Pramana – J. Phys. (2018) 90:17
https://doi.org/10.1007/s12043-017-1501-1

Relativistic effects in the study of weakly bound ^{17}F and ^{11}Be nuclei

FAHIME REZVANI* and MOHAMMAD REZA SHOJAEI

Department of Physics, Shahrood University of Technology, Shahrood, Iran
*Corresponding author. E-mail: rezvani.fa@gmail.com

MS received 4 April 2017; revised 8 July 2017; accepted 28 August 2017; published online 3 January 2018

Abstract. Relativistic effects are employed to describe the weakly bound nuclei of ^{17}F and ^{11}Be. In order to calculate the energy levels of the ground state and the excited states of these nuclei, we solved the Dirac equation with pseudospin symmetry in the shell model by using the basic concept of supersymmetric shape invariance method. The results obtained from this approach are compared with a non-relativistic approach and experiment. It was then seen that the relativistic approach matches more with the experimental results.

Keywords. Relativistic effects; weakly bound nuclei; supersymmetry shape invariance method.

PACS No. **21.10.−k**

1. Introduction

For the past few decades, the relativistic mean field theory has been successful in describing the nuclear phenomena of unstable as well as stable nuclei [1–3]. Compared to the non-relativistic mean field theory, the relativistic mean field theory can provide an account of real nuclear saturation features in the nuclear matter and can also present the spin–orbit coupled potential [4]. The starting point of the relativistic mean field theory is the Lagrangian that describes the nucleons as Dirac spinors moving in the mean field. It includes the interaction between nucleons (protons and neutrons), mesons (σ, ω, ρ), and also the Coulomb field. The main feature of the relativistic nuclear dynamics is the appearance of the attractive scalar field **S** and the repulsive vector field **V**. This feature can lead to simultaneous integration of the attraction and repulsion effects related to long and short distances in the nucleon–nucleon interaction. Owing to the coupling of lower components in the Dirac equation [5], the observed spin-like symmetry in the monoparticle levels of the spherical nuclei can be understood through the relativistic mean field theory.

These properties in nuclear structure, which can be explained by the Dirac equation phonological in the relativistic mean field theory, implies that the relativistic nuclear dynamics can be applied to correct the nuclear structure by studying it even further.

The Dirac equation is one of the most important equations in various fields of physics [6–12]. The equation is used to solve many nuclear and high-energy problems [13–15]. Recently, remarkable efforts have been made to study relativistic wave equations as well as their relativistic effects, in which the solution of the Dirac equation with spin and pseudospin symmetry was important. Within the framework of the Dirac equation, spin symmetry arises if the magnitude of the attractive scalar potential $S(r)$ and repulsive vector potential are nearly equal, $S(r) \sim V(r)$ in the nuclei (i.e., when the difference potential $\Delta(r) = V(r) - S(r) = C_s$ = constant). However, pseudospin symmetry occurs if $S(r) \sim -V(r)$ (i.e., when the sum of the potential $\Sigma(r) = V(r) + S(r) = C_{ps}$ = constant) [16–18]. $\Delta(r) = 0$ and $\Sigma(r) = 0$ correspond to $SU(2)$ symmetries of the Dirac Hamiltonian [19–21]. The spin symmetry is relevant for mesons [22,23]. The pseudospin symmetry concept has been applied to many systems in nuclear physics and related areas [24–27] and used to explain features of deformed nuclei [26] and superdeformation [27], and to establish an effective nuclear shell-model scheme [24,28]. The pseudospin symmetry introduced in nuclear theory refers to a quasidegeneracy of the single-nucleon doublets and can be characterised with the non-relativistic quantum numbers $(n, l, j = l+1/2)$ and $(n-1, l+2, j = l+3/2)$, where n, l and j are respectively the single-nucleon radial, orbital and total angular momentum quantum numbers for a single particle [24,25]. The total angular momentum is given as

$j = l' + s'$, where $l' = l + 1$ is the pseudoangular momentum and $s' = 1/2$ is the pseudospin angular momentum. In real nuclei, the pseudospin symmetry is only an approximation; the quality of approximation depends on the pseudocentrifugal potential and pseudospin orbital potential [29]. The tensor potential will be imported into the Dirac equation by changing the term $\tilde{p} \to \tilde{p} - im\omega\beta \cdot \hat{r}U(r)$ [30–32]. In this way, a spin–orbit coupling term is added to the Dirac Hamiltonian. Tensor couplings and exactly solvable tensor potential have been used to investigate nuclear properties [21,31] and also have some physical applications [33,34]. In this respect, see [35,36].

Other researchers have adopted different approaches to solve this equation with different potentials [37–44]. Since many of the second-order differential equations like Legendre, Hermit, and associated equations have supersymmetry properties and invariance form, we can employ an invariant form and some effective methods in the supersymmetry quantum mechanics to solve such equations [45].

