Pramana – J. Phys. (2018) 90:39
https://doi.org/10.1007/s12043-018-1534-0

Suppression of chaos via control of energy flow

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MS received 8 August 2017; revised 28 October 2017; accepted 31 October 2017;
published online 19 February 2018

Abstract. Continuous energy supply is critical and important to support oscillating behaviour; otherwise, the oscillator will die. For nonlinear and chaotic circuits, enough energy supply is also important to keep electric devices working. In this paper, Hamilton energy is calculated for dimensionless dynamical system (e.g., the chaotic Lorenz system) using Helmholtz's theorem. The Hamilton energy is considered as a new variable and then the dynamical system is controlled by using the scheme of energy feedback. It is found that chaos can be suppressed even when intermittent feedback scheme is applied. This scheme is effective to control chaos and to stabilise other dynamical systems.

Keywords. Hamilton energy; feedback; chaos; phase compression; control.

PACS No. 05.45.–a

1. Introduction

Chaos can be observed in complex and nonlinear systems, and the chaotic series often contains important information [1–3]. In the last decades, relevant topics about synchronisation control of chaos have attracted much attention [4–9]. For example, the emergence of chaos, secure communication based on chaotic systems, synchronisation and control are extensively discussed [10–16] on chaotic systems. For low-dimensional dynamical systems, appropriate parameter regions are detected to reproduce chaotic behaviours [17], and then these chaotic systems are used to investigate the problems related to control and synchronisation. It is found that adaptive track control [18] and intermittent control [19] have some advantages over most of the other control schemes because the controller consumes lower energy and shorter transient period for reaching the target orbits. The dynamical characteristics of chaotic systems are dependent on the setting of initial values though most of the attractors and phase portraits are invariant when the parameters are fixed. However, some nonlinear dynamical systems [20] can be switched between chaotic and periodical attractors by resetting the initial values, and the mechanism may be the memory effect associated with memristor [21,22]. The chaotic oscillator models are often used to investigate the problems about chaos control, synchronisation and switch between different types of attractors [23]. Furthermore, these chaotic models can be verified on PSpice [24,25], analog circuits [26,27], while the collective behaviours of coupled oscillators [28,29] and coupled neuronal circuits [30] are discussed in network with different types of connection.

Within the control problem, it is important to consider the control cost such as power consumption [18,31] and transient period approaching the target orbits. In fact, for any dynamical systems, the oscillating behaviuors are much dependent on the energy supply. Therefore, it is important to discuss the energy transformation and supply in nonlinear dynamical systems. In nonlinear circuits, for example, in inductor-coupled-capacitor circuit, the inductor can store and collect magnetic field energy; the capacitor can collect and release energy of electric field, while the resistance often consumes Joule heat. Continuous electromagnetic radiation can calm down the nonlinear autonomous circuits because electromagnetic energy is released from the inductor and the capacitor when no external inputs are supplied. As a result, it is reliable to control the nonlinear circuits by modulating the supply of electromagnetic energy. For the dimensionless dynamical systems and chaotic oscillators, which can also be simulated by certain electric circuits, the generic Hamilton energy [32,33] can be approached using the famous Helmholtz's theorem.

In this paper, the Hamilton energy is calculated and negative feedback is applied to adjust the energy supply in the Lorenz system. It is found that different periodical orbits can be stabilised in the chaotic Lorenz system. Furthermore, intermittent feed-back in energy is used to control the chaotic behaviours.

2. Scheme and model

Generic dynamical system can also be described by the combination of vector and gradient field according to the physical law defined as Helmholtz's theorem [34], and it reads as

$$\dot{\vec{X}} = \vec{F}(\vec{r}) = \vec{F}_c(\vec{r}) + \vec{F}_d(\vec{r}) = -\nabla\phi(\vec{r}) + \nabla\times\vec{A}(\vec{r}), \quad (1)$$

where the variable is marked as $\boldsymbol{X} = \{x,\ y,\ z, \ldots\}$, and the gradient field is described by

$$\begin{cases} \vec{F}_d(\vec{r}) = -\nabla\phi(\vec{r}) = -\dfrac{1}{4\pi}\nabla\displaystyle\int_V \dfrac{\nabla'\cdot\vec{F}(r')}{|\vec{r}-\vec{r}'|}\mathrm{d}^3r' \\ \qquad +\dfrac{1}{4\pi}\nabla\displaystyle\oint_S \dfrac{\vec{F}(r')}{|\vec{r}-\vec{r}'|}\cdot\hat{n}\mathrm{d}^2r' \\ \nabla\times\vec{F}_d(\vec{r}) = 0 \end{cases} \quad (2)$$

