Pramana – J. Phys. (2018) 90:25
https://doi.org/10.1007/s12043-017-1517-6

Cluster-modified function projective synchronisation of complex networks with asymmetric coupling

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MS received 13 March 2017; revised 9 August 2017; accepted 28 September 2017;
published online 18 January 2018

Abstract. This paper investigates the cluster-modified function projective synchronisation (CMFPS) of a generalised linearly coupled network with asymmetric coupling and nonidentical dynamical nodes. A novel synchronisation scheme is proposed to achieve CMFPS in community networks. We use adaptive control method to derive CMFPS criteria based on Lyapunov stability theory. Each cluster of networks is synchronised with target system by state transformation with scaling function matrix. Numerical simulation results are presented finally to illustrate the effectiveness of this method.

Keywords. Complex networks; cluster-modified function projective synchronisation; adaptive control; asymmetric coupling.

PACS Nos **05.45.Xt; 05.45.Gg; 05.45.Pq**

1. Introduction

In recent years, as a typical collective behaviour, synchronisation of complex dynamical networks has attracted more and more attention. Many scientists from various fields have focussed their attention on the control and synchronisation of complex networks such as biology, mathematics, sociology, and so on. So far, they have proposed many types of synchronisations [1–6], such as complete synchronisation, phase synchronisation, anticipated synchronisation, projective synchronisation, generalised synchronisation, lag synchronisation, etc.

Recently, many researchers were interested in cluster synchronisation of complex networks and some conditions guaranteeing cluster synchronisation have been presented by them [7–11]. The cluster synchronisation of the network has gradually aroused people's wide attention. Cluster synchronisation of a network means that all nodes in that complex network are divided into several clusters. Synchronisation between nodes in each cluster or between the node and the synchronisation target can be achieved, but synchronisation between the nodes in different clusters cannot be achieved. Yang *et al* [9] investigated the cluster synchronisation of network with community structure, and they designed effective controllers to construct an effective control scheme and adjust automatically the adaptive external coupling strength by taking external coupling strength as adaptive variables on a small fraction of network edges. Hu and Jiang [12] focussed on driving a class of directed networks to achieve cluster synchronisation by pinning scheme and answered several challenging problems in pinning control of directed community networks: What communities should be chosen as controlled candidates? How many communities are needed to be controlled and how large should the control gains be in a given community network to achieve cluster synchronisation? A new technique for the cluster synchronisation between uncertain networks with different dynamics has been proposed by Lü *et al* [13]. Based on the Lyapunov theorem and Lipschitz condition, the controllers and the identification laws of uncertain parameters are designed. They are efficiently used to achieve cluster synchronisation and identification of uncertain parameters. Feng *et al* [14] investigated the mean square exponential cluster synchronisation of complex networks with nonlinear hybrid time-varying delays and stochastic noises via periodically intermittent pinning control. Li [15] investigated H_∞ cluster synchronisation and a novel concept of H_∞ cluster synchronisation is proposed to

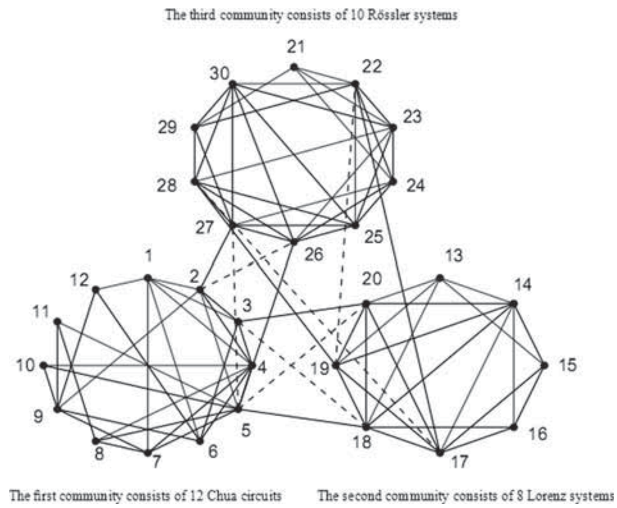

Figure 1. The network with three communities.