In this work, we study the relativistic effects in determining energy levels of the weakly bound nuclei of ^{17}F and ^{11}Be. The ground states of ^{17}F and ^{11}Be are weakly bounded to proton and ^{16}O by a separation energy equal to 600 keV and to neutron and ^{10}Be by a separation energy equal to 501 keV respectively [46,47].

To do so, we solve the Dirac equation through the combination of the deformed Yukawa potential and the first part of the deformed Rosen–Morse potential in the pseudospin symmetry, and then derive the energy levels of these nuclei. We compare these energy levels with the results obtained from experiments and the non-relativistic approach.

2. Nucleon–nucleon interaction

To study the properties of the nucleus, standard potentials like Woods–Saxon and Yukawa can be used [48]. However, central potentials – based on these standard potentials with the same behaviour as nucleus potentials – have also been treated to obtain the nucleus properties more accurately. The suggested potential is a combination of the deformed Yukawa potential and the first part of the deformed Rosen–Morse potential, as follows [49,50]:

$$V(r) = V_0 \left(\frac{a}{r^2} - \frac{b}{(1 - \mathrm{e}^{-2\alpha r})^2} \right) \mathrm{e}^{-\alpha r}, \tag{1}$$

where V_0 is the depth of the potential, α is the range of the potential and a, b are adjustable parameters. The behaviour of this potential is similar to the famous

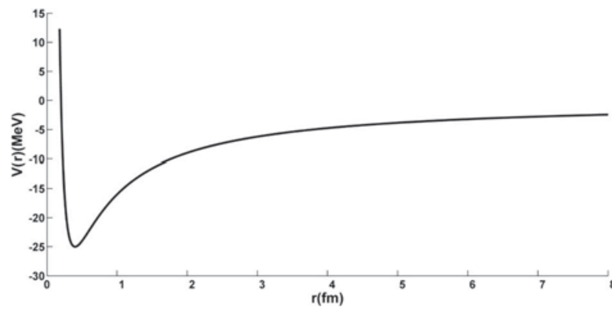

Figure 1. Behaviour of the central potential in eq. (1) for $a = b = 0.04$.

potentials in nuclear physics [51], which is shown in figure 1.

3. Relativistic approach

The Dirac equation—including tensor interaction for spin-1/2 particles in attractive scalar $S(r)$, repulsive vector $V(r)$ and a tensor $U(r)$ potentials – reads as (in atomic units $\hbar = c = 1$) [52]:

$$[\vec{\alpha} \cdot \vec{p} + \beta(M + S(r) - i\beta\vec{\alpha} \cdot \hat{r}U(r))]\psi(\vec{r}) \\ = (E - V(r))\psi(\vec{r}), \tag{2}$$

where α and β are the Dirac matrices. For spherical nuclei, the nucleon angular momentum J and spin–orbit operator $\hat{K} = -\hat{\beta}(\hat{\sigma} \cdot \hat{L} + 1)$ commute with the Dirac Hamiltonian. The eigenvalues of $\hat{K}$ are: $\kappa = \pm(j + 1/2)$ with minus for aligned spin ($s_{1/2}$; $p_{3/2}$, etc.) and plus for unaligned spin ($p_{1/2}$; $d_{3/2}$, etc.). Hence, we use the quantum number κ because it is sufficient to label the orbitals. The wave functions can be classified in accordance with their angular momentum j, κ, and the radial quantum number n, and can be written in the following form:

$$\psi_{n\kappa} = \frac{1}{r} \begin{pmatrix} F_{n\kappa}(r) Y^{l}_{jm}(\theta, \varphi) \\ iG_{n\kappa}(r) Y^{l'}_{jm}(\theta, \varphi) \end{pmatrix} \tag{3}$$

where $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ are the upper and lower radial functions, $Y^{l}_{jm}(\theta, \varphi)$ and $Y^{l'}_{jm}(\theta, \varphi)$ are the spinor spherical harmonic functions. The orbital angular momentum quantum numbers l and l' are respectively the labels of the upper and lower components. We substitute eq. (3) into eq. (2) and obtain two coupled differential equations for the upper and lower radial wave functions, $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$

$$\left(\frac{\mathrm{d}}{\mathrm{d}r} - \frac{\kappa}{r} + U(r)\right) G_{n\kappa}(r) = (M - E_{n\kappa} + \Sigma(r))F_{n\kappa}(r) \tag{4}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}r} + \frac{\kappa}{r} - U(r)\right) F_{n\kappa}(r) = (M + E_{n\kappa} - \Delta(r))G_{n\kappa}(r). \tag{5}$$

By substituting $G_{n\kappa}(r)$ from eq. (4) into eq. (5) and $F_{n\kappa}(r)$ from eq. (5) into eq. (4), two following second-order differential equations for the upper and lower components are obtained:

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{\kappa(\kappa+1)}{r^2} - \frac{2\kappa}{r}U(r) + \frac{\mathrm{d}U(r)}{\mathrm{d}r} + U^2(r) - \frac{\mathrm{d}\Delta/\mathrm{d}r}{E_{n\kappa}+M-\Delta(r)}\left(\frac{\mathrm{d}}{\mathrm{d}r}+\frac{\kappa}{r}-U(r)\right)\right] F_{n\kappa}(r) = [E_{n\kappa} + M - \Delta(r)][M - E_{n\kappa} + \Sigma(r)]F_{n\kappa}(r) \tag{6}$$

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{\kappa(\kappa-1)}{r^2} - \frac{2\kappa}{r}U(r) - \frac{\mathrm{d}U(r)}{\mathrm{d}r} + U^2(r) - \frac{\mathrm{d}\Sigma/\mathrm{d}r}{M-E_{n\kappa}+\Sigma(r)}\left(\frac{\mathrm{d}}{\mathrm{d}r}-\frac{\kappa}{r}+U(r)\right)\right] G_{n\kappa}(r) = [E_{n\kappa} + M - \Delta(r)]\,[M - E_{n\kappa} + \Sigma(r)]G_{n\kappa}(r), \tag{7}$$

where $\Delta(r) = V(r) - S(r)$, $\Sigma(r) = V(r) + S(r)$ and the Coulomb-like potential [53] for the tensor due to a charge Z_{ae} interacting with a charge Z_{be}, distributed uniformly over a sphere of radius R_c, is added [32]

$$U(r) = -H/r, \quad H = Z_a Z_b \mathrm{e}^2/4\pi\varepsilon_0, \quad r \geq R_\mathrm{c}, \tag{8}$$

where $R_\mathrm{c} = 7.78$ fm is the Coulomb radius, Z_a and Z_b denote the charges of nucleons. Considering the pseudospin symmetry, $\Delta(r)$ as the modified potential and $\Sigma(r) = C_s = \mathrm{const.}$ [18], we can reach eq. (9) by substituting eq. (8) into eq. (7)

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{(\kappa+H)(\kappa+H-1)}{r^2} + (E_{n\kappa} + M - C_{ps}) \times \left(\frac{V_1}{r^2} - \frac{V_2}{(1-\mathrm{e}^{-2\alpha r})^2}\right)\mathrm{e}^{-\alpha r}\right] G_{n\kappa}(r) = [E_{n\kappa}^2 - M^2 - C_{ps}(E_{n\kappa} + M)]G_{n\kappa}(r), \tag{9}$$

where $V_1 = aV_0$ and $V_2 = bV_0$. This equation cannot be solved analytically for $\kappa \neq 0$ using standard methods like supersymmetric formalism [54] because, from eq. (9), it is seen that the effective potential is a combination of the exponential and inverse square potentials, and so analytical solutions can be achieved by using an approximation method because of the pseudocentrifugal term.

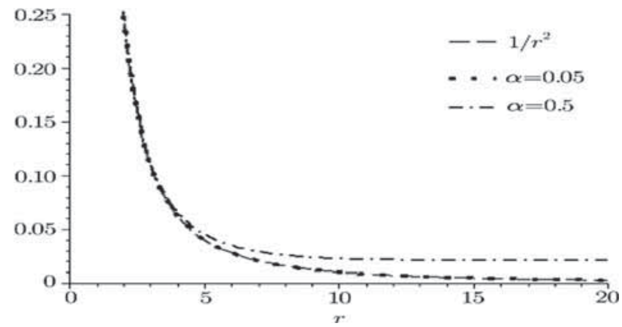

Figure 2. A plot of $1/r^2$ and approximation for various α [56].

We use an approximation for the pseudocentrifugal term similar to the one used by Dong *et al* [55]:

$$\frac{1}{r^2} \approx \alpha^2 \frac{\mathrm{e}^{-2\alpha r}}{(1-\mathrm{e}^{-2\alpha r})^2}. \tag{10}$$

Such an approximation is good for small values of parameter α, which is shown in figure 2.