and the rotational field is described by

$$\begin{cases} \vec{F}_c(\vec{r}) = \nabla\times\vec{A}(\vec{r}) = \dfrac{1}{4\pi}\nabla\times\displaystyle\int_V \dfrac{\nabla'\times\vec{F}(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathrm{d}^3r' \\ \qquad +\dfrac{1}{4\pi}\nabla\times\displaystyle\oint_S \dfrac{\vec{F}(\vec{r}')}{|\vec{r}-\vec{r}'|}\times\hat{n}\mathrm{d}^2r' \\ \nabla\cdot\vec{F}_c(\vec{r}) = 0 \end{cases}, \quad (3)$$

A generic RLC (resistance-coupled-capacitor-inductor) circuit composed of one resistance, one capacitor and one inductor, can be approached by

$$\begin{cases} C\dfrac{\mathrm{d}V}{\mathrm{d}t} = f(V,\ I) + i_{\text{ext}} \\ L\dfrac{\mathrm{d}I}{\mathrm{d}t} = g(V, I) \end{cases}, \quad (4)$$

where C and L are the capacitance and inductance, respectively. V, I and i_{ext} are the voltage of the capacitor, current across the inductor and external forcing current on the capacitor, respectively. As a result, the total energy of the circuit can be approached by

$$H = \frac{1}{2}LI^2 + \frac{1}{2}CV^2. \quad (5)$$

As is well known, the equations for the circuit can be mapped into dimensionless dynamical equations by using scale transformation. For example, eq. (4) is reproduced by

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}\tau} = f(x, y, p) + i'_{\text{ext}} \\ \dfrac{\mathrm{d}y}{\mathrm{d}\tau} = g(x, y, p) \end{cases}; \quad x = V/V_0, \quad y = I/I_0, \quad \tau = t/\sqrt{LC}, \quad (6)$$

where p represents the circuit parameters associated with L, C, and the energy function $H = H(x,\ y,\ p)$ in eq. (5) can be approached by using the scheme proposed in ref. [33]. The generic Hamilton energy can be calculated according to the required criterion as follows:

$$\begin{cases} \nabla H^T F_c(X) = 0, \\ \nabla H^T F_d(X) = \dot{H} = \mathrm{d}H/\mathrm{d}t. \end{cases} \quad (7)$$

For simplicity, the Lorenz system is used to verify the effectiveness of this scheme, and the possible control mechanism will be discussed. The Lorenz system is described by

$$\begin{cases} \dot{x} = \delta y - \delta x, \\ \dot{y} = \rho x - y - xz, \\ \dot{z} = xy - \beta z, \end{cases} \quad (8)$$

where δ, ρ, β are parameters, and chaotic behaviours can be observed at $\delta = 10$, $\rho = 28$, $\beta = 8/3$ even when arbitrary initial values are used for eq. (8). The Lorenz model can be replaced by

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 0 & \delta & 0 \\ \rho & 0 & -x \\ 0 & x & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} -\delta & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\beta \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \delta y \\ \rho x - xz \\ xy \end{pmatrix} + \begin{pmatrix} -\delta x \\ -y \\ -\beta z \end{pmatrix}. \quad (9)$$

As shown in ref. [34], the solution of the Hamilton energy can be found in eq. (10)

$$\frac{\partial H}{\partial x}(\delta y) + \frac{\partial H}{\partial y}(\rho x - xz) + \frac{\partial H}{\partial z}(xy) = 0. \quad (10)$$

As a result, a possible solution for eq. (10) is approached by

$$\begin{cases} H = \dfrac{1}{2}\left(-\frac{\rho}{\delta}x^2 + y^2 + z^2\right), \\ \dfrac{\mathrm{d}H}{\mathrm{d}t} = \rho x^2 - y^2 - \beta z^2. \end{cases} \quad (11)$$

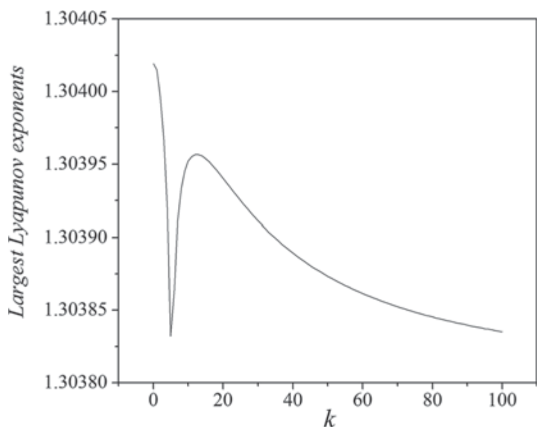

Figure 1. Distribution of the largest Lyapunov exponents vs. feedback gain k.

