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Space-charge solitary waves and double layers in n-type compensated semiconductor quantum plasma

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Abstract. Using quantum hydrodynamic (QHD) model and standard reductive perturbation method, we have investigated the formation and characteristics of space-charge solitary waves and double layers in n-type compensated drifting semiconductor plasma with varying doping profiles. Through numerical analysis, it is shown that the structures of space-charge solitary waves and double layers depend significantly on electron drift and compensation parameter which measures a comparative proportion of the donor, acceptor and intrinsic ion concentrations.

Keywords. Quantum hydrodynamic model; space-charge solitary waves and double layers; KdV and mKdV equations; electron–hole quantum plasma.

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1. Introduction

Generation of space-charge waves in semiconductor plasma is very common in laboratory and simulation experiments and it has been found through investigations that the nonlinear wave propagation phenomena of space-charge waves can be linked well with the modern nanoscale electronic devices, optical sensors, phase shifters, delay lines etc. Some recent experiments and simulations have shown the existence of nonlinear excitation for space-charge waves such as solitons in the semiconductor microcavity [1,2]. In recent years, Barrientos and Palankovskib [3] investigated, through numerical simulation, the amplification process of space-charge waves for n-InP films in the presence of negative differential conductance and showed higher harmonics generation due to nonlinear effect. Moreover, laboratory experiments confirm that optical and spin-wave quantum solitons can be generated from high-intensity wave packets if their spreading due to dispersion is balanced by the nonlinearity of the plasma system [4–6]. Recently, some researchers have shown the generation of envelope solitons within piezoelectric semiconductors [7,8]. Cuesta and Schmeiser [9] considered a semiconductor drift-diffusion model and showed the generation of stable solitary wave-like structures. Earlier, Pawlik and Rowlands [10] numerically confirmed the existence of solitary waves in piezoelectric semiconductors. Bonilla [11] considered Gunn effect for finite geometry semiconductor sample with constant external electric field and described the formation of solitons analytically. In the last decade, Couton *et al* [12] had set up an experimental model in the laboratory to investigate self-generation of multiple dark and white photovoltaic solitons within the iron-doped LiNbO samples. Earlier, Berezhiani *et al* [13] have numerically analysed the generation of a chain of electromagnetic solitary wave structures in narrow-gap n-InSb semiconductors with different doping concentrations. Long ago, Pic and Ligeon [14] experimentally investigated the effect of electron drift velocity on helicon waves for InSb semiconductor sample under static electric and magnetic field and showed that helicon waves are modified by the transverse effect of electron drift.

Further, in semiconductor plasma, de Broglie wavelength of charged carriers is comparable with the characteristic spatial scales of the system and such plasma systems can be described conveniently by quantum hydrodynamical (QHD) model which has been successfully applied to resonant tunnelling diodes, self-consistent quantum electron gas, the metallic and semiconductor nanostructures, carbon nanotubes, charged quantum fluids, plasmonics etc. [15–17]. Moreover, the dispersive effects of fermionic pressure and quantum Bohm potential have been experimentally verified in X-ray scattering experiments for electron laser plasmas [18].

Most of the earlier researchers considered electromagnetic solitary waves in semiconductor plasma both experimentally and theoretically [12,13,19,20]. Considering the applicability of space-charge wave formation in semiconductor plasma, both laboratory-based and simulation-based [1,2,11,21–23], it would be interesting to investigate space-charge solitary waves in semiconductor plasma. Very recently, Banerjee and Ghosh [24] have studied modulational instability of space-charge waves and the formation of envelope solitons in n-type compensated semiconductor with varying doping profiles. So far, we are aware that no one has studied the formation of KdV-type solitary waves for space-charge waves in semiconductor plasma. Apart from solitary wave structure, there is another important nonlinear structure, namely double layer (DL). Semiconductor plasma is also suitable for the formation of double layer [25–29]. But to the best of our knowledge, till date no one has studied space-charge double layers in semiconductor plasma. So, it would be interesting to investigate KdV-type soliton and the double layer formation for space-charge waves in semiconductor plasma.