Using the approximation given in eq. (10) and defining

$$\tilde{V}_1 = -V_1\alpha^2[E_{n\kappa} - M - C_{ps}] \tag{11a}$$

$$\tilde{V}_2 = \alpha^2[\kappa(\kappa+1) - H^2 - H + 2\kappa H] \tag{11b}$$

$$\tilde{V}_3 = -V_2[E_{n\kappa} - M - C_{ps}] \tag{11c}$$

$$\tilde{E}_{n\kappa} = E_{n\kappa}^2 - M^2 - C_{ps}[E_{n\kappa} + M] \tag{11d}$$

we can write down the Schrödinger-like eq. (9) for the lower spinor component as

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \tilde{V}_1\frac{\mathrm{e}^{-3\alpha r}}{(1-\mathrm{e}^{-2\alpha r})^2} + \tilde{V}_2\frac{\mathrm{e}^{-2\alpha r}}{(1-\mathrm{e}^{-2\alpha r})^2} + \tilde{V}_3\frac{\mathrm{e}^{-\alpha r}}{(1-\mathrm{e}^{-2\alpha r})^2}\right] G_{n\kappa}(r) = \tilde{E}_{n\kappa}G_{n\kappa}(r). \tag{12}$$

We solve eq. (12) by using the basic concept of the supersymmetric formalism and supersymmetric shape invariance method [54]. We write the ground-state lower spinor component $G_{0,\kappa}(r)$ as

$$G_{0,\kappa}(r) = \exp\left(-\int W(r)\mathrm{d}r\right), \tag{13}$$

where $W(r)$ is the superpotential in the supersymmetric quantum mechanics [57]. Substituting eq. (13) into eq. (12) leads to the following equation for $W(r)$:

Table 1. The energy levels of the weakly bound ^{17}F nucleus, the parameters $\alpha = 0.01$ fm^{-1} and $V_0 = 0.4$ MeV were used.

States	l	n, κ	$(l, j = l + 1/2)$	E (MeV) (Exp) [62,63]	E (MeV) (Relativistic)	E (MeV) (Non-relativistic)
1	2	1, −3	$1d_{5/2}$	128.2196	128.4925	128.5153
2	0	2, −1	$2s_{1/2}$	127.7243	127.9552	128.0082
3	1	2, 1	$2p_{1/2}$	125.1156	125.0196	125.5214
4	3	1, 3	$1f_{5/2}$	124.3626	124.1484	124.3657
5	1	2, −2	$2p_{3/2}$	123.5796	123.3291	123.3118
6	2	2, 2	$2d_{3/2}$	123.2196	123.2208	123.9409
7	5	1, 5	$1h_{9/2}$	122.9996	122.9013	123.8633
8	1	3, 1	$3p_{3/2}$	122.7366	122.4644	122.3480

‘1’ indicates the ground state of the nucleus, ‘2’ indicates the first excited state and other numbers show higher excited states.

$$W^2(r) - \frac{\mathrm{d}W(r)}{\mathrm{d}r} = \tilde{V}_1 \frac{\mathrm{e}^{-3\alpha r}}{(1-\mathrm{e}^{-2\alpha r})^2} + \tilde{V}_2 \frac{\mathrm{e}^{-2\alpha r}}{(1-\mathrm{e}^{-2\alpha r})^2} + \tilde{V}_3 \frac{\mathrm{e}^{-\alpha r}}{(1-\mathrm{e}^{-2\alpha r})^2} - \tilde{E}_{0,\kappa}, \tag{14}$$

where $W(r)$ is called a superpotential supersymmetric quantum mechanics. Equation (14) is a non-linear Riccati equation. By using the proposed supersymmetric shape invariance method ref. [57], the superpotential of eq. (14) will be as follows:

$$W(r) = -\frac{1}{\sqrt{2\mu}} \left(Q_1 + \frac{Q_2}{1-\mathrm{e}^{-2\alpha r}} + \frac{Q_3 \mathrm{e}^{-\alpha r}}{1-\mathrm{e}^{-2\alpha r}} \right) \tag{15}$$

in which the coefficients of eq. (15) are shown below:

$$Q_1 = \frac{Q_2}{2} \tag{16a}$$

$$Q_2 = -\alpha + \left\{ \frac{1}{2}[(\alpha^2 + 2\mu\tilde{V}_2)^2 - (2\mu(\tilde{V}_1 + \tilde{V}_3))^2]^{1/2} + \frac{1}{2}(\alpha^2 + 2\mu\tilde{V}_2) \right\}^{1/2} \tag{16b}$$

$$Q_3 = \frac{\mu(\tilde{V}_1 + \tilde{V}_3)}{\alpha + Q_2}. \tag{16c}$$

Taking advantage of the basic concept of the six-parameter exponential-type potential (SPEP) method [58], we can calculate the energy levels as follows:

$$E_{n\kappa} = -\frac{1}{8\mu} \left[\alpha + 2n\alpha \pm \sqrt{\frac{1}{2}[(\alpha^2 + 2\mu\tilde{V}_2)^2 - (2\mu(\tilde{V}_3 + \tilde{V}_1))^2]^{1/2} + \frac{1}{2}(\alpha^2 + 2\mu\tilde{V}_2)} \right]^2, \quad n = 0, 1, 2, \ldots. \tag{17}$$

Finally, some numerical results in the relativistic approach are given in tables 1 and 2 for the weakly bound nuclei of ^{17}F and ^{11}Be.