quantify against the exogenous disturbance of the complex networks. H_∞ cluster synchronisation criteria are derived based on the Lyapunov stability theory. Zhou *et al* [16] investigated cluster synchronisation on multiple sub-networks of complex networks with stochastic disturbances and time-varying delays. Based on the leader–follower model, an improved network structure model for realising the cluster synchronisation on multiple sub-networks of complex networks is presented. However, all the above results are concerned with traditional cluster synchronisation of networks with identical or non-identical nodes. As far as we know, the results on cluster-modified function projective synchronisation (CMFPS) of dynamical complex networks with asymmetric coupling are few. Du *et al* [17] discussed a new type of synchronisation phenomenon, modified function projective synchronisation (MFPS), where the drive and the response systems could be synchronised up to a desired scaling function matrix. Particularly, MFPS is the more general definition of MPS and FPS when the scaling function matrix is chosen by a constant matrix and a scaling function, respectively. Qiu *et al* [18] investigated the modified function projective synchronisation problem for general complex networks with multiple proportional delays. Du *et al* [19] investigated MFPS. MFPS means that the drive and the response systems could be synchronised up to a desired scaling function matrix. Zheng [20] investigated MFPS between two different dimensional chaotic systems with fully unknown or partially unknown parameters via increased order. So it is challenging to deal with CMFPS of dynamical complex networks with asymmetric coupling.

Inspired by the above discussions, this paper investigates CMFPS in community networks [21], topological structure of which is shown in figure 1. Compared to the existing literatures about cluster synchronisation of networks, the main contributions of this paper are fourfold: (1) Each cluster has different number of nodes and different topology and the local dynamics systems are nonidentical in different communities. (2) Based on the structure of the networks, the controllers are designed differently between the nodes in one community which have direct connections to the nodes in other communities and the nodes without direct connections with the nodes in other communities. (3) It expands the types of traditional cluster synchronisation and makes the scaling factor develop from a single value to a function matrix. (4) The weight configuration matrix is not assumed to be symmetric and irreducible. By using the methods in [22] to investigate CMFPS of networks with asymmetric coupling, some sufficient conditions are derived. Numerical simulations are performed to verify the effectiveness of the theoretical results.

The paper is organised as follows: The network model is introduced in §2. The CMFPS of the complex networks with nonidentical dynamical nodes is discussed in §3. Simulations are obtained in §4. In §5 various conclusions are discussed.

Some necessary mathematical notations used throughout this paper are first introduced as follows. Let A^T (or x^T) be the transpose of the matrix A (or vector x). The $\otimes$ denotes the Kronecker product of two matrices, and $\lambda_{\max}(A)$ represents the maximum eigenvalue of a square matrix A. $\|A\|$ denotes the Euclidean norm.

2. Model and preliminaries

Consider that a community network consists of N nodes and m communities with $(2 \le m \le N)$, let $\{C_1, C_2, \ldots, C_m\}$ denote m communities of the networks and $\cup_{i=1}^{m} C_i = \{1, 2, \ldots, N\}$. The network can be described by

$$\dot{x}_i(t) = A_{\phi_i} x_i(t) + f_{\phi_i}(t, x_i(t)) + \sum_{j=1}^{N} g_{ij} x_j(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in_k}(t))^T \in R^{n_k}$ is the state vector of node i. $A_{\phi_i} \in R^{n_k \times n_k}$ is a constant matrix. The function ϕ is defined as $\phi: \{1, 2, \ldots, N\} \to \{1, 2, \ldots, m\}$. $f_{\phi_i}: R^{n_k} \to R^{n_k}$ describes smooth nonlinear vector field. We employ $\varphi_{\phi_i}(\cdot) = A_{\phi_i} x_i(t) + f_{\phi_i}(t, x_i(t))$ to represent the local dynamics of all nodes in the ϕ_ith community. For any pair of nodes i and j, if $\phi_i \neq \phi_j$, i.e., nodes i and j belong to different communities, then $\varphi_{\phi_i} \neq \varphi_{\phi_j}$. $G = (g_{ij})_{N \times N} \in R^{N \times N}$ is the weight configuration matrix. If there exists a connection from node i to node $j\,(j \neq i)$, then $g_{ij} > 0$. Otherwise, $g_{ij} = g_{ji} = 0$, the diagonal elements of matrix G are defined by

$$g_{ii} = -\sum_{j=1,\, j\neq i}^{N} g_{ij}, \quad i = 1, 2, \ldots, N.$$

The weight configuration matrix is not assumed to be symmetric and irreducible.