According to eq. (11), the Hamilton energy of Lorenz system is dependent on the selection of time-varying variables and two parameters. Therefore, the dynamical behaviours and outputs of Lorenz system will be changed when the Hamilton energy is controlled, and the controlled system is described by

$$\dot{H} = \frac{\mathrm{d}H}{\mathrm{d}t} = \rho x^2 - y^2 - \beta z^2 - kH, \tag{12}$$

where k is the feedback gain for the Hamilton energy. In fact, positive feedback gain can decrease the energy supply and thus the phase space can be controlled and as a result, the orbits and the attractors can be modulated.

3. Numerical results and discussion

In this section, the fourth-order Runge–Kutta algorithm is used to find solution for eqs (8) and (12). The parameters selected are: $\delta = 10$, $\rho = 28$, $\beta = 8/3$, time step $h = 0.01$, initial values of the variables are set as $(x_0, y_0, z_0, H_0) = (1, -5, 5, 0)$. The estimation of Lyapunov exponent [35] can predict the emergence of chaos and hyperchaos. Surely, other nonlinear analysis schemes are effective to detect the occurrence of chaos. For example, de la Fraga *et al* [35] suggested that meta-heuristics can be more effective for signal analysis and for discerning the chaotic series. The distribution of the largest Lyapunov exponents is calculated by using the Wolf scheme [36] in figure 1 by setting different feedback gains k in eq. (12).

It is found in figure 1 that the largest Lyapunov exponent remains positive even when negative feedback is applied on the Hamilton energy. Extensive numerical results confirmed that chaotic attractors will be alive even when the feedback gain is further increased. The possible mechanism could be that the Hamilton energy is dependent on all the variables which cooperate to support the phase space. As a result, all the variables are suppressed with the same scale and thus the chaotic attractors are kept alive. That is, the energy feedback should impose distinct impact on each variable, and thus the outputs can be modulated and suppressed. Therefore, an improved scheme is used to stabilise chaotic Lorenz system, and the dynamical equations are given by

$$\begin{cases} \dot{x} = \delta y - \delta x - k_1 x H, \\ \dot{y} = \rho x - y - xz, \\ \dot{z} = xy - \beta z, \\ \dot{H} = \rho x^2 - y^2 - \beta z^2 - kH. \end{cases} \tag{13}$$

When the second variable and third variable is respectively coupled with Hamilton energy, we get eqs (14) and (15).

$$\begin{cases} \dot{x} = \delta y - \delta x, \\ \dot{y} = \rho x - y - xz - k_2 y H, \\ \dot{z} = xy - \beta z, \\ \dot{H} = \rho x^2 - y^2 - \beta z^2 - kH, \end{cases} \tag{14}$$

$$\begin{cases} \dot{x} = \delta y - \delta x, \\ \dot{y} = \rho x - y - xz, \\ \dot{z} = xy - \beta z - k_3 z H, \\ \dot{H} = \rho x^2 - y^2 - \beta z^2 - kH, \end{cases} \tag{15}$$

where k_1, k_2, k_3 define the coupling intensity or dependence of each variable on the Hamilton energy. Appropriate feedback and coupling intensity are selected to detect the outputs and suppression of chaos.

The results in figure 2 confirmed that positive Lyapunov exponents can be decreased and chaotic behaviours can be suppressed when negative Hamilton energy is feedbacked into the Lorenz system. When the three-variable Lorenz system is out of control, the variables x, y can switch between negative and positive values, while the third variable z often holds positive value in large scope. Therefore, a negative value is chosen for the coupling intensity k_3 so that the chaotic state can be controlled by negative feedback. Furthermore, the distribution of largest Lyapunov exponents is calculated in two-parameter space for k vs. $k_i (i = 1, 2, 3)$, and the results are plotted in figure 3.