So, the purpose of the present paper is to study the formation of KdV-type solitary waves and DLs for space-charge waves in n-type compensated semiconductor plasma with varying doping profiles and electron drifts. Starting from the QHD model equations we have derived KdV and mKdV equations using standard reductive perturbation techniques and from these equations we have found space-charge solitary wave and double layer solutions. Through numerical analysis it is shown that the profiles of space-charge solitary waves and double layers are significantly modified by doping profile and electron drift.

This paper is organised as follows. Section 2 represents quantum hydrodynamic (QHD) model for semiconductor plasma. The linear dispersion relation is explained numerically in §3. In §4 derivation of KdV equation and solution of solitary waves are presented. In §5, mKdV equation and double layer solution have been analysed. Numerical results are represented in §6. Finally, in §7 some concluding discussions are given.

2. Governing equations

The dynamics of the space-charge waves in the quasineutral electron–hole quantum semiconductor plasma for n-type compensated semiconductor can be described by the following quantum hydrodynamic model equations [24]:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) = 0, \tag{1}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{q_e}{m_e}\frac{\partial \phi}{\partial x} - \frac{1}{m_e n_e}\frac{\partial p_e}{\partial x} + \frac{\hbar^2}{2m_e^2}\frac{\partial}{\partial x}\left\{\left(\frac{1}{\sqrt{n_e}}\right)\frac{\partial^2}{\partial x^2}\left(\sqrt{n_e}\right)\right\}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0}(n_e - n_h), \tag{3}$$

where n_e, u_e, p_e, m_e and q_e are respectively the perturbed number density, velocity, pressure, mass and charge of the electron; n_h is the number density of holes; ϕ is the electrostatic potential and $\hbar$ is the reduced Planck's constant. Equation (1) is the continuity equation. Equation (2) is the momentum equation for the electrons. In the right-hand side of eq. (2) the second and third terms correspond respectively to classical and quantum pressures. The quantum pressure originates from quantum force related to quantum Bohm potential. A detailed discussion regarding this can be found in the paper by Manfredi and Hass [30]. We assume the plasma particles to behave like a one-dimensional Fermi gas at zero temperature. Then, following Manfredi and Hass [30], the pressure law can be expressed as

$$p_e = \frac{m_e V_{Fe}^2}{3n_{e0}^2} n_e^3, \tag{4}$$

where $V_{Fe} = \sqrt{2k_B T_{Fe}/m_e}$ is the Fermi thermal speed and T_{Fe} is the Fermi temperature. Now by normalising velocities by V_{Fe}, electrostatic potential by $k_B T_{Fe}/e$, all lengths by V_{Fe}/ω_{pe}, time by ω_{pe}^{-1} and all densities by h_0, where h_0 is the hole concentration at thermal equilibrium, the basic eqs (1) and (2) can be put in the following normalised form:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) = 0, \tag{5}$$

$$\mu\left\{\frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x}\right\} = \frac{\partial \phi}{\partial x} - n_e\frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left\{\left(\frac{1}{\sqrt{n_e}}\right)\frac{\partial^2}{\partial x^2}\left(\sqrt{n_e}\right)\right\}, \tag{6}$$

where

$$\omega_{pe} = \sqrt{\frac{n_{e0}e^2}{\varepsilon_0 m_e}}$$

is the electron plasma frequency,

$$\mu = \frac{m_e}{m_h}$$

is the electron–hole mass ratio,

$$H = \frac{h\omega_{pe}\sqrt{\delta}}{2k_B T_{Fe}}$$

is a dimensionless parameter proportional to quantum diffraction effect h_0. In most cases μ is taken as one but in the case of semiconductor μ should be taken as electron–hole effective mass ratio. The effective mass is determined by the energy vs. wave number curve for the species in valence/conduction band. Due to the parabolic nature of the curve for semiconductors, the value of μ in our model can differ from unity. In compensated semiconductors the concentration of both the donor and the acceptor impurity atoms are assumed to be equal. But in n-type compensated semiconductors, the donor concentration (n_D) is usually much greater than the acceptor concentration (n_A). So in n-type compensated semiconductor, the charge neutrality condition is