When controlling inputs $u_i(t) \in R^{n_k}$ $(i = 1, 2, \ldots, N)$ are introduced, the controlled dynamical network with respect to network (1) can be written as

$$\dot{x}_i(t) = A_{\phi_i} x_i(t) + f_{\phi_i}(t, x_i(t)) + \sum_{j=1}^{N} g_{ij} x_j(t) + u_i(t), \quad i = 1, 2, \ldots, N. \tag{2}$$

DEFINITION 1

Assume that $s(t) \in R^n$ is any smooth dynamics. For a dynamical network model with asymmetric coupling and nonidentical dynamical nodes, it is said that the dynamical network model (1) is CMFPS, if there exists a scaling function matrix $M_{\phi_i}(t)$, such that

$$\lim_{t\to\infty} \|e_i(t)\| = \lim_{t\to\infty} \left\| x_i(t) - M_{\phi_i}(t)s(t) \right\| = 0,\ i = 1, 2, \ldots, N. \tag{3}$$

Let the vector error state be $e_i = x_i(t) - M_{\phi_i}(t)s(t)$, $i = 1, 2, \ldots, N$, where $M_{\phi_i}(t)$ is an n-order real diagonal matrix, i.e. $M_{\phi_i}(t) = \mathrm{diag}\big(m_1^{\phi_i}(t), m_2^{\phi_i}(t), \ldots, m_n^{\phi_i}(t)\big)$ and $m_k^{\phi_i}(t), k = 1, \ldots, n$ is a continuously bounded differentiable function and $\big(\dot{m}_1^{\phi_i}(t), \dot{m}_2^{\phi_i}(t), \ldots, \dot{m}_n^{\phi_i}(t)\big)^T = (S_1(m_1^{\phi_i}, m_2^{\phi_i}, \ldots, m_n^{\phi_i}, t), S_2(m_1^{k}, m_2^{\phi_i}, \ldots, m_n^{\phi_i}, t), \ldots, S_n(m_1^{\phi_i}, m_2^{\phi_i}, \ldots, m_n^{\phi_i}, t))^T$. There exists a stable equilibrium point, a stable periodic orbit or a chaotic attractor in the phase space.

3. Controller design for CMFPS

Let synchronisation errors $e_i(t) = x_i(t) - M_{\phi_i}(t)s(t)$ for $i = 1, 2, \ldots, N$. According to networks (1), the error dynamical system can be derived as

$$\begin{aligned}\dot{e}_i(t) = {} & A_{\phi_i} e_i(t) + A_{\phi_i}(M_{\phi_i}(t)s(t)) - M_{\phi_i}(t)\dot{s}(t) \\ & - \dot{M}_{\phi_i}(t)s(t) + f_{\phi_i}(t, x_i(t)) \\ & + \sum_{j=1}^{N} g_{ij} e_j(t) + \sum_{j=1}^{N} g_{ij}(M_{\phi_i}(t)s(t)) + u_i(t).\end{aligned} \tag{4}$$

According to the diffusive coupling condition of the matrix G,

$$\sum_{j=1}^{N} g_{ij}(M_{\phi_i}(t)s(t)) = 0,\ i \in J_{\phi_i} - \bar{J}_{\phi_i}, \tag{5}$$

where J_{ϕ_i} denotes all the nodes in the ϕ_ith community and $\bar{J}_{\phi_i}$ represents the nodes in the ϕ_ith community which have direct links with the nodes in other communities. We have

$$\begin{cases} \dot{e}_i(t) = A_{\phi_i} e_i(t) + A_{\phi_i}(M_{\phi_i}(t)s(t)) - M_{\phi_i}(t)\dot{s}(t) \\ \qquad - \dot{M}_{\phi_i}(t)s(t) + f_{\phi_i}(t, x_i(t)) \\ \qquad + \sum_{j=1}^{N} g_{ij} e_j(t) + \sum_{j=1}^{N} g_{ij}(M_{\phi_i}(t)s(t)) \\ \qquad + u_i(t), \qquad i \in \bar{J}_{\phi_i} \\ \dot{e}_i(t) = A_{\phi_i} e_i(t) + A_{\phi_i}(M_{\phi_i}(t)s(t)) - M_{\phi_i}(t)\dot{s}(t) \\ \qquad - \dot{M}_{\phi_i}(t)s(t) + f_{\phi_i}(t, x_i(t)) \\ \qquad + \sum_{j=1}^{N} g_{ij} e_j(t) + u_i(t),\ i \in J_{\phi_i} - \bar{J}_{\phi_i}. \end{cases} \tag{6}$$

It is easy to see that the CMFPS of the controlled complex network (2) is achieved if the zero solution of the error system (4) is globally asymptotically stable which is ensured by the following theorem.

