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Anisotropic stars obeying Chaplygin equation of state

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Abstract. In this work we provide a framework for modelling compact stars in which the interior matter distribution obeys a generalised Chaplygin equation of state. The interior geometry of the stellar object is described by a spherically symmetric line element which is simultaneously co-moving and isotropic with the exterior space–time being vacuum. We are able to integrate the Einstein field equations and present closed form solutions which adequately describe compact strange star candidates such as 4U 1538-52, PSR J1614-2230, Vela X-1 and Cen X-3 (Gangopadhyay *et al*, *Mon. Not. R. Astron. Soc.* **431**, 3216 (2013)).

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1. Introduction

The search for exact solutions of the Einstein field equations has generated a rich field of models describing relativistic compact objects. Since the pioneering work of Schwarzschild [1] who obtained the first interior solution describing a uniform density sphere, the modelling of relativistic stars has moved from the regime of toy models to sophisticated, realistic stellar structures. With the discovery of pulsars, neutron stars and strange stars, there was a need to obtain relativistic analogues of Newtonian stars, particularly when the central densities of the stellar material was of the order of $10^{15}\ \mathrm{g\cdot cm^{-3}}$. At these ultrahigh densities, the radial and transverse stresses within the stellar interior may be unequal, i.e., the pressure inside the stellar object may be anisotropic in nature [2]. Various mechanisms have been proposed to explain the existence of anisotropic pressure within a stellar object. These range from treating the stellar fluid as a type-3A superfluid [3], the stellar material undergoing various phase transitions [4], pion condensation [5], to highlight just a few. An excellent review of the role played by anisotropy in self-gravitating systems is provided by Herrera and Santos [6]. Anisotropic effects may also originate from slow rotation of the core [7,8]. All static and spherically symmetric solutions to Einstein equations can be obtained by a general method introduced by Herrera *et al* [9] (see the references therein for a complete review). The simplistic model of a static uniform density star has been generalised to include the effects of pressure anisotropy, electric charge, scalar field, dark energy and the cosmological constant in [10–20].

In order to close the system of equations governing the gravitational and thermodynamical behaviour of bounded objects, various techniques were employed by researchers working in this field of study: (i) imposition of symmetry, (ii) ad-hoc assumptions of the gravitational potentials, (iii) specific choices of the fall-off behaviour of the pressure, density or the anisotropy, to name a few [21–23]. To construct a physically motivated stellar model, it is important to impose an equation of state which relates the pressure to the density of the star, i.e., $p = p(\rho)$. Most of the earlier works were centred on imposing a linear equation of state of the form $p = \alpha\rho$ where α is a constant. This was later generalised to $p = \alpha\rho - \beta$, where $\beta = \alpha\rho_{\mathrm{s}}$ and ρ_{s} is the surface density [24]. The conditions were relaxed by allowing for anisotropic pressure. Investigations in fundamental particle physics led to the MIT-bag model which hinged on an equation of state of the form $p = \alpha\rho - 4B$ where B is the Bag constant. The linear equation of state was further generalised to the quadratic equation of state of the form $p = \alpha\rho - \beta + \sigma\rho^2$ by Ngubelanga *et al* [25]. One of the first successful attempts to obtain a generalisation of the Newtonian polytrope was achieved by Buchdahl [26] in which he obtained a pseudorelativistic version of the Lane–Emden polytrope of index 5. Herrera and Barreto [27] presented a general formalism to generate relativistic polytropes with anisotropic pressure in Schwarzschild coordinates. Their findings also prompted further investigations into the origins of anisotropy, cracking in relativistic stellar models and stability [28]. In order to fine-tune these models with observations, some researchers employed a mixed polytrope equation of state in which two or more species of particles made up the stellar fluid. The inclusion of charge within the stellar core led to a plethora of static stellar models in which the role of the electromagnetic field on the stability, mass–radius ratio and red-shift was demonstrated by Ray *et al* [29,30]. The gravastar model first proposed by Mazur and Mottola [31] sought to address many of the problems encountered during the late stages of collapse. The construction of a gravastar as a composite body and its subsequent stability depends on the various equations of state of the different layers [32,33].

Motivated by the existence of dark energy, Lobo and coworkers [34] have proposed stellar models, the so-called 'dark stars' in which the equation of state is of the form $p = \alpha\rho$ in which $-1 < \alpha < -1/3$. It has been proposed that in the phantom regime ($\alpha = -1$), the extremely high pressures may invoke a topological change rendering the dark energy star to a wormhole. An interesting proposal regarding dark energy and dark matter is treating them as different manifestations of a single entity. This proposal leads to the Chaplygin gas model in which the equation of state derives from string theory. Various applications of the Chaplygin gas model have been pursued in order to account for cosmological observations such as acceleration of the cosmic fluid and structure formation. The Chaplygin equation of state has been subsequently modified to a more generalised Chaplygin gas equation of state. The generalised Chaplygin equation of state has been employed to model dark stars which are remnants of continued gravitational collapse. The idea here is that the dark energy provides sufficient repulsion to halt collapse leading to stable bounded configurations free of horizons and singularities [35,36].

