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Inducement and enhancement of multiple coherence resonances in unidirectionally coupled neural systems: Random and time-periodic coupling strength

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Abstract. The effect of cooperative coupling strength (CCS), i.e., random coupling strength and time-periodic coupling strength, on multiple coherence resonances in unidirectionally coupled neural system has been investigated. Results show that noise, frequency and amplitude play efficient roles for the enhancement of various coherent behaviours. There exist an optimal frequency and an optimal amplitude which make the system to display the best coherent behaviours. Furthermore, the novel coherence biresonance (CBR) induced by frequency of CCS and coherence multiresonances (CMR) induced by amplitude of CCS, are found.

Keywords. Random and time-periodic coupling strength; multiple coherence resonances; unidirectionally coupled neurons.

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1. Introduction

As we know, the coupled neural system is formed by a great number of neurons coupled via synapses [1–3], and noise is ubiquitous in coupled neurons, which originates from both the fluctuations of synapses and the random switching of ion channels [4]. Up to now, several resonance phenomena have been intensively studied in coupled neural system [5–10]. Among these, stochastic resonance (SR) [7,11] and coherence resonance (CR) [5,6,8,9] are important phenomena and have significant effects on the encoding and processing of information in coupled neural system.

Meanwhile, many studies have focussed on how the coupling type and coupling strength affect the dynamical behaviours of the neural network [12–17]. For example, the effective connectivity in networks of coupled neuronal oscillators was detected in [12]; the oscillations and synchronization status of two different network connectivity patterns in Izhikevich model were demonstrated in [13]; the noise-sustained synchronization in a FitzHugh–Nagumo ring with electrical phase-repulsive coupling was studied in [14].

However, the above studies assumed that coupling strength among coupled neurons is constant [12–17]. In fact, synapses is plastic, i.e., the coupling among neurons is time-varying and random so that the coupling can instantly adjust the firing behaviour of neurons to achieve new synchronization of coupled neurons [18–22]. Therefore, the effect of time-periodic coupling strength (TPCS) on the firing dynamics of a globally coupled array of FitzHugh–Nagumo oscillators was studied in [18]. The enhancement of temporal coherence via time-periodic coupling strength in a scale-free network of stochastic Hodgkin–Huxley neurons was reported in [19]; the study on random coupling strength-induced synchronization transitions in neuronal network was carried out in [21]. Furthermore, it is well recognized that the unidirectionally coupled way is motivated by the dynamics of central pattern generators, which are responsible for the control of every rhythmic activity in animals [23–30].

However, to our knowledge, few researches are focussed on how the cooperative coupling strength (CCS), i.e., random coupling strength and time-periodic coupling strength, affects the coherent behaviours in coupled neurons. Our purpose is, therefore, to investigate the cooperative effect of random coupling strength (RCS) and TPCS on the internal information flow transduction through the CR phenomenon in unidirectionally coupled neural system. We expect that the present work will help us to reveal the importance of RCS and TPCS in improving the time precision of the signal transmission in coupled neural system.

The remainder of this paper is structured as follows: In §2 we introduce the model and mathematical methods presently in use. Results and discussion are presented in §3, and in §4 we summarize our results.

2. Dynamical model and methods

The FitzHugh–Nagumo (FHN) model, proposed by FitzHugh [31], is a representative example of excitable systems. Its dynamic equations are given by

$$\varepsilon\frac{\mathrm{d}x}{\mathrm{d}t} = x - x^3/3 - y,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x + a, \tag{1}$$

where $\varepsilon = 0.01$ and a is the control parameter. For $|a| > 1$, the system has only a stable fixed point, while for $|a| < 1$, a limit cycle appears.