$$e_0 + (e_A - h_A) = h_0 + (n_D - e_D). \tag{7}$$

where e_0 and h_0 are the thermal equilibrium concentrations of electrons and holes in the conduction band and valence band respectively; e_D is the concentration of electrons in the donor energy level and h_A is the concentration of holes in the acceptor energy level. By assuming complete ionisation, the above charge neutrality equation becomes

$$e_0 + n_A = h_0 + n_D. \tag{8}$$

Using the law of mass-action

$$h_0 \cdot e_0 = i_i^2, \tag{9}$$

where i_i is the intrinsic ion concentration in which

$$i_i^2 = N_c N_v \mathrm{e}^{-E_g/k_B T},$$
$$N_c = 2\left(\frac{2\pi m_e k_B T}{\hbar^2}\right)^{3/2}, \quad N_v = 2\left(\frac{2\pi m_h k_B T}{\hbar^2}\right)^{3/2}, \tag{10}$$

E_g is the band gap of the semiconductor, $\hbar$ is the reduced Planck's constant, k_B is the Boltzmann constant, m_e and m_h are the effective masses of electron and hole respectively.

So, using eqs (8) and (9) we get

$$e_0 = \frac{n_D - n_A}{2} + \sqrt{\left(\frac{n_D - n_A}{2}\right)^2 + i_i^2}. \tag{11}$$

Normalising with respect to equilibrium hole concentration h_0, the charge neutrality equation becomes

$$C_p = 1 + \delta - \lambda, \tag{12}$$

where

$$C_p = \frac{\frac{n_D - n_A}{2} + \sqrt{\left(\frac{n_D - n_A}{2}\right)^2 + i_i^2}}{h_0} \tag{13}$$

is the compensation parameter,

$$\delta = \frac{n_D}{h_0} \quad \text{and} \quad \lambda = \frac{n_A}{h_0}.$$

As in the case of compensated semiconductor the concentration of the donor impurity and the acceptor impurity and density of electrons and holes at equilibrium are equal and so $C_p = 1$.

But for n-type compensated semiconductor where electrons are the majority carriers and holes are the minority carriers, $C_p > 1$ and for p-type semiconductor where holes are majority carriers and electrons are minority carriers, $C_p < 1$.

Hence, using eqs (12) and (13) the above system of equations is closed by the normalised Poisson's equation:

$$\frac{\partial^2 \phi}{\partial x^2} = n_e \cdot C_p^{-1} - n_h, \tag{14}$$

where n_h is the minority hole concentration in the semiconductor. As there is no external applied field, the depletion phase hole concentration within an n-type semiconductor will be given by

$$n_h = \delta \mathrm{e}^{-\phi}. \tag{15}$$

3. Dispersion relation

To study the linear and nonlinear propagation of space charge waves, we make the following perturbation expansion for field quantities n_e, u_e and ϕ about their equilibrium values:

$$n_e = C_p + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \varepsilon^3 n_{e3} + \cdots$$
$$u_e = u_{e0} + \varepsilon u_{e1} + \varepsilon^2 u_{e2} + \varepsilon^3 u_{e3} + \cdots$$
$$\phi = 0 + \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \cdots, \tag{16}$$

where u_{e0} is the initial drift velocity of electrons.

Substituting expansion (16) in eqs (5), (6) and (14) and then linearising and considering that all field quantities are varying as $\mathrm{e}^{i(kx-\omega t)}$, we get for normalised wave frequency ω and wave number k, the following linear dispersion relation:

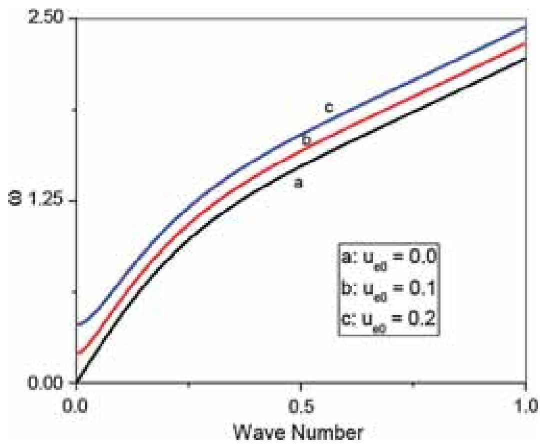

Figure 1. Linear dispersion characteristics for different values of electron drift ($u_{e0} = 0.00, 0.1, 0.2$) with other parameters fixed at $\delta = 0.25$, $\mu = 0.5$, $H = 0.2$, $C_p = 2.25$.

$$\omega^2 \cdot \left[8\mu C_p\left(\delta + k^2\right)\right] - \omega \cdot [16\mu C_p u_{e0}(\delta + k^2)] \\ +[H^2 k^4(\delta + k^2)(1 - 2\sqrt{C_p}) \\ -8C_p k^2\{1 + C_p^2(\delta + k^2)\} \\ -8\mu C_p k^2 u_{e0}^2(\delta + k^2)] = 0. \tag{17}$$

Solving eq. (17) for ω we get

$$\omega = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \tag{18}$$

where

$$A = 8\mu C_p\left(\delta + k^2\right), \\ B = -16\mu C_p u_{e0}\left(\delta + k^2\right), \\ C = H^2 k^4\left(\delta + k^2\right)(1 - 2\sqrt{C_p}) \\ -8C_p k^2\{1 + C_p^2\left(\delta + k^2\right)\} \\ -8\mu C_p k^2 u_{e0}^2\left(\delta + k^2\right). \tag{19}$$

In the long wavelength limit ($k \to 0$) the phase speed v_p of the fast mode is given by

$$v_p = \frac{\omega}{k} = \sqrt{\frac{(1 + C_p^2\delta)}{\mu\delta}} + u_{e0}. \tag{20}$$

Obviously phase speed depends on C_p and δ and hence on the doping level.

Figure 1 shows the dependence of the linear dispersion character (ω vs. k) of the wave on the electron drift velocity (u_{e0}). It shows that the wave frequency increases with increase in electron drift velocity. Physically, the drifting electrons interact with the electric field associated with the wave in an intricate way; as we are considering wave propagation in the direction of electron drift it is likely to favour wave propagation and thus increases the wave speed.

4. KdV equations and solitary waves

To study the formation of solitary wave structures, we derive a KdV equation by following standard reductive perturbation method. We use the following standard stretching for the space and time variables

$$\xi = \varepsilon^{1/2}\left(x - v_p t\right) \quad \text{and} \quad \tau = \varepsilon^{3/2} t, \tag{21}$$

where v_p is the normalised linear long wave phase velocity given by eq. (20) and ε is a small dimensionless parameter measuring smallness of dispersion and nonlinearity. Now rewriting the basic eqs (5), (6) and (14) in terms of these dimensionless variables ξ and τ and then solving the lowest-order equations we obtain

$$n_{e1} = -C_p\delta\alpha, \\ u_{e1} = -\delta\left(v_p - u_{e0}\right)\alpha, \tag{22}$$

where

$$\alpha = \phi_1. \tag{23}$$

Then going to next higher order of ε we obtain Kortweg–de Vries (KdV) equation:

$$a\frac{\partial\alpha}{\partial\tau} + b\alpha\frac{\partial\alpha}{\partial\xi} + c\frac{\partial^3\alpha}{\partial\xi^3} = 0, \tag{24}$$

where

$$a = -2\mu\delta\left(v_p - u_{e0}\right), \\ b = 3\mu\delta^2\left(v_p - u_{e0}\right)^2 + C_p^2\delta^2, \\ c = C_p^2 + \frac{H^2\delta}{4\sqrt{C_p}} - \mu\left(v_p - u_{e0}\right)^2. \tag{25}$$

The KdV eq. (24) has the following solitary wave solution:

$$\alpha = \alpha_s \mathrm{sech}^2\left(\frac{\eta}{\Delta}\right), \tag{26}$$

in which $\eta = \xi - M\tau$, M is the Mach number, $\alpha_s = 3M/b$ is the amplitude and $\Delta = \sqrt{4c/M}$ is the width of the solitary waves.