This paper is structured as follows: In §2 we introduce the field equations necessary for the modelling of a spherically symmetric star within the framework of general relativity. In §3 we present a particular solution describing the interior of the star in which the matter content obeys a generalised Chaplygin equation of state. The junction conditions required for the smooth matching of the interior space–time to the exterior Schwarzschild solution are worked out in §4. A detailed physical analysis of the geometrical and thermodynamical behaviour of our model is presented in §5. We discuss the mass–radius relation in §6. We conclude with a discussion of our results in §7.

2. Spherically symmetric space–time

We consider a model which represents a static spherically symmetric anisotropic fluid configuration obeying a generalised Chaplygin equation of state. The interior space–time is described by a spherically symmetric line element which is simultaneously co-moving and isotropic

$$\mathrm{d}s^2 = -A^2(r)\mathrm{d}t^2 + B^2(r)[\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2], \tag{1}$$

where $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2$ and the metric functions, $A(r)$ and $B(r)$ are yet to be determined. The energy–momentum tensor for the stellar fluid is

$$T_{ab} = \mathrm{diag}(-\rho, p_\mathrm{r}, p_\mathrm{t}, p_\mathrm{t}), \tag{2}$$

where ρ, p_r and p_t are the proper energy density, radial pressure and tangential pressure, respectively. The fluid four-velocity $\mathbf{u}$ is co-moving and is given by

$$u^a = \frac{1}{A}\delta^a_0 . \tag{3}$$

The Einstein field equations for the line element (1) are

$$8\pi\rho = -\frac{1}{B^2}\left(2\frac{B''}{B} - \frac{B'^2}{B^2} + \frac{4}{r}\frac{B'}{B}\right), \tag{4}$$

$$8\pi p_\mathrm{r} = \frac{1}{B^2}\left[\frac{B'^2}{B^2} + 2\frac{A'}{A}\frac{B'}{B} + \frac{2}{r}\left(\frac{A'}{A} + \frac{B'}{B}\right)\right], \tag{5}$$

$$8\pi p_\mathrm{t} = \frac{1}{B^2}\left[\frac{A''}{A} + \frac{B''}{B} - \frac{B'^2}{B^2} + \frac{1}{r}\left(\frac{A'}{A} + \frac{B'}{B}\right)\right], \tag{6}$$

where primes denote differentiation with respect to the radial coordinate r. We have utilised geometrised units in deriving the above system of equations in which the coupling constant and the speed of light are taken to be unity.

The mass of the spherical object is obtained from Santos [37]

$$m(r) = -r^2B'(r) - \frac{r^3}{2B}(B'(r))^2. \tag{7}$$

In seeking solutions of the system of equations, we assume that the interior matter distribution obeys a generalised Chaplygin equation of state (EoS) of the form

$$p_{\rm r} = H\rho - \frac{K}{\rho}, \tag{8}$$

where H and K are positive constants. The Chaplygin EoS was used by Bhar [14], Rahaman *et al* [20], Benaoum [38] to model compact stars within the framework of general relativity.

Substituting (4) and (5) into (8) we obtain

$$\frac{A'}{A} = \frac{1}{2}\left(\frac{B'}{B} + \frac{1}{r}\right)^{-1}[G(r) - F(r)], \tag{9}$$

where

$$G(r) = \frac{(8\pi)^2 K B^4}{2(B''/B) - (B'^2/B^2) + (4/r)(B'/B)}, \tag{10}$$

$$F(r) = 2H\frac{B''}{B} + (1-H)\left(\frac{B'}{B}\right)^2 + (1+2H)\frac{2}{r}\frac{B'}{B}. \tag{11}$$

On integrating (9) we obtain

$$A = d\exp\left[\frac{1}{2}\int D(r){\rm d}r\right], \tag{12}$$

where

$$D(r) = \left(\frac{B'}{B} + \frac{1}{r}\right)^{-1}[G(r) - F(r)], \tag{13}$$

and d is a constant of integration. Therefore, the line element (1) can now be written as

$${\rm d}s^2 = -d^2\exp\left[\int D(r)\,{\rm d}r\right]{\rm d}t^2 + B^2[{\rm d}r^2 + r^2{\rm d}\Omega^2], \tag{14}$$

where $D(r)$ is given in (13). Hence, any solution describing a static spherically symmetric anisotropic matter distribution obeying a generalised Chaplygin equation of state in isotropic coordinates can be easily determined by a single generating function $B(r)$.

3. Generating solutions

In order to close the system of equations, several choices for $B(r)$ can be made. It is interesting to note that the choice of the metric potential $B(r)$ determines the gravitational and thermodynamical behaviour of the model. Hence the choice of $B(r)$ must satisfy all the requirements for a realistic stellar model. Recent work by Naidu and Govender [39] has shown that the end-state of gravitational collapse resulting from a dynamically unstable static core is 'sensitive' to the choice of the initial metric functions. They show that for the same $B(r)$ but with two distinct initially static cores; (i) vanishing radial pressure within the static configuration and (ii) uniform density interior, the final outcome of dissipative collapse leads to very different temperature profiles. Following Govender and Thirukkanesh [40] we utilise the physically motivated choice for $B(r)$ as

$$B(r) = \frac{a}{\sqrt{1+br^2}}, \tag{15}$$

where a and b are constants. One can easily verify that the gravitational potential B in (15) satisfies the regularity conditions, $B(0)$ = constant and $B'(r) = 0$ at the origin. The same expression of $B(r)$ was previously utilised to model compact objects in curvature coordinates by Einstein [41] and de Sitter [42] and more recently in co-moving coordinates by Govender and Thirukkanesh [40] and Thirukkanesh *et al* [43].