Theorem 1. *If the weight configuration matrix G is an asymmetric matrix, the off-diagonal elements of G are not assumed to be nonnegative and $g_{ij} + g_{ji} \geq 0$, $i \neq j$. The matrix G can be decomposed into three matrices*

$$G = \bar{G} + \tilde{G} + \hat{G} \tag{7}$$

where $\bar{G}$ is a symmetric and zero row sum matrix given by

$$\bar{G} = [\bar{g}_{ij}]: \begin{cases} \bar{g}_{ij} = (g_{ij} + g_{ji})/2,\ i \neq j \\ \bar{g}_{ij} = -\sum_{k=1,k\neq i}^{N} \bar{g}_{ik},\ i = j \end{cases}$$

$\tilde{G}$ is a antisymmetric matrix given by

$$\tilde{G} = [\tilde{g}_{ij}]: \begin{cases} \tilde{g}_{ij} = (g_{ij} - g_{ji})/2,\ i \neq j \\ \tilde{g}_{ij} = 0,\ i = j \end{cases}$$

and $\hat{G}$ is a diagonal matrix given by

$$\hat{G} = [\hat{g}_{ij}]: \begin{cases} \hat{g}_{ij} = 0,\ i \neq j \\ \hat{g}_{ij} = -\sum_{k=1,k\neq i}^{N} \hat{g}_{ik},\ i = j \end{cases}.$$

The controlled complex networks (2) *is CMFPS via the control law as below:*

$$\begin{aligned} v_i(t) &= -A_{\phi_i}(M_{\phi_i}(t)s(t)) + M_{\phi_i}(t)\dot{s}(t) \\ &\quad +\dot{M}_{\phi_i}(t)s(t) - f_{\phi_i}(t, x_i(t)) \\ u_i(t) &= \begin{cases} v_i(t) - d_i e_i(t) - \sum_{j=1}^{N} g_{ij}(M_{\phi_i}(t)s(t)), & i \in \bar{J}_{\phi_i} \\ v_i(t) - d_i e_i(t), & i \in J_{\phi_i} - \bar{J}_{\phi_i} \end{cases} \end{aligned} \tag{8}$$

where $\dot{d}_i = \theta_i e_i^T(t)\, e_i(t)$, d_i *are the feedback strength and* θ_i *are arbitrary positive constants.*

Proof. Construct the Lyapunov functional

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{(d_i - d^*)^2}{\theta_i},$$

where d^* is a positive value to be determined. Calculating the derivative of $V(t)$, we have

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{N} e_i^T(t)\left[A_{\phi_i}e_i(t) + \sum_{j=1}^{N} g_{ij}e_j(t) - d_i e_i(t)\right] \\ &\quad + \sum_{i=1}^{N}(d_i - d^*)e_i^T(t)\, e_i(t) \\ &= \sum_{i=1}^{N} e_i^T(t)A_{\phi_i}e_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} g_{ij}e_i^T(t)e_j(t) \\ &\quad - \sum_{i=1}^{N} d_i e_i^T(t)e_i(t) + \sum_{i=1}^{N}(d_i - d^*)e_i^T(t)\, e_i(t) \\ &= \sum_{i=1}^{N} e_i^T(t)A_{\phi_i}e_i(t) + \bar{M} + \tilde{M} + \hat{M} \\ &\quad - \sum_{i=1}^{N} d^* e_i^T(t)\, e_i(t). \end{aligned}$$

Let $\tilde{e}_j(t) = (e_{1j}(t), \ldots, e_{Nj}(t))$, $j = 1, 2, \ldots, n$, and $a_i = \sum_{j=1}^{N} g_{ji}$. Shi *et al* in [22] have established the following inequality:

$$\begin{aligned} &\bar{M} + \tilde{M} + \hat{M} - \sum_{i=1}^{N} d^* e_i^T(t)\, e_i(t) \\ &\leq \frac{1}{2}\Bigg(\sqrt{\lambda_{\min}(G^T G)} \\ &\quad - \min_{i\in\{1,\ldots,N\}}\{a_i\} - 2d^*\Bigg)\sum_{j=1}^{n}\tilde{e}_j^T(t)\tilde{e}_j(t). \end{aligned}$$

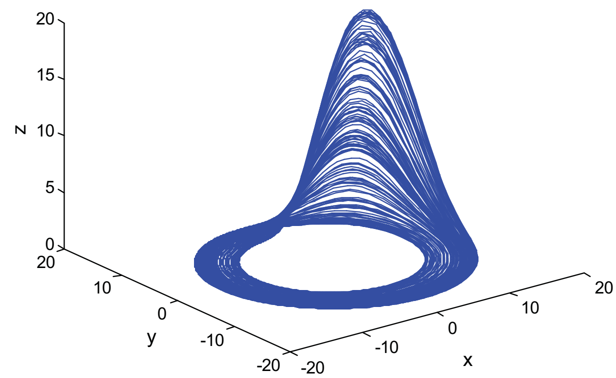

Figure 2. The chaotic behaviour of the target system.