Due to the plasticity among synapses, $f_0 D\xi(t)$ and $f_0 A\sin(wt)$ are employed in 20 unidirectionally coupled FHN neurons, where $f_0 D$ denotes RCS and $f_0 A$ denotes TPCS:

$$\varepsilon\frac{\mathrm{d}x_i}{\mathrm{d}t} = x_i - \frac{x_i^3}{3} - y_i + (f_0 + f_0 D\xi(t) + f_0 A\sin(wt))\,(x_{i-1} - x_x)$$
$$\frac{\mathrm{d}y_i}{\mathrm{d}t} = x_i + a_i. \tag{2}$$

Here, i ranges from 1 to 20. We set a_i $(i = 1, \ldots, 20)$ to be $a_0 = 1.05$, which is slightly larger than 1 such that all neurons in the coupled system are in steady state. $f = f_0 + f_0 D + f_0 A$ denotes CCS, f_0 is the constant coupling strength, D is the intensity of common noise $\xi(t)$ with zero mean value $\langle\xi(t)\rangle = 0$ and unit variance $\langle\xi(t)\xi(t+\tau)\rangle = \delta(\tau)$, A and w are the amplitude and frequency of $f_0 A\sin(wt)$, respectively [2,19–21].

To qualitatively and quantitatively characterize the coherent behaviours of noise-induced oscillations in coupled neural system, the signal-to-noise ratio (SNR) and the correlation time (τ_c) are usually used, and identical results could be obtained by using two characteristic methods [5,32]. In the present work, τ_c is mainly employed to characterize the coherent behaviours of the coupled neural system. The normalized autocorrelation function $C(\tau)$ of the variable y and the corresponding characteristic τ_c are calculated as [23,32,33]:

$$C(\tau) = \frac{\langle\tilde{y}(t)\tilde{y}(t+\tau)\rangle}{\langle\tilde{y}^2\rangle},$$
$$\tilde{y} = y - \langle y\rangle, \quad \tau_c = \int_0^\infty C^2(t)\mathrm{d}t, \tag{3}$$

where $\langle\cdot\rangle$ denotes average over time. τ_c quantitatively characterize the regularity of the pulses: a larger value of τ_c represents better coherent behaviours of coupled neural system [23,32,33]. Equation (2) is integrated using Euler's method with a fixed time step of 0.01 time units. In each calculation, the time evolution of the system lasts from 0 to 500 time units, the first 100 time units are discarded due to transient behaviour. The value of τ_c is calculated by averaging the results of 20 runs.

3. Results and discussion

3.1 *The cooperative effect of random coupling strength (RCS) and time-periodic coupling strength (TPCS) on coupling strength-induced coherence resonance (CR) in coupled neural system*

In this section, we are interested in investigating the cooperative effect of RCS and TPCS on CR. To address the problem, we investigate the dependence of τ_c on constant coupling strength f_0. From figures 1a and 1b, two interesting features are clearly revealed:

(i) It is seen from figure 1a (or figure 1b) that the curve passes through a 'peak' at $f_0 = 0.07$ first and then show a 'plateau' at $f_0 > 0.12$, which implies that CR can be achieved at $f_0 = 0.07$, and the change of coupling level would not influence τ_c any more at $f_0 > 0.12$. The reason might lie in the fact that the information transmission among neurons reaches an optimal value at $f_0 = 0.07$, and is robust and cannot be influenced by coupling at $f_0 > 0.12$. The result implies that the information transmission among neurons reaches saturation at $f_0 > 0.12$, which could be called CR without tuning [23,33–36]. Our previous work [33] demonstrated that noise-induced CR without tuning could occur in coupled neural system. In this letter, RCS CR without tuning and CCS-induced CR without tuning phenomena could both occur, which are important phenomena in coupled neural system because they could be favourable to obtain the optimal or stable system exhibition to the external stimuli by adjusting the coupling [36].