With this choice of $B(r)$, we obtain from (12)

$$A(r) = d\exp\left[\left\{\frac{1+H}{4} - \frac{16a^4K\pi^2}{b^2}\right\}(1+br^2)\right] \times (1+br^2)^{(1+5H)/4}(6+br^2)^{80a^4K\pi^2/b^2},$$

where d is a constant of integration.

Subsequently, the field equations yield

$$\rho = \frac{b(6+br^2)}{8\pi a^2(1+br^2)}, \tag{16}$$

$$p_{\rm r} = \frac{bH}{8\pi a^2}\left(\frac{6+br^2}{1+br^2}\right) - \frac{8\pi a^2K}{b}\left(\frac{1+br^2}{6+br^2}\right), \tag{17}$$

$$p_{\rm t} = \frac{C_1 + C_2r^2 + C_3r^4 + C_4r^6 + C_5r^8 + C_6r^{10}}{32\pi a^2b^2(6+br^2)^2(1+br^2)}, \tag{18}$$

where C_i's ($i = 1, 2, \ldots, 6$) are given by

$$C_1 = 96b(9b^2H - K_1)$$
$$C_2 = 16[9b^4\{3 + H(10+9H)\} - 6b^2K_1(5+3H) + 16K_1^2]$$
$$C_3 = 8b[3b^4\{18 + H(47+36H)\} - 2b^2K_1(47+42H) + 128K_1^2]$$
$$C_4 = 8b^2[b^4\{18 + H(43+27H)\} - b^2K_1(57+61H) + 192K_1^2]$$
$$C_5 = 4b^3\{b^2(1+H) - 4K_1\}\{b^2(5+6H) - 4K_1\}$$
$$C_6 = b^4\{b^2(1+H) - 4K_1\}^2,$$

where $K_1 = 16a^4K\pi^2$.

The anisotropic factor is defined as

$$\Delta = p_{\rm t} - p_{\rm r} \tag{19}$$

which is repulsive in nature if $\Delta > 0$ and attractive if $\Delta < 0$.

4. Junction conditions

In order to smoothly match the interior line element to the vacuum Schwarzschild exterior we invoke the junction conditions due to Israel [44]. We consider a spherical surface described by a time-like three-space Σ. The surface Σ divides space–time into two distinct regions $\mathcal{M}^-$ and $\mathcal{M}^+$. Let g_{ij} be the intrinsic metric to Σ so that

$$\mathrm{d}s^2_\Sigma = g_{ij}\mathrm{d}\xi^i\mathrm{d}\xi^j. \tag{20}$$

The intrinsic coordinates to Σ are given by ξ^i where $i = 1, 2, 3$. The line elements in the regions $\mathcal{M}^\pm$ are of the form

$$\mathrm{d}s^2_\pm = g_{ab}\mathrm{d}\mathcal{X}^a_\pm\mathrm{d}\mathcal{X}^b_\pm. \tag{21}$$

The coordinates in $\mathcal{M}^\pm$ are $\mathcal{X}^a_\pm$ where $a = 0, 1, 2, 3$. We require that the interior metric (1) and the exterior metric (20) match smoothly across Σ. This generates the first junction condition

$$(\mathrm{d}s^2_-)_\Sigma = (\mathrm{d}s^2_+)_\Sigma = \mathrm{d}s^2_\Sigma. \tag{22}$$

We are using the notation $(\ )_\Sigma$ to represent the value of $(\ )$ on Σ. Consequently, the coordinates of Σ in $\mathcal{M}^\pm$ are given by $\mathcal{X}^a_\pm = \mathcal{X}^a_\pm(\xi^i)$. The second junction condition is obtained by requiring continuity of the extrinsic curvature of Σ across the boundary. This gives

$$K^+_{ij} = K^-_{ij}, \tag{23}$$

where

$$K^\pm_{ij} \equiv -n^\pm_a \frac{\partial^2\mathcal{X}^a_\pm}{\partial\xi^i\partial\xi^j} - n^\pm_a \Gamma^a{}_{cd}\frac{\partial\mathcal{X}^c_\pm}{\partial\xi^i}\frac{\partial\mathcal{X}^d_\pm}{\partial\xi^j} \tag{24}$$

and $n^\pm_a(\mathcal{X}^b_\pm)$ are the components of the vector normal to Σ.

The first junction condition (22) for the interior metric (1) and the exterior metric (20) gives

$$A(R) = \frac{(1-(M/2R))}{(1+(M/2R))}, \tag{25}$$

$$B(R) = \left(1+\frac{M}{2R}\right)^2. \tag{26}$$

The second junction condition (23) gives us

$$m_\Sigma = \left(-r^2B_r - \frac{r^3}{2B}B_r^2\right)_\Sigma \tag{27}$$

$$(p_r)_\Sigma = 0. \tag{28}$$

Relation (27) represents the total gravitational mass inside Σ. As pointed out by Bonnor *et al* [45], (27) can be interpreted as the mass function of a sphere of radius $r = r_\Sigma$. The vanishing of the radial pressure at the boundary determines the radius of the star.

