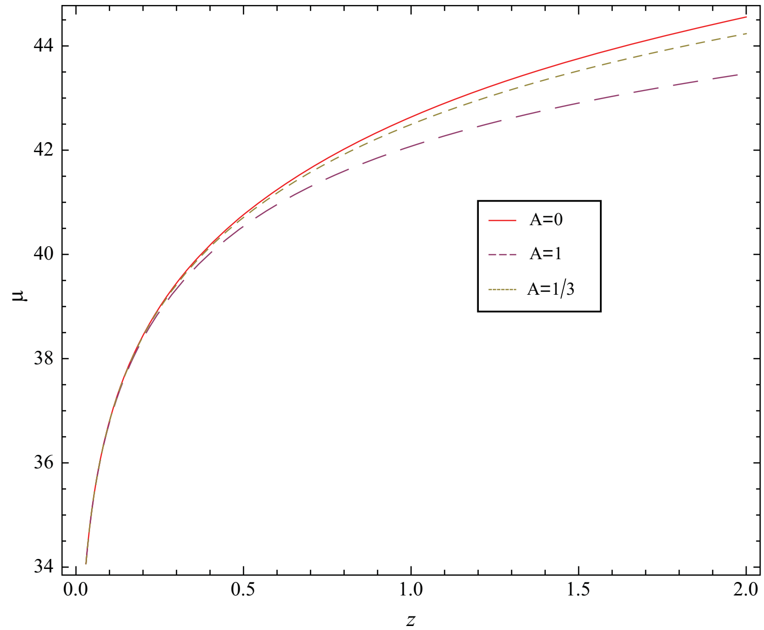

Figure 2. μ vs. z curve for different EU models.

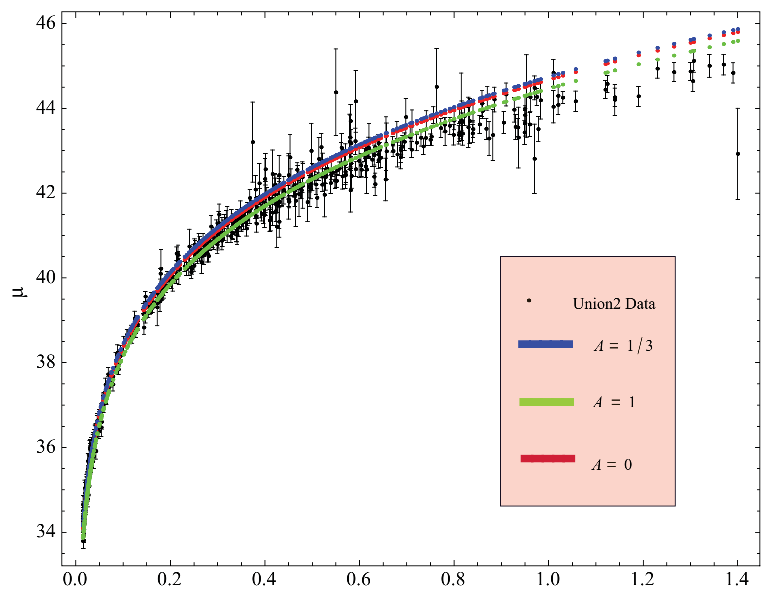

Figure 3. μ vs. z curve for different EU models in late-time approximation.

$A = 1/3$ can be distinguished from $A = 0$ model with the data. The difference shows prominence over the uncertainty in measurement from around $z = 0.5$. It has been shown that any distinction between $A = 1$ and $A = 1/3$ models cannot be made from SNIa data as the difference remains within observational uncertainty. However, it should be noted that SNIa data become more uncertain at red-shifts above $z = 1$ and any distinction is not viable. At the present epoch, the EU models cannot be distinguished from SNIa data as the difference becomes too small compared to uncertainties in the observed data. The behaviour of all EU models in the present era are typical for any asymptotically de Sitter model. It is noted that the EU models are independent of the parameter K under late-time approximation. However, as claimed in [7], the parameter K is fixed from the initial conditions. So, late-time behaviour of all the EU models are, as they should be, independent of the initial conditions. The best-fit values, obtained in the present work, differ from those in [13–15]. These results are more accurate in the sense that significantly more data points have been considered here. It has been shown, in a recent work on EU models [16], that these models are acceptable from the present observations. Present findings are in agreement with the conclusions of ref. [16]. It would be interesting to further check these constraints from growth parameter measurement which will be taken up elsewhere.

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