It is confirmed that the largest Lyapunov exponent can become negative by setting appropriate coupling intensity $k_i (i = 1,\ 2,\ 3)$ and feedback gain k. The phase portraits are plotted in figure 4.

Figure 4 shows that the Lorenz system decreases to a stable point when appropriate coupling intensity and feedback gain on energy function are applied. The sampled time series for variables and Hamilton energy are also calculated; extensive numerical results confirmed the stability approach when energy is controlled. We

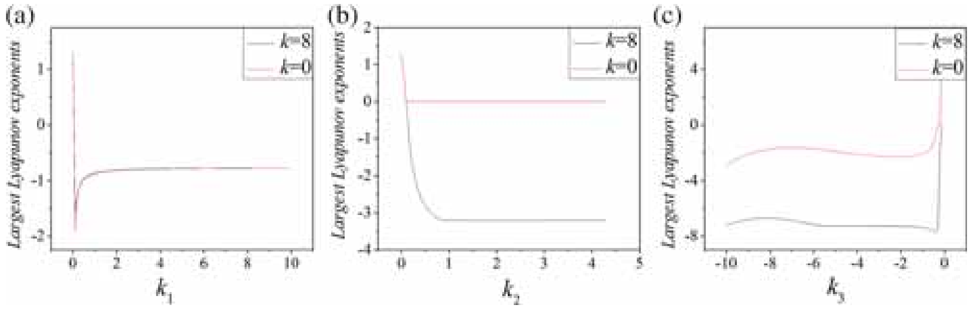

Figure 2. Distribution of the largest Lyapunov exponents by setting different feedback gains for (**a**) $k_2 = k_3 = 0$, (**b**) $k_1 = k_3 = 0$ and (**c**) $k_1 = k_2 = 0$.

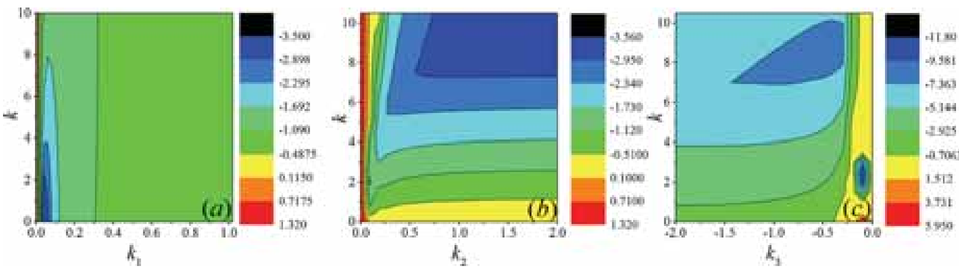

Figure 3. Distribution for the largest Lyapunov exponents in parameter space for (**a**) $k_2 = k_3 = 0$, (**b**) $k_1 = k_3 = 0$ and (**c**) $k_1 = k_2 = 0$. The snapshots are plotted in colour scale.

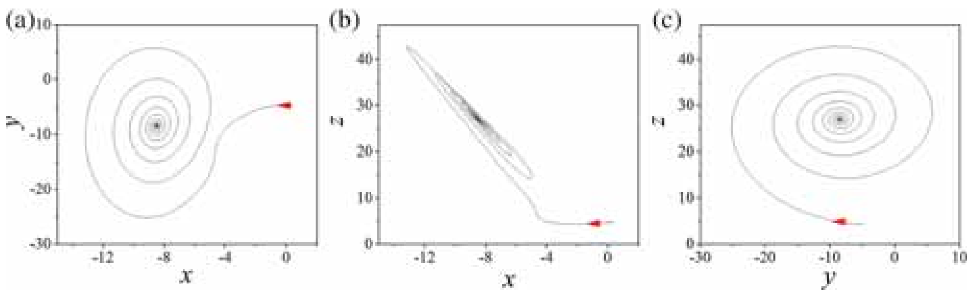

Figure 4. Phase portraits (**a**) x–y space, (**b**) x–z space and (**c**) y–z space for the variables are calculated by setting appropriate coupling intensity and feedback gain when $k = 8$, $k_1 = 1$, $k_2 = k_3 = 0$.