The sign of b, the coefficient of nonlinearity in KdV eq. (24), determines the nature of the solitary waves. It is clear from eq. (25) that the value of b can be negative only when μ is negative and then rarefactive solitons may form. But, we know that in a solid-state medium the sign of effective mass is determined by the E-k diagram and a negative effective mass corresponds to valence band electrons where all energy states are occupied and the particles go the wrong way when an external force is applied. As the QHD model that we are using in this paper is an approximate one, it cannot fully describe the effect of negative masses. For this, we have restricted our discussions to positive effective masses only. Thus, only compressive solitons are possible for our plasma model.

5. Double layers

KdV equation describes the behaviour of small-amplitude solitary waves. The KdV equation is modified and one derives modified KdV (mKdV) equation in order to incorporate additional effects such as higher-order nonlinearity, plasma inhomogeniety etc. In our model, the KdV equation is modified by including cubic nonlinearity to describe the time evolution of double layers. In order to derive the mKdV equation, the independent field variables are stretched as follows [31,32]:

$$\xi = \varepsilon\left(x - v_p t\right) \quad \text{and} \quad \tau = \varepsilon^3 t. \tag{27}$$

Applying these stretchings to the basic eqs (5), (6) and (14) and collecting smallest order terms of ε, second-order quantities of field variables are obtained as

$$\begin{aligned} n_{e2} &= \frac{C_p}{2}\alpha^2 - C_p\delta\phi_2, \\ u_{e2} &= \left(v_p - u_{e0}\right)\left(\frac{1}{2} - \delta^2\right)\alpha^2 \\ &\quad -\delta\left(v_p - u_{e0}\right)\phi_2. \end{aligned} \tag{28}$$

Going to the next higher-order terms of ε we obtain modified Kortweg–de Vries (mKdV) equation:

$$a\frac{\partial \alpha}{\partial \tau} + b\alpha\frac{\partial \alpha}{\partial \xi} + d\alpha^2\frac{\partial \alpha}{\partial \xi} + c\frac{\partial^3 \alpha}{\partial \xi^3} = 0, \tag{29}$$

where

$$\begin{aligned} d = &-3\mu\delta\left(v_p - u_{e0}\right)^2\left(\frac{1}{2} - \delta^2\right) \\ &-\frac{C_p^2}{2} - 2C_p\delta^2 \\ &+\frac{3\mu\delta}{2}\left(v_p - u_{e0}\right)\left(2\delta^2 + 1\right). \end{aligned} \tag{30}$$

Using the transformation $\eta = \xi - M\tau$ and then integrating eq. (29) with respect to η under the boundary conditions $\psi, \mathrm{d}\psi/\mathrm{d}\eta, \mathrm{d}^2\psi/\mathrm{d}\eta^2 \to 0$ as $|\eta| \to \pm\infty$ we obtain the following energy equation:

$$\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right)^2 + \psi_s(\alpha, M) = 0, \tag{31}$$

where $\psi_s(\alpha, M)$ is Sagdeev's pseudopotential and it is given by

$$\psi'(\alpha, M) = \frac{\alpha^2}{2c}\left(\frac{d}{6}\alpha^2 + \frac{b}{3}\alpha - M\right). \tag{32}$$

Now for the existence of double layers the boundary conditions are

$$\begin{aligned} &\text{(i) } \psi'(\alpha, M)\big|_{\alpha=0} = \psi'(\alpha, M)\big|_{\alpha=\alpha_{\max}} = 0, \\ &\text{(ii) } \left.\frac{\mathrm{d}\psi'(\alpha, M)}{\mathrm{d}\eta}\right|_{\alpha=0} = \left.\frac{\mathrm{d}\psi'(\alpha, M)}{\mathrm{d}\eta}\right|_{\alpha=\alpha_{\max}} = 0, \\ &\text{(iii) } \left.\frac{\mathrm{d}^2\psi'(\alpha, M)}{\mathrm{d}\eta^2}\right|_{\alpha=0,\alpha_{\max}} < 0, \end{aligned} \tag{33}$$

where $\alpha = 0$ and $\alpha_{\max}$ are two extreme points of Sagdeev's potential $\psi'(\alpha, M)$.