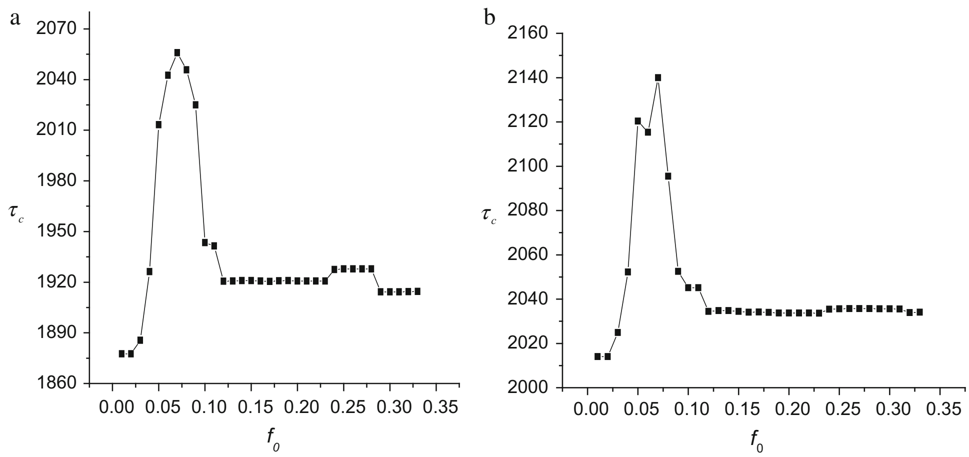

Figure 1. The dependence of correlation time (τ_c) on constant coupling strength (f_0). (**a**) Random coupling strength (f_1) at $D = 0.002$; (**b**) cooperative coupling strength (f) at $D = 0.002$, $A = 0.1$ and $w = 0.05$.

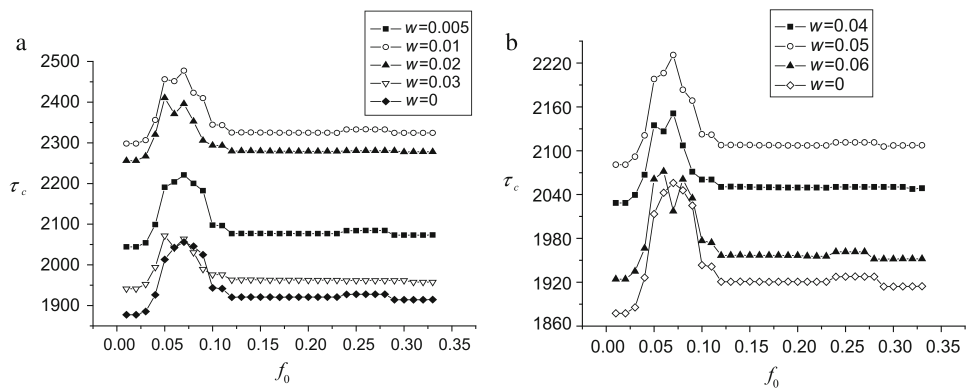

Figure 2. The dependence of correlation time (τ_c) on constant coupling strength (f_0) at various frequencies (w), at $A = 0.1$ and $D = 0.002$ when (**a**) $w = 0, 0.005, 0.01, 0.02, 0.03$ and (**b**) $w = 0.04, 0.05$ and 0.06.

(ii) It is observed from figures 1a and 1b that the maximal τ_c for f_0 is about 2055.9 for RCS (see figure 1a), while the maximal τ_c for f_0 is about 2140 for CCS (see figure 1b). That is to say, CCS-induced CR is stronger than RCS-induced CR, which suggests that CCS plays an active role for the enhancement of CR.

In order to strengthen our arguments, the dependence of τ_c on f_0 at fixed noise intensity D of $f_0 D\xi(t)$, various frequencies (w) and amplitudes (A) of $f_0 A \sin(wt)$ should be investigated. Here, the dependence of τ_c on f_0 at fixed $D = 0.002$, $A = 0.1$ and various w ($w = 0$, 0.005, 0.01, 0.02, 0.03, 0.04, 0.05 and 0.06) are investigated as shown in figure 2. In order to clearly exhibit the results, $w = 0$, 0.005, 0.01, 0.02 and 0.03 are displayed in figure 2a, and $w = 0.04$, 0.05 and 0.06 are shown in figure 2b. It is seen from figure 2a that the CCS-induced CR is enhanced as w is increased from 0.001 to 0.01, reaches the strongest at $w = 0.01$, and then is decreased as w is further increased from 0.01 to 0.03, which displays that 0.01 is the optimal frequency (w_{opt}) for the enhancement of CCS-induced CR.

From figure 2b, we can see that CCS-induced CR increases with the increment of w from 0.04 to 0.05, reaches the strongest at $w = 0.05$, and then drops as w is further increased from 0.05 to 0.06, which suggests that 0.05 is another optimal frequency (w_{opt}) for the enhancement of CCS-induced CR.