4. Non-relativistic approach

The Schrödinger equation for a spherically symmetric potential in D dimensions [59] (in atomic units $\hbar = c = 1$) reads as

$$\frac{-1}{2\mu}[\nabla_D^2 + V(r)]\psi_{nlm}(r, \Omega_m) = E_{nl}\psi_{nlm}(r, \Omega_m), \tag{18}$$

where the Laplacian operator is defined as

$$\nabla_D^2 = \frac{1}{r^{D-1}} \frac{\partial}{\partial r} \left[r^{D-1} \frac{\partial}{\partial r} \right] - \frac{\Lambda_D^2(\Omega_D)}{r^2}. \tag{19}$$

Here, $V(r)$ is the potential, μ is the reduced mass, E_{nl} is the energy spectrum and Ω_D represents the angular coordinate. The hyperspherical harmonic functions are the eigenfunctions of the operator $\Lambda_D^2(\Omega_D)$. Thus, we write

$$\psi_{nlm}(r, \Omega_m) = R_{nl}(r) Y_l^m(\Omega_D), \tag{20}$$

where $Y_l^m(\Omega_D)$ is the hyperspherical harmonic function and $R_{nl}(r)$ is the hyper-radial wave function. It is well known that $\Lambda_D^2(\Omega_D)/r^2$ is a generalisation of the centrifugal barrier for the D-dimensional space and involves the angular coordinate (Ω_D) and the eigenvalues of the hyperspherical harmonic functions $\Lambda_D^2(\Omega_D)$ are given by

$$\Lambda_D^2(\Omega_D) Y_l^m(\Omega_D) = l(l + D - 2) Y_l^m(\Omega_D), \tag{21}$$

where l is the arbitrary angular momentum quantum number. Choosing a common ansatz for the wave function in the form

$$R_{nl}(r) = r^{-(D-1)/2} U_{nl}(r) \tag{22}$$

Table 2. The energy levels of the weakly-bound ^{11}Be nucleus, the parameters $\alpha = 0.05$ fm^{-1} and $V_0 = 0.34$ MeV were used.

Stats	l	n, κ	$(l, j = l + 1/2)$	E (MeV) (Exp) [62,63]	E (MeV) (Relativistic)	E (MeV) (Non-relativistic)
1	0	1, −1	$1s_{1/2}$	65.4779	65.8746	66.9553
2	1	1, 1	$1p_{1/2}$	65.1579	65.3799	65.7382
3	2	1, −3	$1d_{5/2}$	63.3749	63.3651	65.7214
4	1	1, −2	$1p_{3/2}$	60.7209	60.7345	60.5481
5	1	2, −2	$2p_{3/2}$	57.3209	57.3625	57.6218
6	2	1, 2	$1d_{3/2}$	53.4319	53.6480	52.6409
7	3	3, −2	$3p_{3/2}$	49.4769	49.4102	51.9633
8	1	1, 3	$1f_{5/2}$	44.2219	44.1958	46.5110

‘1’ indicates the ground state of the nucleus, ‘2’ indicates the first excited state and other numbers show higher excited states.

we obtain the Schrödinger equation in D dimensions as follows:

$$\frac{\mathrm{d}^2U_{nl}(r)}{\mathrm{d}r^2} + 2\mu[E - V(r)]U_{nl}(r) + \frac{1}{r^2}\left[\frac{(D-1)(D-3)}{4} + l(l+D-2)\right]U_{nl}(r) = 0. \tag{23}$$

To study any quantum physical system, we solve the D-dimensional Schrödinger equation given in eq. (23) [60]. In order to have a two-particle system, we take the two-dimensional Schrödinger equation. By substituting eq. (1) into eq. (23), we obtain

$$\frac{\mathrm{d}^2U_{nl}(r)}{\mathrm{d}r^2} + 2\mu\left[E - V_0\left(\frac{a}{r^2} - \frac{b}{(1-\mathrm{e}^{-2\alpha r})^2}\right)\mathrm{e}^{-\alpha r}\right] \times U_{nl}(r) + \frac{1}{r^2}[l(l+1)]U_{nl}(r) = 0. \tag{24}$$

Using eq. (10) and defining parameter as follows:

$$V_1' = 2\mu\alpha^2 V_1 \tag{25a}$$
$$V_2' = -l(l+1)\alpha^2 = -\kappa\alpha^2 \tag{25b}$$
$$V_3' = -2\mu V_2 \tag{25c}$$

using the basic concept of the supersymmetric formalism and supersymmetric shape invariance method [58], similar to the relativistic approach, the superpotential of eq. (24) will be as below:

$$W(r) = -\frac{1}{\sqrt{2\mu}}\left(Q_1 + \frac{Q_2}{1-\mathrm{e}^{-2\alpha r}} + \frac{Q_3\mathrm{e}^{-\alpha r}}{1-\mathrm{e}^{-2\alpha r}}\right) \tag{26}$$

in which the coefficients in eq. (26) are as follows:

$$Q_1 = \frac{Q_2}{2} \tag{27a}$$
$$Q_2 = -\alpha + \left\{\frac{1}{2}[(\alpha^2 + 2\mu V_2')^2 - (2\mu(V_1' + V_3'))^2]^{1/2} + \frac{1}{2}(\alpha^2 + 2\mu V_2')\right\}^{1/2} \tag{27b}$$
$$Q_3 = \frac{\mu(V_1' + V_3')}{\alpha + Q_2}. \tag{27c}$$