Equation (29) has the following double layer solution:

$$\alpha = \frac{\alpha'_{\max}}{2}\left[1 - \tanh\left(\frac{2\eta}{\Delta'}\right)\right], \tag{34}$$

where

$$\Delta' = \frac{4\sqrt{6c/-d}}{|\alpha_{\max}|} \tag{35}$$

is the width of the double layer.

For a double layer solution to exist, the following condition must be satisfied:

$$-\frac{d}{c} > 0 \quad \text{or} \quad d < 0. \tag{36}$$

The double layer structure depends on several plasma parameters such as compensation parameter (C_p), electron drift velocity (u_{e0}), minority carrier concentration (δ), electron–hole mass ratio (μ) and quantum diffraction parameter (H).

6. Numerical results

The solitary wave profile given by eq. (26) has been analysed numerically for different values of the compensation parameter (C_p) and the electron drift (u_{e0}).

Figure 2 shows the solitary wave profile for different values of the compensation parameter (C_p). From eq. (25) we find that both the nonlinear coefficient b and the dispersive coefficient c depend upon compensation parameter (C_p) and hence nature, amplitude and width of the solitary waves are expected to depend sensitively on the compensation parameter. Figure 2 shows that amplitude and width of the compressive solitons decrease with the increase of compensation parameter (C_p). Increase in C_p means increase in donor impurity concentration which leads to the generation of excess free electrons and as a result system nonlinearity increases and hence narrower solitons are formed. Similar results have also been reported for envelope solitons [24].

Dependence of the KdV solitary wave profile on electron drift has also been studied numerically. In figure 3

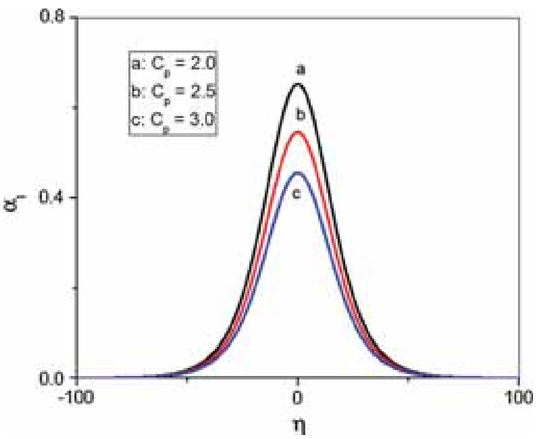

Figure 2. Solitary wave profile for different values of compensation parameter $\left(C_p = 2.0, 2.5, 3.0\right)$, with other parameters fixed at $\delta = 0.25$, $\mu = 0.5$, $H = 0.2$, $u_{\mathrm{e}0} = 0.1$.

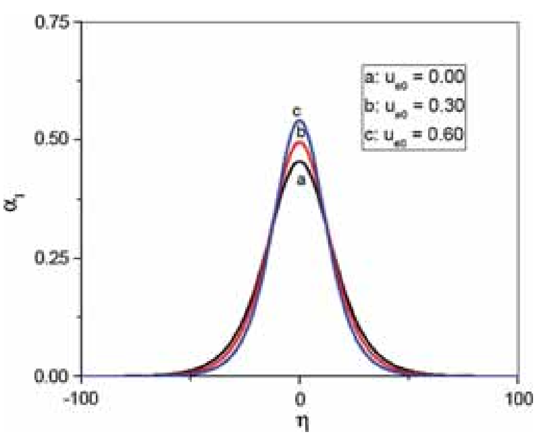

Figure 3. Dependence of the KdV solitary wave profile on electron drift ($u_{\mathrm{e}0} = 0.00, 0.30, 0.60$) with other parameters fixed at $\delta = 0.25$, $\mu = 0.5$, $H = 0.2$, $C_p = 2.5$.