Let $u = \max(\|A_{\phi_i}\|_2)$ we can obtain

$$\begin{aligned} \dot{V}(t) &\leq \Bigg[u + \frac{1}{2}\Bigg(\sqrt{\lambda_{\min}(G^T G)} \\ &\quad - \min_{i\in\{1,\ldots,N\}}\{a_i\} - 2d^*\Bigg)\Bigg]\sum_{j=1}^{n}\tilde{e}_j^T(t)\tilde{e}_j(t). \end{aligned}$$

Taking

$$d^* \geq u + \frac{1}{2}\sqrt{\lambda_{\min}(G^T G)} - \frac{1}{2}\min_{i\in\{1,\ldots,N\}}\{a_i\},$$

we obtain

$$\dot{V}(t) < -e^T(t)e(t).$$

According to the Lyapunov stability theory, the proof is completed. □

Remark 1. In the networks, each cluster has different topology and number of nodes, and to achieve CMFPS, we first need to detect the community structure of the networks. Next, the controllers are designed differently between the nodes in one community which have direct connections to the nodes in other communities and the nodes without direct connections with the nodes in other communities. It indeed provides more new insights for the future practical engineering design than the previous papers. Community networks with delays can also be well-investigated.

Remark 2. In this paper, each community is formed by a group of identical oscillators, but different from the oscillators of the other communities. Node dynamics of networks have identical dimensions, which are more general than the network model in the previous papers. The proposed strategy is applicable to almost all types of complex community networks.

COROLLARY 1

When the scaling factor $M_{\phi_i} \in R^{n\times n}$, the controlled complex networks (2) *is CMFPS via the control law as below*:

$$v_i(t) = -A_{\phi_i}(M_{\phi_i}s(t)) + M_{\phi_i}\dot{s}(t) - f_{\phi_i}(t, x_i(t))$$
$$u_i(t) = \begin{cases} v_i(t) - d_i e_i(t) - \sum_{j=1}^{N} g_{ij}(M_{\phi_i}s(t)), i \in \bar{J}_{\phi_i} \\ v_i(t) - d_i e_i(t)\,, \quad i \in J_{\phi_i} - \bar{J}_{\phi_i} \end{cases} \tag{9}$$

where $\dot{d}_i = k_i e_i^T(t)\, e_i(t)$, d_i are the feedback strength and k_i are arbitrary positive constants.

4. Illustrative examples

In this section, a numerical example will be given to demonstrate the validity of the synchronisation criteria obtained in the previous section. In order to simplify the calculation, consider a community network consisting of 30 three-dimensional chaotic systems. Consider a dynamical network with three communities.

The first community consists of 12 Chua circuits [23], which are described by

$$A_1x_i(t) + f_1(t, x_i(t)) = \begin{bmatrix} -\frac{10}{7} & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -12 & 0 \end{bmatrix} \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \\ x_{i3}(t) \end{bmatrix} + \begin{bmatrix} -\frac{20}{7}(x_{i1}(t))^3 \\ 0 \\ 0 \end{bmatrix},$$
$$i = 1, 2, \ldots, 12.$$

The second community consists of 8 Lorenz systems [24], which are described by

$$A_2x_i(t) + f_2(t, x_i(t)) = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix} \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \\ x_{i3}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -x_{i1}(t)x_{i3}(t) \\ x_{i1}(t)x_{i2}(t) \end{bmatrix},$$
$$i = 13, 14, \ldots, 20.$$

The third community consists of 10 Rössler systems [25], which are described by

$$A_3x_i(t) + f_3(t, x_i(t)) = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & -5.7 \end{bmatrix} \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \\ x_{i3}(t) \end{bmatrix}$$