Furthermore, in figures 3a and 3b the dependence of τ_c on f_0 is also investigated at fixed $D = 0.002$, $w = 0.05$ and various amplitudes A ($A = 0$, 0.01, 0.05, 0.1, 0.2, 0.3, 0.4 and 0.5). The results at $A = 0$, 0.01, 0.05 and 0.1 are shown in figure 3a and those at $A = 0.2$, 0.3, 0.4 and 0.5 are exhibited in figure 3b. It is easy to observe from figure 3a that CCS-induced CR rises with the increment of A from 0 to 0.05, reaches the strongest at $A = 0.05$, and then drops as A is increased from 0.05 to 0.1. From figure 3b, CCS-induced CR is increased as A is increased from 0.2 to 0.4, reaches the strongest at

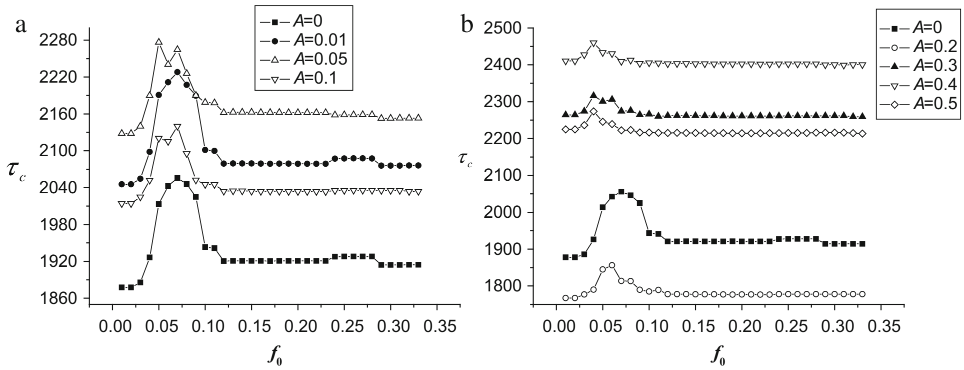

Figure 3. The dependence of correlation time (τ_c) on constant coupling strength (f_0) at (**a**) $A = 0, 0.01, 0.05$ and 0.1 and (**b**) $A = 0.2, 0.3, 0.4$ and 0.5, when $w = 0.05$ and $D = 0.002$.

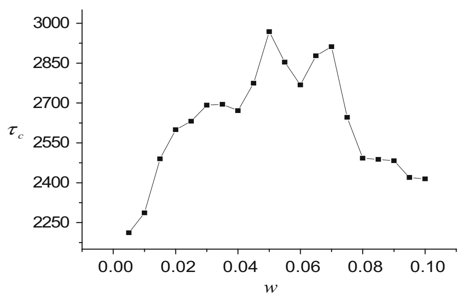

Figure 4. The dependence of correlation time (τ_c) on frequency (w) of cooperative coupling strength (CCS) at $D = 0.002$, $A = 0.1$ and $f_0 = 0.07$.

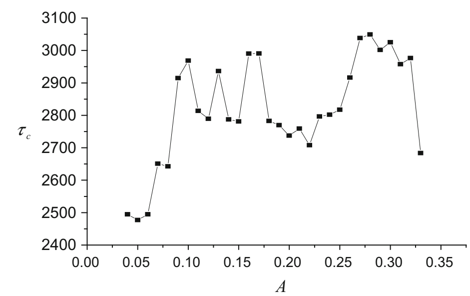

Figure 5. The dependence of correlation time (τ_c) on amplitude (A) of cooperative coupling strength (CCS) at $D = 0.002$, $w = 0.05$ and $f_0 = 0.07$.

$A = 0.4$, and then is decreased as A is increased from 0.4 to 0.5. The results in figures 3a and 3b show that there exist two optimal A (A_{opt}) for the enhancement of CCS-induced CR.