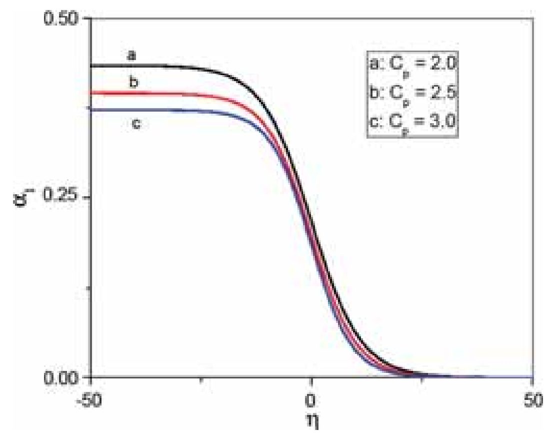

Figure 4. Dependence of DL profile on compensation parameter $\left(C_p = 2.0, 2.5, 3.0\right)$, with other parameters fixed at $\delta = 0.25$, $\mu = 0.5$, $H = 0.2$, $u_{\mathrm{e}0} = 0.1$.

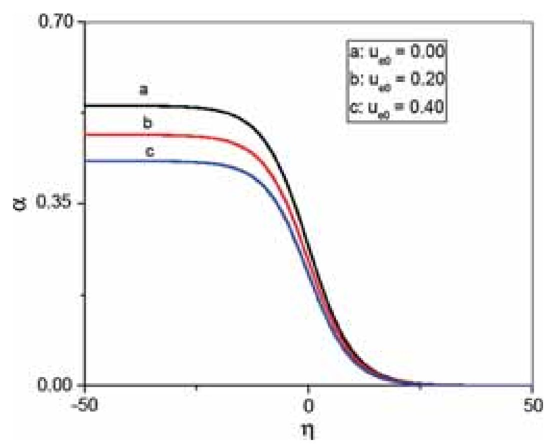

Figure 5. Double layer profile for different values of electron drift velocity ($u_{\mathrm{e}0} = 0.00, 0.20, 0.40$), with other parameters fixed at $\delta = 0.25$, $\mu = 0.5$, $H = 0.2$, $C_p = 2.5$.

we show the solitary wave profile for different values of electron drift. It shows that the effect of electron drift is to steepen the solitary wave profile.

The dependence of DL profile given by eq. (34) on compensation parameter (C_p) and electron drift velocity (u_{e0}) has also been studied numerically. Figure 4 shows the dependence of DL profile on compensation parameter (C_p). It is clear from the figure that both the amplitude and width of the compressive double layers decrease with increase in the value of the compensation parameter (C_p). Physically, higher values of compensation parameter (C_p) can generate higher potential difference within shorter distance.

In figure 5 we show the profile of double layers for different values of electron drift velocity ($u_{\mathrm{e}0}$). It shows that increase in electron drift velocity ($u_{\mathrm{e}0}$) decreases the amplitude of the compressive double layers. Experimentally, it has been observed that in the positive column of low-pressure discharges, free DLs are usually formed as a result of plasma instability driven by the current and this instability is mostly dependent upon drift velocity of the electrons [33–35].

7. Concluding remarks

In this paper, we have studied space-charge solitary waves and double layers in n-type compensated semiconductor quantum plasma by using QHD model equations and reductive perturbation technique considering electron drift and varying doping profile. We have derived and used KdV and mKdV equations to investigate the properties of space-charge solitary waves and space-charge double layers. Through numerical analysis it is shown that the structure of space-charge solitons and double layers depend significantly on the compensation parameter (C_p) and electron drift velocity (u_{e0}).

Finally, we would like to point out that the plasma model used in this paper may be generalised to study the effects of external electric field, magnetic field or optical illumination which is beyond the scope of the present study. However, we believe that the results of the present investigation will be useful in modern nanoscale semiconductor technology.

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