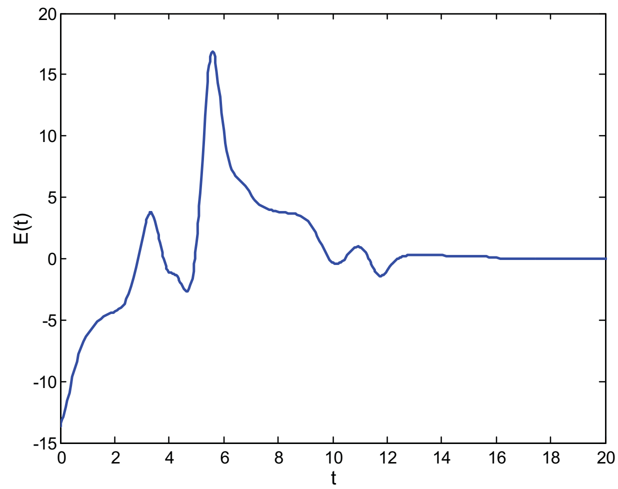

Figure 3. Time evolution of total synchronisation errors $E(t)$.

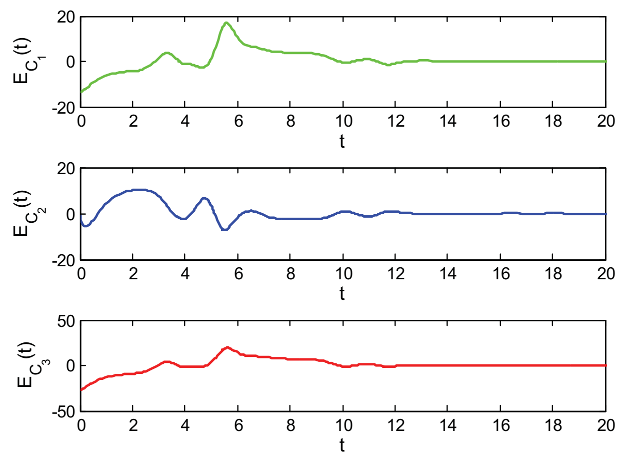

Figure 4. Time evolution of synchronisation errors $E_{C_i}(t), i = 1, 2, 3$.

$$+ \begin{bmatrix} 0 \\ 0 \\ 0.2 + x_{i1}(t)x_{i3}(t) \end{bmatrix}, \quad i = 21, 22, \ldots, 30.$$

The target system is described as follows:

$$s(t)\colon \begin{cases} \dot{x}_1 = -x_2 - x_3 \\ \dot{x}_2 = x_1 - 0.1x_2 \\ \dot{x}_3 = x_1x_3 - 10x_3 + 0.1 \end{cases}.$$

It is the Rossler system.

We take scaling function matrix

$$M_1(t) = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & -5.7 \end{bmatrix} \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \\ x_{i3}(t) \end{bmatrix}$$

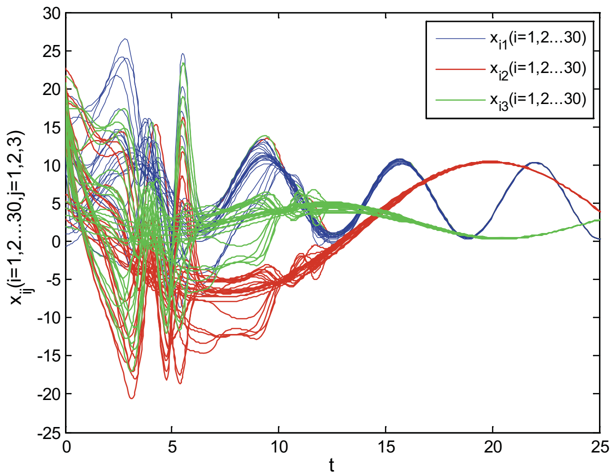

Figure 5. Orbits of state variables $x_i(t)$. The blue, red and green lines denote the orbits of $x_{i1}(t)$, $x_{i2}(t)$ and $x_{i3}(t)$ respectively.

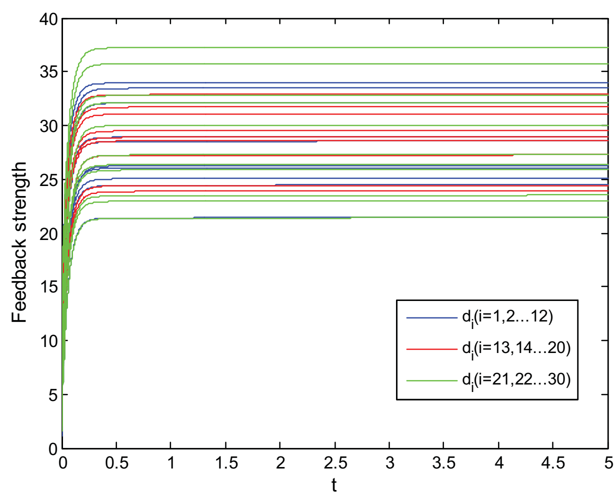

Figure 6. Orbits of the adaptive feedback strength d_i. The blue, red and green lines denote the orbits in the first, second and third community, respectively.