Based on the results in figures 2a and 2b and figures 3a and 3b, we could intuitively deduce that the CCS frequency-induced coherence multiresonance (CMR) and CCS amplitude-induced CMR might occur. In order to confirm the result, the dependence of τ_c on frequency (w) and amplitude (A) are investigated, respectively. As shown in figure 4, the CCS frequency-induced coherence biresonance (CBR) is obtained. In figure 5, the CCS amplitude-induced CMR occurs.

Now let us turn to the problem of CCS frequency-induced CBR in figure 4 and CCS amplitude-induced CMR in figure 5. As presented in refs [11,23,34], when the time-scale of the noise control and the time-scale of the action of noise-induced internal oscillations reach one optimal matching, the single peak CR would usually be achieved. But when the stochastic time-scale and the determinate time-scale reach the optimal multimatching, the system would exhibit CMR behaviour [37]. In our paper, the CCS frequency-induced CBR and CCS amplitude-induced CMR indicate that the amplitude of the noise-induced oscillations increases by multiple values of frequency (w) or amplitude (A) of CCS in coupled neural system.

Noise is often thought to have undesirable effects on dynamical systems. In fact, it could also play an active role for the coherent behaviours of dynamical systems [38–40]. So we are also interested in investigating how noise intensity (D) of CCS affects CR. In figures 6a–6d the dependence of τ_c on f_0 is investigated at fixed $A = 0.1$, $w = 0.05$ and various D. One can see that different coherent behaviours could be induced by D, i.e., CMR at $D = 0.001$ (see figure 6a), CBR at $D = 0.002$ (see figure 6b), CR without tuning at $D = 0.006$ (see figure 6c) and 0.008 (see figure 6d). In particular, the maximal correlation time $\tau_{c(\max)}$ is 3072 in figure 6a, 2115 in figure 6b, 725 in figure 6c and 487 in figure 6d. That is to say, $\tau_{c(\max)}$ is decreased as D is increased, which implies that D plays a negative role for the CCS-induced coherent behaviours.

It is because that there exists a competition between noise-induced oscillations propagation induced by coupling and noise-induced oscillation process induced by noise in coupled neural system. When the noise intensity is too weak, noise-induced oscillation propagation