We can calculate the energy levels as follows:

$$E_{n\kappa} = -\frac{1}{8\mu}\left[\alpha + 2n\alpha \pm \sqrt{\frac{1}{2}\left[\alpha^4(1+2\mu\kappa^2)^2 - (4\mu^2(V_2 - \alpha^2V_1))^2\right]^{1/2} + \frac{\alpha^2}{2}(1+2\mu\kappa^2)}\right]^2, \quad n = 0, 1, 2, \dots. \tag{28}$$

Finally, the energy spectrum with the spin–orbit term can be expressed by

$$E_{nl} = E + \Delta = E + \int R_{nl}^*(r)V_{L\cdot s}(r)R_{nl}(r)\mathrm{d}^3r, \tag{29}$$

where the spin–orbit term is given by [61]

$$V_{L\cdot s} = -\lambda\left(\frac{1}{2mc}\right)^2 \mathbf{L}\cdot\mathbf{S} \tag{30}$$

$$\mathbf{L}\cdot\mathbf{S} = \frac{1}{2}(\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2) = \frac{\hbar^2}{2}[j(j+1) - l(l-1) - s(s-1)], \tag{31}$$

where $\lambda = 35$ is a dimensionless coupling constant and m and c are the nucleon mass and the speed of light respectively. The ground-state wave function can be written as [58]

$$R_0(r) = N\frac{e^{Q_1 r}}{r}\left(-\frac{e^{-2\alpha r}}{1-e^{-2\alpha r}}\right)^{-Q_2/2\alpha} \times \exp\left[-\frac{Q_3}{\alpha}\text{arc coth}(e^{-\alpha r})\right], \quad (32)$$

where N is a normalised constant. Consequently, some numerical results in the non-relativistic approach are briefly described in tables 1 and 2 for the weakly bound nuclei of ^{17}F and ^{11}Be.

5. Result and discussion

Relativistic analyses – based on the Dirac equation – have shown that they can achieve better agreement with experimental data than the non-relativistic analyses based on Schrödinger equation. One of the merits of the Dirac approach – instead of the use of the non-relativistic approach – is that the spin–orbit potential appears naturally in the Dirac approach when the Dirac equation is reduced to a Schrödinger-like second-order differential equation, while the spin–orbit potential should be inserted by hand into the non-relativistic Schrödinger approach. According to the numerical results calculated from the energy levels of ^{17}F, shown in table 1, it can be seen that – in most states – the relativistic approach has a better agreement with experimental results. It particularly includes the states of 3, 6 and 7 in the $1d_{5/2}$, $1d_{3/2}$, $1p_{3/2}$ levels respectively. The highest error corresponds to experimental results for the last calculated excited state, i.e., $1f_{3/2}$.

For a better comparison, the results of the calculation are shown in figure 3.

For ^{11}Be (table 2), the relativistic approach is in better agreement with experimental results in all states.

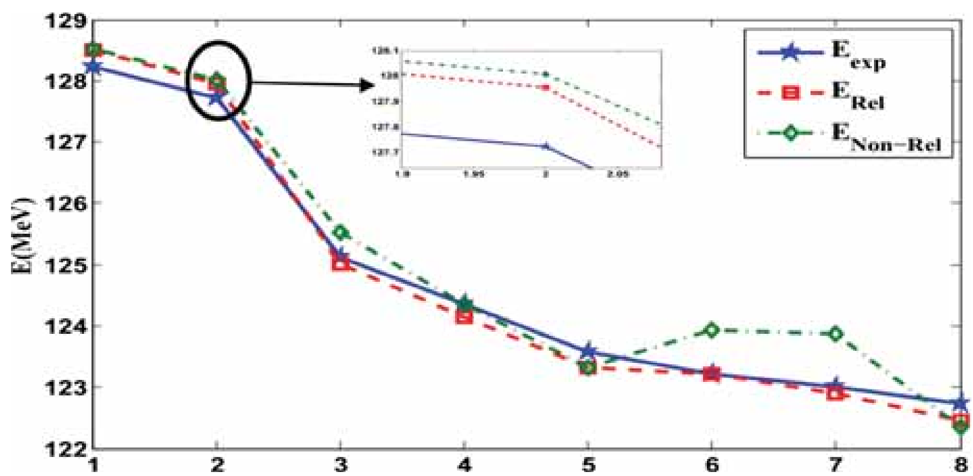

Figure 3. The energy levels derived from the relativistic approach are compared with the non-relativistic approach and experiments for the weakly bound ^{17}F nucleus.