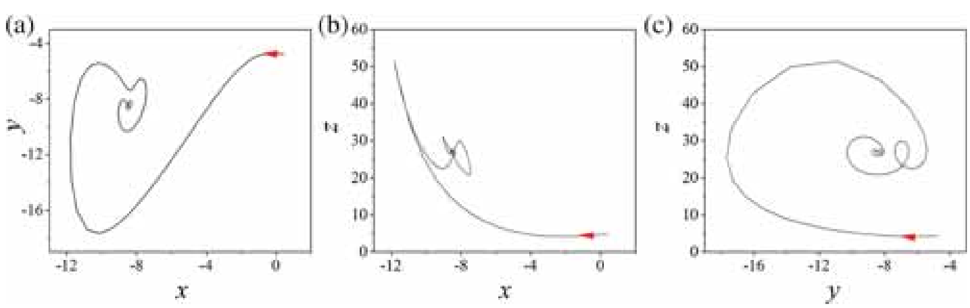

Figure 5. Phase portraits (**a**) x–y space, (**b**) x–z space and (**c**) y–z space for the variables are calculated by setting appropriate coupling intensity and feedback gain when $k = 8$, $k_3 = -0.4$, $k_1 = k_2 = 0$.

also investigated the case shown in eq. (15), and the results are plotted in figure 5.

Similar to the results shown in figure 4, the unstable orbits can be stabilised completely and the chaotic behaviours are suppressed. When the control cost is considered, it is feasible to adopt intermittent feedback scheme, and for simplicity, the feedback is switched on–off by using a rectangular square wave with a period T.

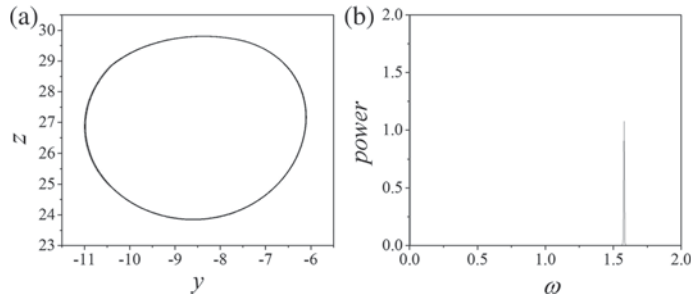

Figure 6. Period-1 orbit is approached and power spectrum is calculated from sampled time series for variable x, $H_{\max} \geq 470$, $x = -9.5$; $H_{\min} \leq 110$, $x = -10$. (**a**) Phase portrait and (**b**) power spectrum.

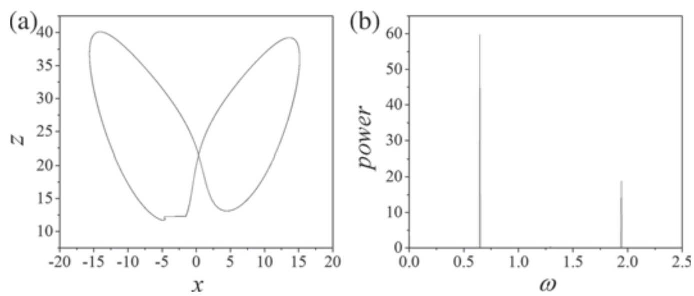

Figure 7. Period-2 orbit is approached and the power spectrum is calculated from sampled time series for variable x, $H_{\max} \geq 950$, $x = 0.22$; $H_{\min} \leq 79$, $x = -4.65$. (**a**) Phase portrait and (**b**) power spectrum.

The numerical results confirmed that the chaotic state can also be controlled by applying appropriate value for T. The mechanism could be that appropriate intermittent period T decreases energy cost because the system can automatically approach stable orbit in some transient period.

It is confirmed that enough phase space is critical to support chaotic attractors. Therefore, the chaotic states can be controlled to reach periodical orbits by applying phase-space compression [37,38] on the dynamical systems, and synchronisation [39] can be realised as well. Ma *et al* [40] confirmed that the phase-space scheme is equivalent to intermittent feedback which can be realised by using Heaviside function. Inspired by this idea, it is interesting to discuss this problem by using compression in energy. For example,

$$x = x_{\max},\ H \geq H_{\max};\quad x = x_{\min},\ H \leq H_{\min};$$
$$x = x,\ H_{\min} < H < H_{\max}, \tag{16}$$

where $H_{\max}$, $H_{\min}$, $x_{\max}$, $x_{\min}$ are the upper and lower thresholds for Hamilton energy, and variable x, respectively. That is, the outputs are suppressed to approach maximal value when the Hamilton energy is beyond the fixed threshold. On the other hand, the outputs are enhanced to keep a minimal value when the Hamilton energy is below the fixed threshold. Otherwise, the system will be developed without external constraint. In figure 6, the first variable is controlled according to the fluctuation in Hamilton energy, and periodical orbit is stabilised completely.