To examine the behaviour of the thermodynamical parameters such as matter density, radial and transverse pressures, we assume $a = 0.843$, $b = 0.006$ and $H = 0.3$. By using the matching conditions together with $p_{\rm r}(r = R) = 0$, we obtain the constant $K = 7.29 \times 10^{-7}$ for a star of radius 7.87 km. The mass of the stellar configuration turns out to be $0.90M_\odot$ which is very close to the observed mass of the strange star candidate 4U 1538-52 proposed by Gangopadhyay *et al* [46]. By assigning particular values to the constants a, b, H, K we obtain the masses and radii of compact star candidates PSR J1614-2230, Vela X-1, Cen X3 (refer to table 1) which approximate the observational data presented by Gangopadhyay *et al* [46] to a very good degree.

5. Physical analysis

We are now in a position to discuss the physical features of the model generated in the preceding section. In order to describe a realistic stellar structure, our model must satisfy the following physical requirements:

1. Regularity of the gravitational potentials at the origin:

   In our model,

   $$A^2(0) = d^2 6^{160a^4K\pi^2/b^2}\mathrm{e}^{[b^2(1+H)-64a^4K\pi^2]/2b^2}$$

   $$B^2(0) = a^2$$

   which are constants and $(A^2(r))' = 0,\ (B^2(r))' = 0$ at the origin $r = 0$, which indicate that the gravitational potentials are regular at the origin.

2. Positive definiteness of the energy density and pressure at the centre:

   Since $\rho(0) = 3b/4\pi a^2$, the energy density is positive and regular at the origin. We also have $p_{\rm r}(0) = (9b^2H - 16\pi^2a^4K)/12\pi a^2b$. To ensure that the radial pressure is positive at the centre we must have

   $$\frac{K}{H} < \left(\frac{3b}{4\pi a^2}\right)^2.$$

Table 1. Comparison of theoretical values for radii and $M/M_{\odot}$ with observational data (Gangopadhyay *et al* [46]).

Compact star	a	b	H	K	R (km) observed	$M/M_{\odot}$ observed	R (km) calculated	$M/M_{\odot}$ calculated
4U 1538-52	0.843	0.006	0.3	7.29×10^{-7}	7.866 ± 0.21	0.87 ± 0.07	7.87	0.90
PSR J1614-2230	1.212	0.007	0.3	1.74×10^{-7}	9.69 ± 0.2	1.97 ± 0.04	9.7	1.97
Vela X-1	1.078	0.008	0.3	3.4×10^{-7}	9.56 ± 0.08	1.77 ± 0.08	9.56	1.77
Cen-X3	1.058	0.006	0.3	2.55×10^{-7}	9.178 ± 0.13	1.49 ± 0.08	9.18	1.5

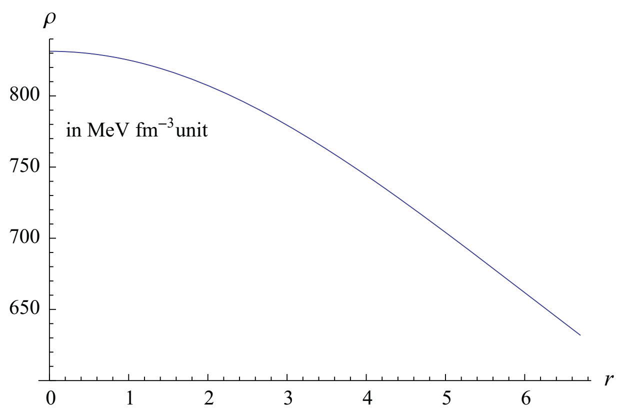

Figure 1. Matter density ρ is plotted against the radial distance r inside the fluid for a particular configuration with $a = 0.843$, $b = 0.006$, $H = 0.3$ and $K = 7.29\times10^{-7}$.

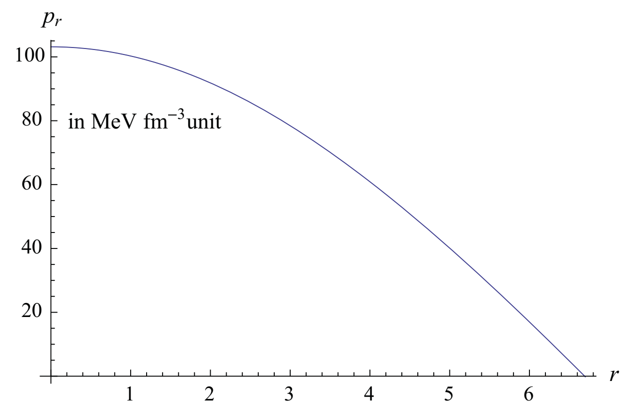

Figure 2. Radial pressure $p_{\rm r}$ is plotted against the radial distance r inside the stellar interior by employing the same values of the constants as mentioned in figure 1.