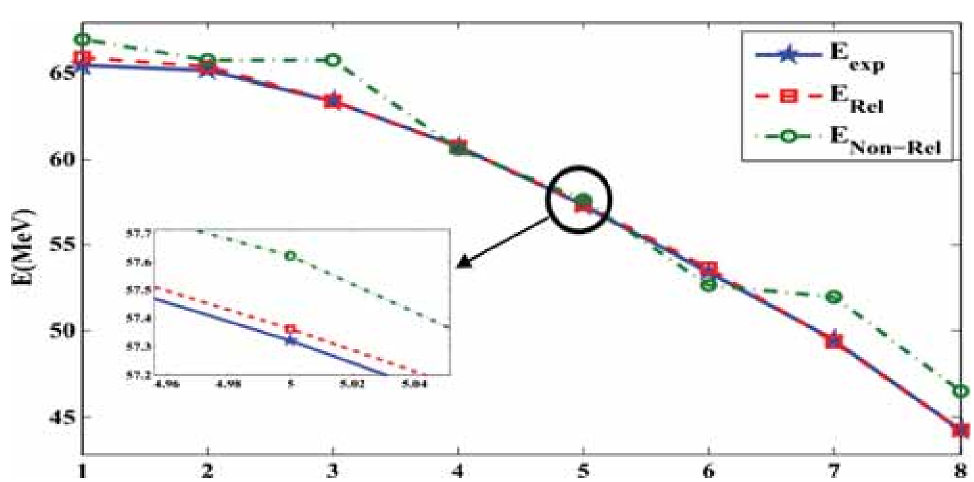

Figure 4. The energy levels derived from the relativistic approach are compared with the non-relativistic approach and experiments for the weakly bound ^{11}Be nucleus.

The second excited state, calculated for this nucleus ($1d_{5/2}$), indicates the greatest discrepancy against experiments.

For a better comparison, the results of the calculation are shown in figure 4.

6. Conclusion

In this work, we calculated energy levels of two weakly bound nuclei of ^{17}F and ^{11}Be in both relativistic and non-relativistic models. For this purpose, we first introduced our selected nuclear potential, which is a combination of the deformed Yukawa potential and the first part of the deformed Rosen–Morse potential. We next calculated energy levels of these two nuclei by solving the Dirac equation in the relativistic model and by solving the Schrödinger equation with related potential in non-relativistic model. Also, we utilised Dong approximation and supersymmetry methods to solve the equations. The results obtained with the help of the calculations and their comparison with experimental results are provided in tables 1 and 2 and figures 3 and 4. Finally, it can be said that the relativistic approach agrees more with the experimental results.