$$
+\begin{bmatrix} 0 \\ 0 \\ 0.2 + x_{i1}(t)x_{i3}(t) \end{bmatrix},
$$

$$
M_2(t) = \begin{bmatrix} -\frac{10}{7} & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -12 & 0 \end{bmatrix} \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \\ x_{i3}(t) \end{bmatrix}
+ \begin{bmatrix} -\frac{20}{7}(x_{i1}(t))^3 \\ 0 \\ 0 \end{bmatrix},
$$

$$
M_3(t) = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix} \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \\ x_{i3}(t) \end{bmatrix}
$$

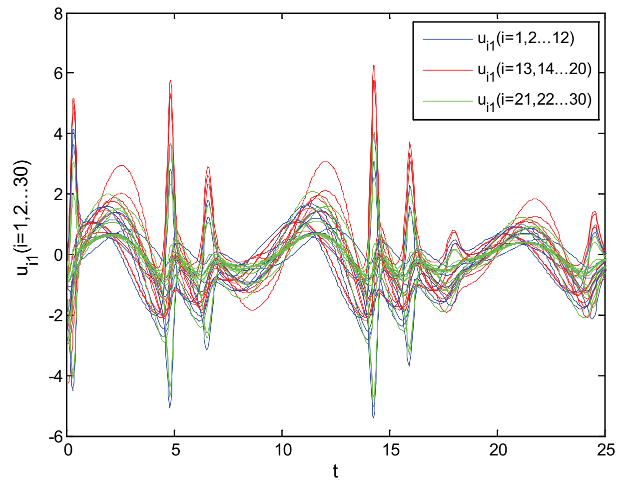

Figure 7. Time series of the controller u_{i1}, $i = 1, 2, \ldots, 30$. The blue, red and green lines denote the orbits in the first, second and third communities, respectively.

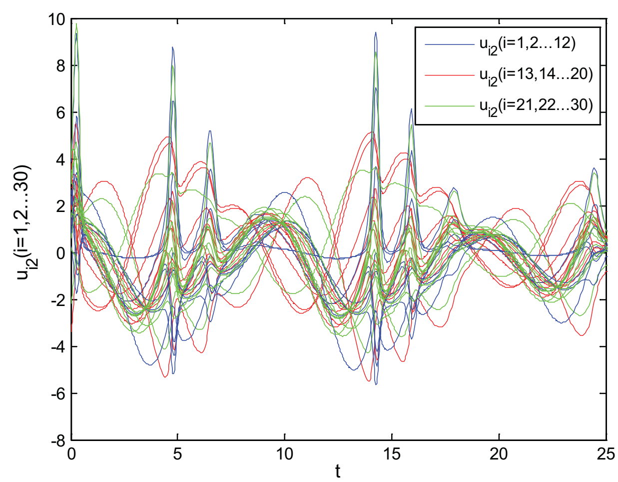

Figure 8. Time series of the controller u_{i2}, $i = 1, 2, \ldots, 30$. The blue, red and green lines denote the orbits in the first, second and third communities, respectively.

$$
+\begin{bmatrix} 0 \\ -x_{i1}(t)x_{i3}(t) \\ x_{i1}(t)x_{i2}(t) \end{bmatrix}.
$$

The coupling matrix G is given by

$$
G = (g_{ij})_{30\times 30}
= \begin{bmatrix} -5 & 1 & 1 & \cdots & \cdots & 0 \\ 1 & -7 & 1 & \cdots & \cdots & 0 \\ 1 & 1 & -7 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & -5 & 1 \\ 0 & 0 & 0 & \cdots & 1 & -7 \end{bmatrix}_{30\times 30}
$$

and the diagonal elements of matrix G are defined by

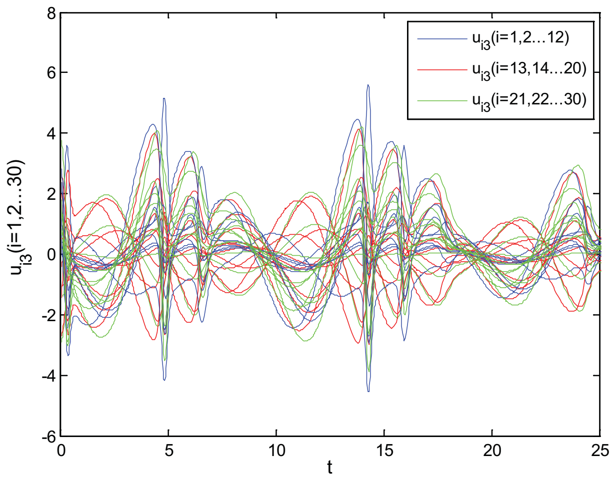

Figure 9. Time series of the controller $u_{i3}, i = 1, 2, \ldots, 30$. The blue, red and green lines denote the orbits in the first, second and third communities, respectively.