Moreover,

$$\frac{d\rho}{dr} = -\frac{5b^2 r}{4\pi a^2(1+br^2)^2} < 0,$$

i.e., the energy density is a decreasing function of r. We also note that

$$\frac{dp_{\rm r}}{dr} = -\frac{80a^2 K\pi r}{(6+br^2)^2} - \frac{5b^2 H r}{4\pi a^2(1+br^2)^2},$$

which implies that $p_{\rm r}$ is a decreasing function of r. The matter density, radial and transverse pressures are shown in figure 1 and figure 2 respectively. The anisotropic factor Δ is plotted in figure 3.

3. Continuity of the extrinsic curvature across the matching hypersurface, $K_{ij}^{-} = K_{ij}^{+}$:

   Continuity of the extrinsic curvature across the matching hypersurface, $r = R$ yields

$$(p_{\rm r})_{(r=R)} = 0 \tag{29}$$

   which gives

$$R = \sqrt{\frac{2(32a^4K\pi^2 + 20a^2b\pi\sqrt{HK} - 3b^2H)}{b(b^2H - 64a^4K\pi^2)}},$$

   which is finite for appropriate choice of parameters a, b, H and K.

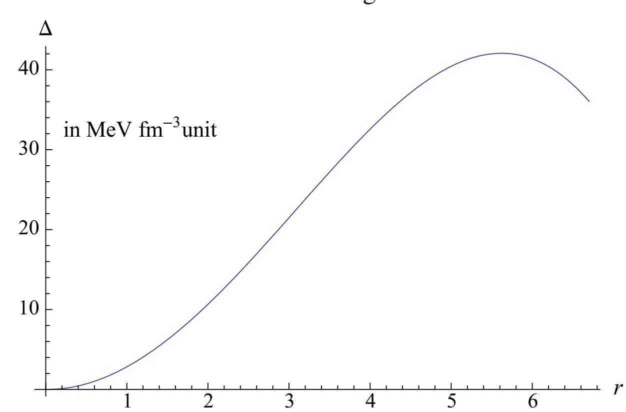

Figure 3. Anisotropic factor is plotted against the radial distance r inside the stellar interior by employing the same values of the constants as mentioned in figure 1.

4. Ratio of trace of stress tensor to energy density $(p_{\rm r} + 2p_{\rm t})/\rho$:

   We require that the ratio of trace of stress tensor to energy density $(p_{\rm r} + 2p_{\rm t})/\rho$ decreases radially outwards. This is confirmed graphically in figure 4.

5. Velocity of sound:

   For causality to be obeyed, the radial and transverse velocities of sound should be lie between [0, 1]. The radial velocity $(v_{\rm sr}^2)$ and transverse velocity $(v_{\rm st}^2)$ of

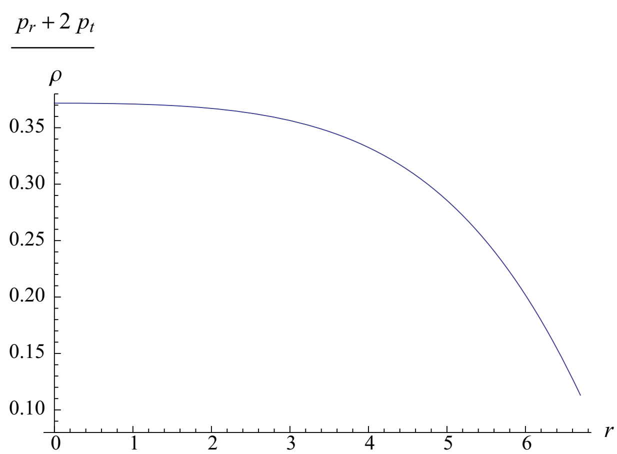

Figure 4. $(p_r + 2p_t)/\rho$ is plotted against the radial distance r inside the stellar interior by employing the same values of the constants as mentioned in figure 1.

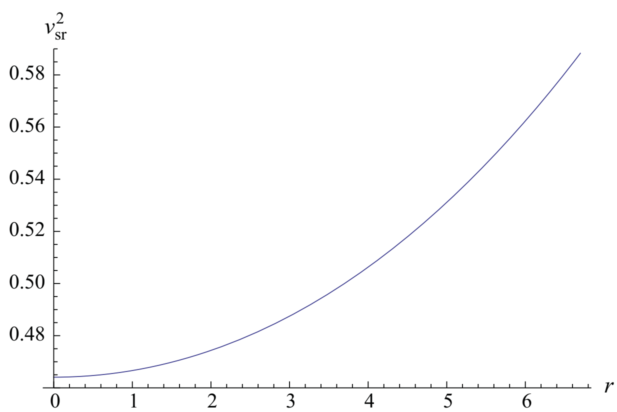

Figure 5. Square of the radial velocity of sound is plotted against the radial distance r inside the stellar interior by employing the same values of the constants as mentioned in figure 1.