From figures 6, 7 and 8 we can see that many periodical orbits can be reached by using intermittent energy control and feedback. Extensive results confirmed the emergence of distinct periodicity when fast Fourier transform (FFT) algorithm is carried out on the sampled time series. Surely, appropriate thresholds can be selected to reach the control target.

In summary, energy supply is important and critical for supporting oscillating behaviours and motion. As a result, the oscillating behaviours can be controlled when the energy flow is modulated carefully. For financial systems, this scheme is helpful to enhance safety of monetary market by restricting the supply of currency, credit and loan. Finally, some open problems on this topic can be suggested for further discussion. This paper just discusses the case of an isolated oscillator without external disturbance. Noise and mixed signals are often imposed on dynamical systems, and phase transition can be induced. Indeed, in physical view, noise driving can change the Hamilton energy of the dynamical systems and the dynamics can be modulated. For example, noise can cause transition between sleep and wake states in nervous system [41]. Noise can also trigger bursting behaviour in a chaotic optic system [42]. Furthermore, noise-like electromagnetic radiation [43–45] can change the modes selection in electrical activities of neurons, and thus the biological function of the nervous system

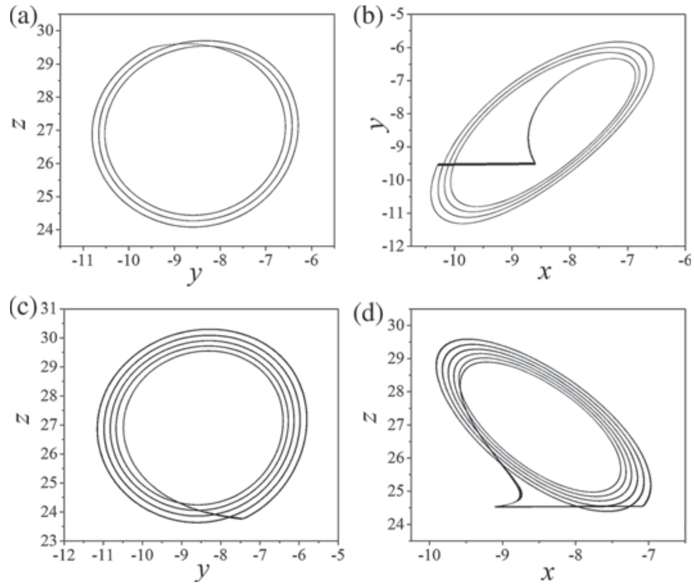

Figure 8. Period-n ($n = 3, 4, 5, 6$) orbit is approached and the power spectrum is calculated from the sampled time series for the variable x for (**a**) $n = 3$, $H_{\max} \geq 480$, $x = -9$, $H_{\min} \leq 110$, $x = -10$, (**b**) $n = 4$, $H_{\max} \geq 500$, $x = -8.6$, $H_{\min} \leq 300$, $x = -10$, (**c**) $n = 5$, $H_{\max} \geq 505$, $x = -9$, $H_{\min} \leq 310$, $x = -9.1$ and (**d**) $n = 6$, $H_{\max} \geq 500$, $x = -9$, $H_{\min} \leq 330$, $x = -9.1$.

can be modulated greatly. Particularly, noise can induce synchronisation between neurons and coupled oscillators [46,47], and the possible mechanism could be that enough energy supply can enhance orbit selection and ergodicity. Therefore, this question can be further investigated for chaotic system under noise. On the other hand, pinning control with energy feedback can also be attractive for network synchronisation and control. With respect to network problems, synchronisation and control are appreciated while consensus control is more important for multiagent systems [48]. The authors wish that this scheme can be further discussed for consensus control of multiagent systems and network stability.

4. Conclusions

In this paper, a general Hamilton energy for low-dimensional dynamical systems is introduced. The chaotic Lorenz system is used to check the effectiveness of this scheme, where the Hamilton energy is modulated intermittently. The Hamilton energy is modulated in an appropriate region by adjusting the energy supply and release, and it is found that the chaotic orbits can be controlled to reach arbitrary periodical states completely.

Acknowledgements

This work is partially supported by National Natural Science Foundation of China under Grant No. 11372122.

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