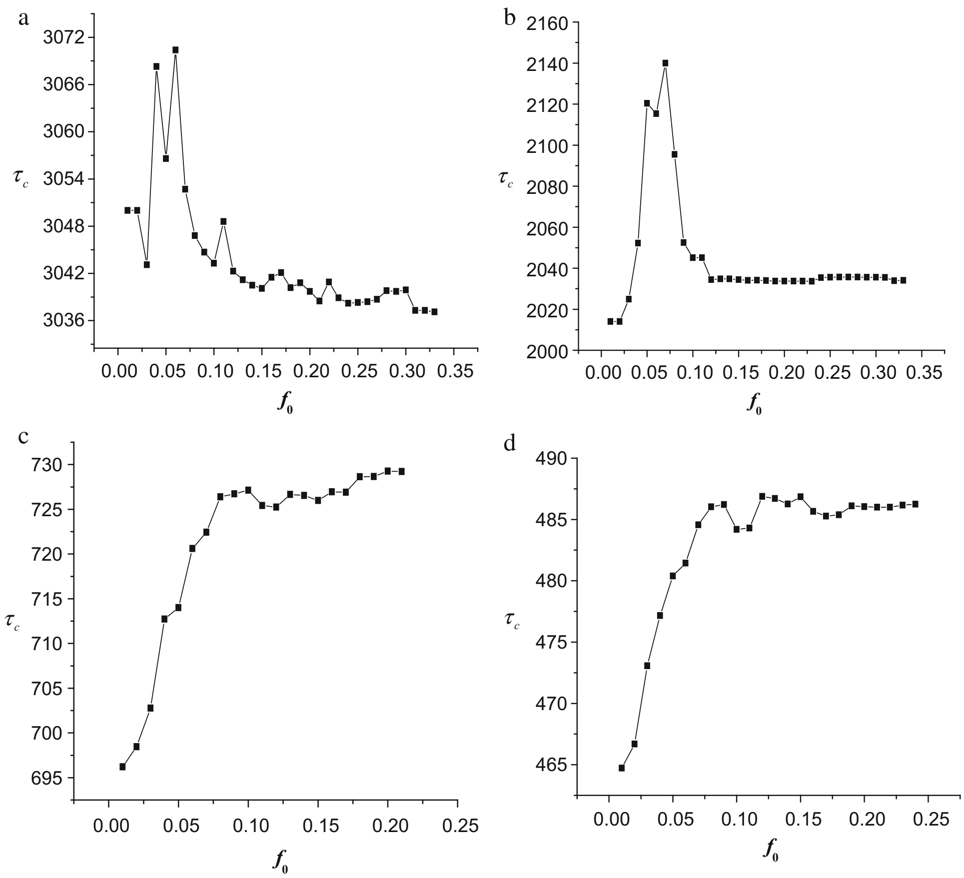

Figure 6. The dependence of correlation time (τ_c) on constant coupling strength (f_0) at various noise intensities (D), $A = 0.1$ and $w = 0.05$. (**a**) $D = 0.001$; (**b**) $D = 0.002$; (**c**) $D = 0.006$; (**d**) $D = 0.008$.

induced by coupling is stronger than noise-induced oscillation process induced by noise, indicating that the coupled neurons are highly coherent; When noise intensity is too strong, noise-induced oscillation propagation induced by coupling is weaker than noise-induced oscillation process induced by noise so that the coupled neurons are in low coherence [41].

3.2 *The cooperative effect of random coupling strength (RCS) and time-periodic coupling strength (TPCS) on noise-induced coherence resonance (CR) in coupled neural system*

In what follows, let us consider the cooperative effect of RCS and TPCS on noise-induced CR. In figure 7 the dependence of τ_c on D for random coupling strength (f_1) and cooperative coupling strength (f) are investigated. It is clearly observed that noise-induced CBR can be exhibited for f_1 and f respectively. Furthermore, the noise-induced CBR for f is stronger than that for f_1, which suggests that the cooperative effect of RCS and TPCS plays an active role for the enhancement of noise-induced CBR.

Herein, let us further investigate the effect of CCS on noise-induced CBR at various frequencies (w) and amplitudes (A). In figure 8, we show the dependence of τ_c on D at various amplitudes (A). One can see that the peak 2 is increased as A is increased from 0.05 to 0.1, reaches the strongest at $A = 0.1$, and then is decreased as A is increased from 0.1 to 0.12, which implies that there exists an optimal amplitude (A_{opt}) for the enhancement of peak 2, and A_{opt} is about 0.1. However, the peak 1 hardly changes with the increment of A. The result reveals that the CBR enables neurons to selectively respond to the signal at the preferred noise intensity, which significantly improves the ability of neurons to effectively transmit signals. We argue that the phenomenon of signal-selective enhancement is novel and potentially important, in the noisy background, to study the role of coupling information processing and transduction in the nervous system [41–44].

Furthermore, in figure 9 the dependence of τ_c on D at various w is also investigated. It is seen that the noise-induced CBR increases as w is increased from 0.01 to 0.07, reaches the strongest at $w = 0.07$,

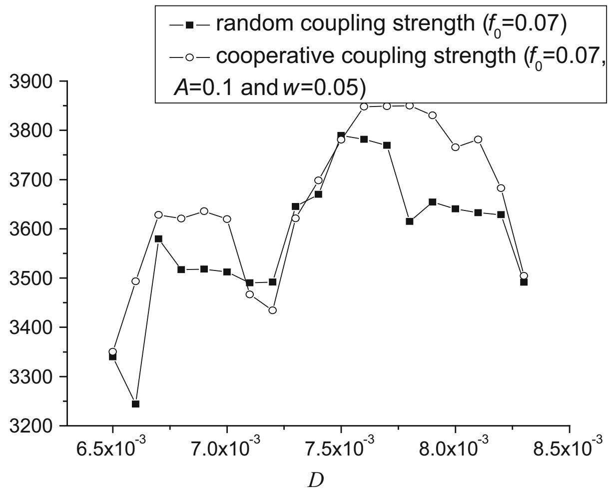

Figure 7. The dependence of correlation time (τ_c) on noise intensity (D) of random coupling strength (RCS) and cooperative coupling strength (CCS).