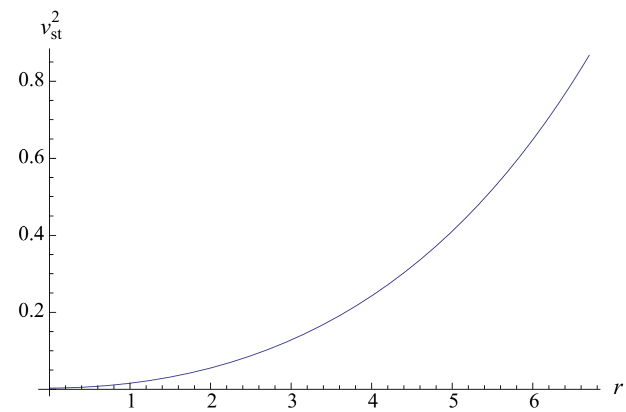

Figure 6. Square of the transverse velocity of sound is plotted against the radial distance r inside the stellar interior by employing the same values of the constants as mentioned in figure 1.

sound can be obtained as

$$v_{sr}^2 = \frac{dp_r}{d\rho}, \tag{30}$$

$$v_{st}^2 = \frac{dp_t}{d\rho}. \tag{31}$$

Due to the complexity of these expressions, we illustrate the causality conditions with the help of graphical representations. Figures 5 and 6 clearly show that $0 < v_{sr}^2 \leq 1$ and $0 < v_{st}^2 \leq 1$ everywhere within the stellar configuration.

6. Stability:

The stability of a stellar configuration depends on the adiabatic index Γ given by

$$\Gamma_r = \frac{\rho + p_r}{p_r}\frac{dp_r}{d\rho}. \tag{32}$$

Now $\Gamma > 4/3$ is the condition for the stability of a Newtonian sphere and $\Gamma = 4/3$ is the condition for a neutral equilibrium proposed by Bondi [47]. This condition changes for a relativistic isotropic sphere due to the regenerative nature of pressure, which renders the sphere more unstable. For an anisotropic general relativistic sphere the situation becomes more complicated, because the stability will depend on the type of anisotropy. For an anisotropic relativistic sphere the stability condition is given by Chan *et al* [48]

$$\Gamma_r > \frac{4}{3} + \left[\frac{4}{3}\frac{p_{t0} - p_{r0}}{|p'_{r0}|r} + \frac{8\pi}{3}\frac{\rho_0 p_{r0} r}{|p'_{r0}|}\right]_{max}, \tag{33}$$

where p_{r0}, p_{t0} and ρ_0 are the initial radial and tangential pressure and energy density. The profile of the adiabatic index Γ is shown in figure 7 which indicates that the particular configuration developed in this paper is stable.

7. Energy conditions:

A realistic star should satisfy the energy conditions namely, the weak energy condition (WEC), null energy condition (NEC) and strong energy condition (SEC) as given below:

(i) NEC: $\rho \geq 0$, (34)

(ii) WEC: $\rho - p_r \geq 0$, $\rho - p_t \geq 0$, (35)

(iii) SEC: $\rho - p_r - 2p_t \geq 0$. (36)

For the specific stellar configuration developed here, validity of the inequalities (34)–(36) have been demonstrated with the help of graphical representations in figure 9.

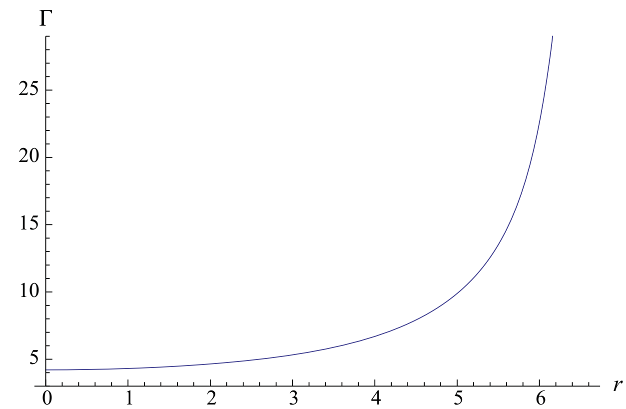

Figure 7. The adiabatic index Γ is plotted against the radial distance r inside the stellar interior by employing the same values of the constants as mentioned in figure 1.

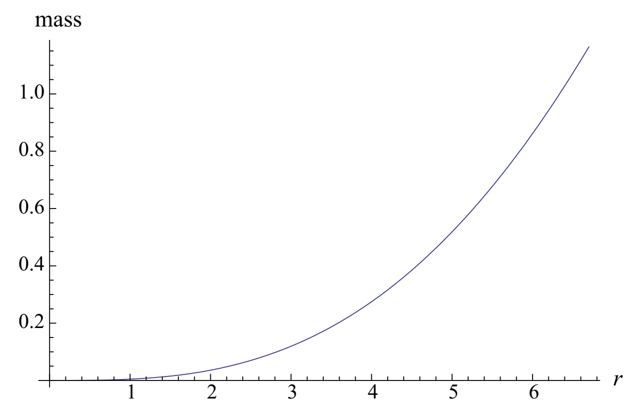

Figure 8. Mass function is plotted against the radial distance r inside the stellar interior by employing the same values of the constants as mentioned in figure 1.

6. Mass–radius relation

The mass function in our model is obtained from (7) as

$$m(r) = \frac{abr^3(2 + br^2)}{2(1 + br^2)^{5/2}}. \quad (37)$$

The profile of the mass function is shown in figure 8. Since as $r \to 0$ we have $m(r) \to 0$, it implies that the mass function is free from any central singularity.

Buchdahl [49] obtained an upper bound on the mass to radius ratio, i.e., compactness u of a relativistic compact star such that $M/r < 4/9$. In our model, we have

$$u(r) = \frac{abr^2(2 + br^2)}{2(1 + br^2)^{5/2}}. \quad (38)$$

The compactness u for different compact star models are given in table 2. We note that the compactness of the stars we are considering are within the Buchdahl limit [49].