$$g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij},\ i = 1, 2, \ldots, N.$$

$u = \max(\|A_1\|_2, \|A_2\|_2, \|A_3\|_2) = \max(15.6833, 29.8423, 5.7897) = 29.8423$, $\lambda_{\min}(G^T G) = 0.3028$. Taking $d^* = 35$, according to the Lyapunov stability theory, we can obtain

$$d^* \geq u + \frac{1}{2}\sqrt{\lambda_{\min}(G^T G)} - \frac{1}{2}\min_{i\in\{1,\ldots,N\}}\{a_i\}.$$

The following quantities are utilised to measure the CMFPS:

$$\begin{cases} E(t) = \sum_{i=1}^{12}[x_i(t) - M_1(t)s(t)] \\ \qquad + \sum_{j=13}^{20}\left[x_j(t) - M_2(t)s(t)\right] \\ \qquad + \sum_{k=21}^{30}[x_k(t) - M_3(t)s(t)], \\ E_{C_1}(t) = \sum_{i=1}^{12}\sum_{j=1}^{3} e_{ij} \\ E_{C_2}(t) = \sum_{i=13}^{20}\sum_{j=1}^{3} e_{ij} \\ E_{C_3}(t) = \sum_{i=21}^{30}\sum_{j=1}^{3} e_{ij} \end{cases} \tag{10}$$

where $E(t)$ is the total synchronisation error of CMFPS for this controlled network (2). $E_{C_1}(t)$ is the synchronisation error of CMFPS for the first community, and $E_{C_2}(t)$ is the synchronisation error of CMFPS for the second community and $E_{C_3}(t)$ is the synchronisation error of CMFPS for the third community. CMFPS is achieved if the synchronisation error $E(t)$ and $E_{C_i}(t)$, $i = 1, 2, 3$ converge to zero. Figure 2 shows the chaotic behaviour of the target system. Figure 3 shows the time evolution of the total synchronisation errors $E(t)$. Figure 4 shows the time evolution of synchronisation errors $E_{C_i}(t)$, $i = 1, 2, 3$. Figure 5 shows the orbits of state variables $x_i(t)$. In figure 6 we see orbits of the adaptive feedback strength. From figures 7, 8 and 9, we see that the external control input vector is really small, and correspondingly, it is seen that the control inputs are feasible to implement in practice. The numerical results show that Theorem 1 is effective.

Remark 3. In this section, the dynamical network with three communities is studied. Node dynamics in different communities are three-dimensional systems. Each cluster is asymptotically modified function projective synchronised with the same target system. The synchronised state is similar to the target system, and it is possible to use the method to get any desired synchronised dynamics or oscillations.

Remark 4. Complex networks are huge networks with very irregular structure, and the size of the network should be larger than 1000. But, in order to simplify the calculation, we consider a community network consisting of 30 three-dimensional chaotic systems. Community networks have different topology and number of nodes in different communities.

5. Conclusions

In this paper, a CMFPS scheme is proposed for dynamical networks with nonidentical dynamical nodes. Based on Lyapunov stability theory, a simple adaptive controller is designed to achieve CMFPS by the community structure of networks and each cluster is asymptotically modified function projective synchronised with the target system. In addition, numerical simulations are performed to verify the effectiveness of the theoretical results. But the controllers $u(t)$ are designed to rely on specific cases like nodal dynamics, nature of coupling, topology of communities etc., and so the method cannot be very general. In our future work, we shall consider CMFPS of the complex dynamical networks with node dynamics in different communities that are nonidentical, dimensional, and the controller will be designed to be independent of specific cases.

Acknowledgements

This research is partially supported by the National Nature Science Foundation of China (Grant

No. 11402226), the Fundamental Research Funds for the Central Universities, China (No. 2017B17914) and the Preeminent Youth Fund of Zhejiang University of Finance and Economics.

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