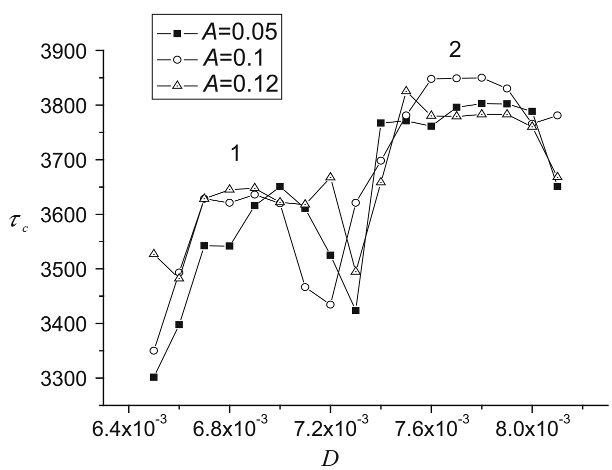

Figure 8. The dependence of correlation time (τ_c) on noise intensity (D) of cooperative coupling strength (CCS) for different amplitudes (A) at $w = 0.05$ and $f_0 = 0.07$.

and then drops with the increment of w from 0.07 to 0.09. The result suggests that the optimal frequency (w_{opt}) for the strongest noise-induced CBR is about 0.07.

Taken together, depending on noise intensity, frequency and amplitude, CCS-induced CR without tuning and CCS-induced CR, noise-induced CBR, CCS frequency-induced CBR and CCS amplitude-induced CMR can occur. These coherent behaviours could be briefly explained as follows: The CCS could act as an external drive signal (energy storage). If the amplitude or frequency of CCS is suitable, it can be used to make coupled neural system to achieve the most regular coherent behaviours and CR phenomenon [22].

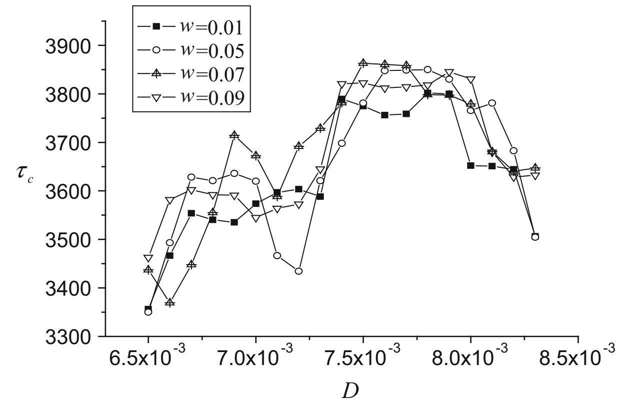

Figure 9. The dependence of correlation time (τ_c) on noise intensity (D) of cooperative coupling strength (CCS) for different frequencies (w) at $A = 0.1$ and $f_0 = 0.07$.

For all the above results, special attention is paid to know how a chain of 20 unidirectionally coupled FHN neurons, all tuned to be excitable, exhibits various coherent behaviours by modulating RCS and TPCS. Now we are interested in investigating how the number of systems affects the coherent behaviours. To do it, three cases of the number of systems are investigated: (1) $i = 20$, (2) $i = 40$; (3) $i = 60$. One can see from figures 10a–10c that, for three cases, τ_c increases from the first neuron to the second neuron, and then τ_c is unchanged along the coupling chain, which implies that the noise-induced oscillation (NIOs) signals might propagate along the coupling chain for a long distance, an example of undamped signal transmission [33,45].

In ref. [18], the dynamics of a population of globally coupled FHN oscillators with a time-periodic coupling strength was studied. It was found that, when a population of globally coupled oscillators with desynchronizing coupling is periodically forced, resonance responses occur predominantly with a 2n:1 subharmonic locking. In contrast to their study, we investigate how the cooperative coupling strength (CCS, i.e., random coupling strength and time-periodic coupling strengh) affects CR in unidirectionally coupled FHN system, which shows that noise intensity, frequency and amplitude could effectively control the occurrence of different CR behaviours. Furthermore, 60 globally coupled FHN oscillators are employed, and the coupled neurons are not subjected to noise in ref. [18], while 20 unidirectionally coupled FHN oscillators are used and the coupled neurons are subjected to common noise of CCS in our present work.