We have also determined the surface red-shift using the formula

$$1 + z_s = \left(1 - \frac{2m}{R}\right)^{-1/2},$$

which for our model turns out to be

$$z_s = \left[1 - \frac{abr^2(2 + br^2)}{(1 + br^2)^{5/2}}\right]^{-1/2} - 1. \quad (39)$$

The values of the surface red-shift parameter for different stellar configurations are given in table 2. For an isotropic star, in the absence of a cosmological constant, Buchdahl [49] and Straumann [50] have shown that $z_s \leq 2$. Böhmer and Harko [51] showed that for an anisotropic star, in the presence of a cosmological constant, the surface red-shift can take on a much higher value $z_s \leq 5$. The restriction was subsequently modified by Ivanov [52] who showed that the maximum permissible value could be as high as $z_s = 5.211$. In our case, we have $z_s \leq 1$ for different compact star models developed in this paper (see table 2).

7. Discussion

In this paper we have presented a new model of a compact star in isotropic coordinates which is free from central singularity with the exterior being the vacuum Schwarzschild space–time. To solve the Einstein field equations we have employed the modified Chaplygin gas equation of state which is inspired by the current observation of the expanding Universe and its connection to the existence of dark energy (figure 9).

By considering the observed radius of the compact star 4U 1538-52 as an input parameter (we have

Table 2. Values of physical parameters for different compact star models.

Compact star	Central density ($g\cdot cm^{-3}$)	Surface density ($g\cdot cm^{-3}$)	Central pressure ($dyne\cdot cm^{-2}$)	u	z_s
4U 1538-52	2.71×10^{15}	2.10×10^{15}	2.94×10^{35}	0.168	0.228
PSR J1614-2230	1.535×10^{15}	1.027×10^{15}	2.29×10^{35}	0.299	0.579
Vela X-1	2.22×10^{15}	1.44×10^{15}	3.47×10^{35}	0.273	0.484
Cen-X3	1.73×10^{15}	1.24×10^{15}	2.24×10^{35}	0.241	0.389

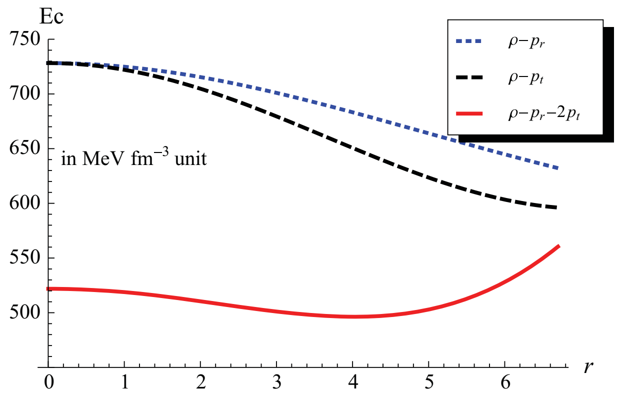

Figure 9. Weak, null and strong energy conditions are plotted against the radial distance r inside the stellar interior by employing the same values of the constants as mentioned in figure 1.

assumed $R = 7.87$ km), we have analysed the physical viability of our model. We note that the metric coefficients are free from any singularity. The variations of ρ, $(p_r + 2p_t)/\rho$, p_r, p_t are plotted in figures 1, 4 and 2 respectively which clearly show that matter density, radial and transverse pressures are positive within the stellar interior and they are monotonically decreasing functions of the radial coordinate. At the boundary of the star, the matter density and transverse pressure are non-negative and the radial pressure vanishes as expected. From the plot of $(p_r + 2p_t)/\rho$ (figure 4) we note that it is non-zero and monotonically decreasing from the centre towards the stellar surface. The anisotropic factor $\Delta = p_t - p_r$ is plotted against r in figure 3. Since $\Delta > 0$, the anisotropic factor is repulsive in nature in our model which is a desirable feature of a compact star proposed by Gokhroo and Mehra [53]. Moreover, at the centre of the star Δ vanishes which is also an essential feature of a realistic star. In order to investigate the relevance of our model in the studies of compact stars, we have considered a number of compact stars, namely 4U 1538-52, PSR J1614-2230, Vela X-1 and Cen X-3, and showed that the estimated radii and masses determined from our model are very close the observed values of these stars (see ref. [46]). We have demonstrated that our solution can be used as a viable model for describing ultracompact stars within the framework of general relativity.