References

[1] J D Waleca, *Ann. Phys*. **83**, 491 (1974)
[2] B D Serot and J D Waleca, *Adv. Nucl. Phys*. **16**, 1 (1986)
[3] P Ring, *Prog. Part. Nucl. Phys*. **37**, 193 (1996)
[4] H Kucharek and P Ring, *Z. Phys. A* **339**, 23 (1991)
[5] J Meng and P Ring, *Phys. Rev. Lett.* **77**, 3963 (1996)
[6] H Egrifes and R Sever, *Phys. Lett. A* **344**, 117 (2005)
[7] W C Qiang, R S Zhou and Y Gao, *Phys. Lett. A* **333**, 8 (2004)
[8] H Motavali, *Mod. Phys. Lett. A* **24**, 1227 (2009)
[9] S M Ikhdair and R Sever, *Appl. Math. Comp*. **46**, 545 (2010)
[10] M R Setare and Z Nazari, *Acta. Phys. Polon. B* **40**, 2809 (2009)
[11] L H Zhang, X P Li and C S Jia, *Phys. Lett. A* **372**, 2201 (2008)
[12] F Zhou, Y Wu and J Y Guo, *Commun. Theor. Phys. (Beijing, China)* **52**, 813 (2009)
[13] O Aydogdu and R Sever, *Ann. Phys.* **325**, 373 (2010)
[14] G F Wei and S H Dong, *Phys. Lett. B* **686**, 288 (2010)
[15] J Y Guo and Z Q Sheng, *Phys. Lett. A* **338**, 90 (2005)
[16] J N Ginocchio, *Phys. Rev. C* **69**, 034318 (2004)
[17] J N Ginocchio, *Phys. Rev. Lett*. **78**, 436 (1997)
[18] J N Ginocchio, *Phys. Rep.* **414**, 165 (2005)
[19] P Alberto *et al*, *J. Phys. Conf. Ser.* **490**, 012069 (2014)
[20] J S Bell and H Ruegg, *Nucl. Phys. B* **98**, 151 (1975)
[21] P Alberto *et al*, *Phys. Rev. A* **92**, 062137 (2015)
[22] P R Page, T Goldman and J N Ginocchio, *Phys. Rev. Lett*. **86**, 204 (2001)
[23] G B Smith and L J Tassie, *Ann. Phys*. **65**, 352 (1971)
[24] A Arima, M Harvey and K Shimizu, *Phys. Lett. B* **30**, 517 (1969)
[25] K T Hecht and A Adler, *Nucl. Phys. A* **137**, 129 (1969)
[26] A Bohr, I Hamamoto and B R Mottelson, *Phys. Scr*. **26**, 267 (1982)
[27] J Dudek *et al*, *Phys. Rev. Lett.* **59**, 1405 (1987)
[28] D Troltenier, C Bahri and J P Draayer, *Nucl. Phys. A* **586**, 53 (1995)
[29] J Meng *et al*, *Phys. Rev. C* **58**, R628 (1998)
[30] M Moshinsky and A Szczepaniak, *J. Phys. A: Math. Gen*. **22**, 817 (1989)
[31] G Mao, *Phys. Rev. C* **67**, 044318 (2003)
[32] H Akcay and C Tezcan, *Int. J. Mod. Phys. C* **20**, 931 (2009)
[33] M Moshinsky and Y Smirnov, *The harmonic oscillator in modern physics* (Hardwood Academic Publishers, The Netherlands, 1996) pp. 289–404
[34] M H Pacheco, R R Landim and C A S Almedia, *Phys. Lett. A* **311**, 93 (2003)
[35] O Aydogdu and R Sever, *Few-Boby Syst*. **47**, 193 (2010)
[36] S Zarrinkamar, A A Rajabi and H Hassanabadi, *Ann. Phys.* **325**, 2522 (2010)
[37] M R Shojaei and A A Rajabi, *Int. J. Phys. Sci.* **6**, 33 (2011)
[38] H Feizi, A A Rajabi and M R Shojaei, *Acta Phys. Polon. B* **42**, 2143 (2011)
[39] H Feizi, M R Shojaei and A A Rajabi, *Eur. Phys. J. Plus* **127**, 41 (2012)
[40] A Soylu, O Bayrak and I Boztosun, *J. Math. Phys*. **48**, 082302-9 (2007)
[41] M Farrokh, M R Shojaei and A A Rajabi, *Eur. Phys. J. Plus* **128**, 14 (2013)
[42] X Zou, L Z Yi and C S Jia, *Phys. Lett. A* **346**, 54 (2005)
[43] J Y Gou and Z Q Sheng, *Phys. Lett. A* **338**, 90 (2005)
[44] Y Xu, S He and C S Jia, *J. Phys. A: Math. Theor.* **41**, 255302 (2008)
[45] L E Gendenshtein, *Sov. Phys-JETP Lett.* **38**, 356 (1983)
[46] C A Bertulani and P Danielewicz, *Nucl. Phys. A* **717**, 19 (2003)
[47] R Morlock *et al*, *Phys. Rev. Lett*. **79**, 3837 (1997)
[48] S M Ikhdair and R Sever, *Cent. Eur. J. Phys.* **8(4)**, 652 (2010)
[49] M Hamzavi, S M Ikhdair and B I Ita, *Phys. Scr.* **85**, 045009 (2012)
[50] Y Sun, S He and C S Jia, *Phys. Scr*. **87**, 025301 (2013)
[51] R D Woods and D S Saxon, *Phys. Rev.* **95**, 577 (1954)
[52] P Alberto *et al*, *Phys. Rev. C* **71**, 034313 (2005)
[53] G R Satchler, *Direct nuclear reactions* (Oxford University Press, London, 1983)
[54] F Cooper, J N Ginocchio and A Khare, *Phys. Rev. D* **36**, 2458 (1987)
[55] S H Dong *et al*, *J. Phys. A: Math. Theor.* **40**, 10535 (2007)
[56] N K Akpan *et al*, *Commun. Theor. Phys.* **61**, 457 (2014)
[57] F Cooper, A Khare and U Sukhatme, *Phys. Rep.* **251**, 267 (1995)

[58] C S Jia *et al*, *Commun. Theor. Phys.* **37**, 523 (2002)
[59] W Greiner, *Quantum mechanics* (Springer, Berlin, 2001)
[60] S H Dong, *Wave equation in higher dimension* (Springer, Berlin, 2012)
[61] K S Krane, *Introductory nuclear physics* (Kim Hup Lee Printing Co, 1955)
[62] www.nndc.bnl.gov
[63] G Audi, A H Wapstra and C Thibault, *Nucl. Phys. A* **729**, 337 (2003)