Our previous work [33] has reported the selectivity of noise and coupling for coherence biresonance and array-enhanced coherence biresonance in coupled FHN neural systems, which mainly studies the effect of external

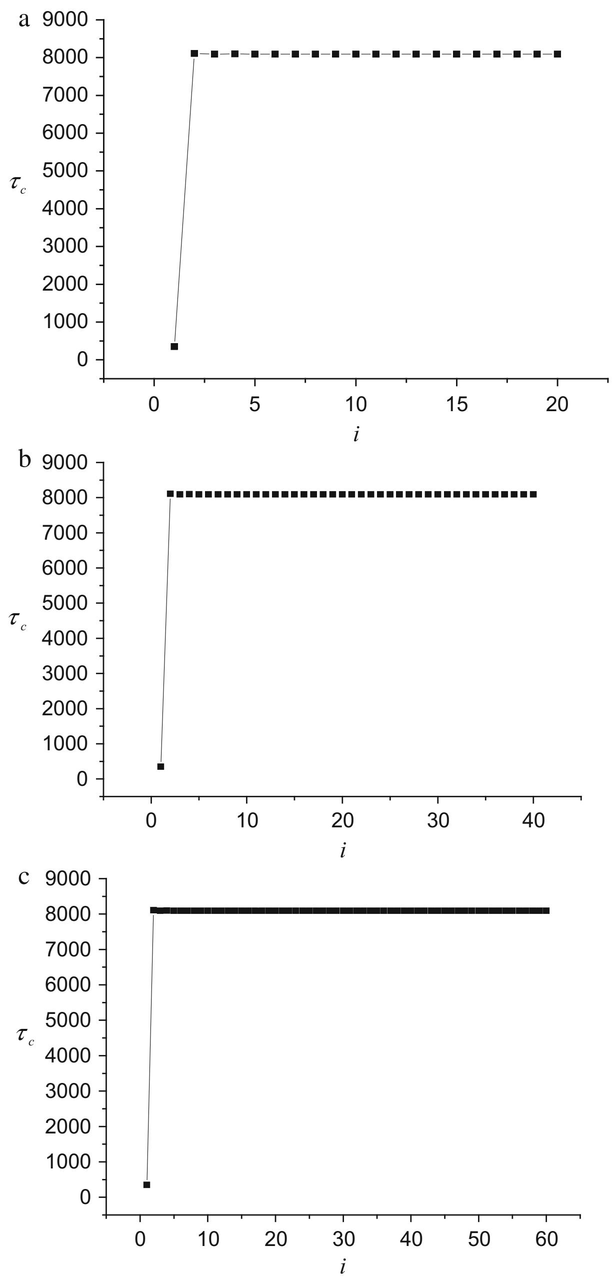

Figure 10. The dependence of correlation time (τ_c) on the number of systems (i) at $D = 0.002$, $f_0 = 0.01$, $A = 0.1$ and $w = 0.01$. (**a**) $i = 20$; (**b**) $i = 40$; (**c**) $i = 60$.

noise on the transimission of noise-induced oscillation (NIO) signal and coherent behaviours for each neuron one by one. However, in the present work, our purpose is to investigate how noise, amplitude and frequency affect the collective coherent behaviours of coupled neural system. Furthermore, in our previous work [33], noise is local and it is added to the control parameter of the first neuron. However, in the present work, noise is common and it is added to coupling strength.

4. Conclusions

The effect of CCS, i.e., random coupling strength and time-periodic coupling strength, on multiple coherence resonances (MCR) in unidirectionally coupled neural system has been investigated. Results show that noise intensity, the frequency and amplitude play efficient roles for the enhancement of coherence behaviours. There exists an optimal frequency and an optimal amplitude which makes the system to display the best CCS-induced CR, CCS-induced CR without tuning and noise-induced CBR. Furthermore, we find that novel CCS frequency-induced CBR and CCS amplitude-induced CMR can occur. We expect that the results in this paper might be important in signal transmission in human brain, and might be exploited in improving the time precision of the signal transmission in coupled neural system.

Acknowledgements

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