References

[1] K Schwarzschild, *Sitz. Deut. Akad. Wiss. Math. Phys. Berlin* **24**, 424 (1916)
[2] R Ruderman, *Rev. Astronom. Astrophys.* **10**, 427 (1972)
[3] R Kippenhahn and A Weigert, *Stellar structure and evolution* (Springer, Berlin, 1990)
[4] A I Sokolov, *JETP* **79**, 1137 (1980)
[5] R F Sawyer, *Phys. Rev. Lett.* **29**, 823 (1972)
[6] L Herrera and N O Santos, *Phys. Rep.* **286**, 53 (1997)
[7] L Herrera, A Di Prisco, J Ospino and J Carot, *Phys. Rev. D* **94**, 064072 (2016)
[8] L Herrera, A Di Prisco and J Ospino, *Phys. Rev. D* **89**, 127502 (2014)
[9] L Herrera, J Ospino and A Di Prisco, *Phys. Rev. D* **77**, 027502 (2008)
[10] S K Maurya, Y K Gupta, S Ray and S R Chowdhury, *Eur. Phys. J. C* **75**, 389 (2015)
[11] P Mafa Maharaj, *Astrophys. Space Sci.* **354**, 463 (2014)
[12] S D Maharaj, J M Sunzu and S Ray, *Eur. Phys. J. Plus* **129**, 3 (2014)
[13] P Bhar, *Astrophys. Space Sci.* **356**, 309 (2015)
[14] P Bhar, *Astrophys. Space Sci.* **359**, 41 (2015)
[15] P Bhar and B S Ratanpal, *Astrophys. Space Sci.* **361**, 217 (2016)
[16] K N Singh, P Bhar and N Pant, *Int. J. Mod. Phys. D* **25**, 1650099 (2016)
[17] K Jotania and R Tikekar, *Int. J. Mod. Phys. D* **8**, 1175 (2006)
[18] R Tikekar and K Jotania, *J. Phys.* **68**, 397 (2007)
[19] P Bhar, *Eur. Phys. J. C* **75**, 123 (2015)
[20] F Rahaman, S Ray, A K Jafry and K Chakraborty, *Phys. Rev. D* **82**, 104055 (2010)
[21] K Dev and M Gleiser, *Gen. Relativ. Gravit.* **34**, 1793 (2002)
[22] K Dev and M Gleiser, *Gen. Relativ. Gravit.* **35**, 1435 (2003)
[23] B V Ivanov, *J. Math. Phys.* **43**, 1029 (2002)
[24] R Sharma and S D Maharaj, *Mon. Not. R. Astron. Soc.* **375**, 1265 (2007)
[25] S A Ngubelanga, S D Maharaj and S Ray, *Astrophys. Space Sci.* **357**, 74 (2015)
[26] H A Buchdahl, *Phys. Rev. D* **116**, 1027 (1959)
[27] L Herrera and W Barreto, *Phys. Rev. D* **88**, 084022 (2013); *Eur. Phys. J. C* **75**, 123 (2015)
[28] M Azam, S A Mardan and M A Rehman, *Astrophys. Space Sci.* **359**, 14 (2015)
[29] S Ray, D Ray and R N Tiwari, *Astrophys. Space Sci.* **199**, 333 (1993)
[30] S Ray, B Das, F Rahaman and S Ray, *Int. J. Mod. Phys. D* **16**, 1745 (2007)
[31] P O Mazur and E Mottola, *Proc. Natl Acad. Sci.* **101**, 9545 (2004)
[32] A A Usmani, F Rahaman, S Ray, K K Nandi, P K F Kuhfittig, S A Rakib and Z Hasan, *Phys. Lett. B* **701**, 388 (2011)
[33] F Rahaman *et al*, *Phys. Lett. B* **707**, 319 (2012)
[34] F S N Lobo, *Class. Quantum Grav.* **23**, 1525 (2006)
[35] P Bhar, *Eur. Phys. J. C* **75**, 123 (2015)
[36] P Bhar, *Astrophys. Space Sci.* **359**, 41 (2015)
[37] N O Santos, *Mon. Not. R. Astron. Soc.* **216**, 403 (1985)
[38] H B Benaoum. arXiv:hep-th/0205140 (2002)

[39] N Naidu and M Govender, *Int. J. Mod. Phys.* **25**, 1650092 (2016)
[40] M Govender and S Thirukkanesh, *Astrophys. Space Sci.* **358**, 39 (2015)
[41] A Einstein, *Sitz. Deut. Akad. Wiss. Math. Phys. Berlin* **8**, 142 (1917)
[42] W de Sitter, *Proc. R. Acad. Amst.* **19**, 1217 (1917)
[43] S Thirukkanesh, M Govender and D B Lortan, *Int. J. Mod. Phys. D* **24**, 1550002 (2015)
[44] W Israel, *Nuovo Cimento B* **44**, 1 (1966); **48**, 463 (1966)
[45] W B Bonnor, A K G de Oliveira and N O Santos, *Phys. Rep.* **181**, 269 (1989)
[46] T Gangopadhyay, S Ray, X-D Li, J Dey and M Dey, *Mon. Not. R. Astron. Soc.* **431**, 3216 (2013)
[47] H Bondi, *Proc. R. Soc. London A* **281**, 39 (1964)
[48] R Chan, L Herrera, N O Santos, *Mon. Not. R. Astron. Soc.* **265**, 533 (1993)
[49] H A Buchdahl, *Phys. Rev.* **116**, 1027 (1959)
[50] N Straumann, *General relativity and relativistic astrophysics* (Springer, Berlin, 1984) p. 43
[51] C G Böhmer and T Harko, *Class. Quantum Gravit.* **23**, 6479 (2006)
[52] B V Ivanov, *Phys. Rev. D* **65**, 104011 (2002)
[53] M K Gokhroo and A L Mehra, *Gen. Relativ. Gravit.* **26**, 75 (1994)