



Quantum eraser for three-slit interference

NAVEED AHMAD SHAH¹ and TABISH QURESHI² *

¹Department of Physics, Jamia Millia Islamia, New Delhi 110 025, India

²Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi 110 025, India

*Corresponding author. E-mail: tabish@ctp-jamia.res.in

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Abstract. It is well known that in a two-slit interference experiment, if the information, on which of the two paths the particle followed, is stored in a quantum path detector, the interference is destroyed. However, in a set-up where this path information is ‘erased’, the interference can reappear. Such a set-up is known as a quantum eraser. A generalization of quantum eraser to a ‘three-slit’ interference is theoretically analysed. It is shown that three complementary interference patterns can arise out of the quantum erasing process.

Keywords. Quantum eraser; wave–particle duality; quantum interference.

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1. Introduction

The first double-slit interference experiment was performed with light by Thomas Young in 1801, thereby demonstrating the wave nature of light [1]. With the advent of quantum theory, it was realized that all quantum particles, although considered indivisible, should show wave nature. The first double-slit interference with electrons was demonstrated by Jönsson [2].

A double-slit experiment with massive quantum particles brings in a whole new concept, that of a particle interfering with itself. That the interference of electrons in a double-slit experiment involves an electron interfering with itself, and not with other electrons, was conclusively demonstrated for the first time by Tonomura *et al* [3].

Niels Bohr had stated that the wave aspect and the particle aspect are complementary, in the sense that if an experiment clearly reveals the wave nature, it will completely hide the particle aspect and vice-versa [4]. The complementarity principle came under attack right after its inception when Einstein proposed his famous recoiling slit experiment (see e.g. [5]). Since then, it has been a subject of debate regarding its various aspects and its validity too [6–8]. The current understanding is that in the context of the two-path interference experiment, either as a two-slit experiment,

or as a Mach–Zehnder interferometer, the principle of complementarity is quantitatively represented by the so-called duality relation [9]

$$\mathcal{V}^2 + \mathcal{D}^2 \leq 1, \quad (1)$$

where \mathcal{D} is the path distinguishability and \mathcal{V} is the visibility of the interference pattern.

A very interesting consequence of this wave–particle duality is the so-called quantum eraser [10,11]. The idea of quantum eraser is that if the which-way information can, in some way, be ‘erased’, the lost interference pattern can be made to reappear. This phenomenon holds even when the which-path information has been erased ‘well after’ the particle has been registered on the screen. In such a scenario, the phenomenon is called ‘delayed-choice’ quantum eraser. In course of time, quantum eraser was experimentally demonstrated for two-path interference in several different ways [12–23].

The concept of complementarity should hold in multi-path or multislit experiments too. It is an issue which has been, and continues to be, the focus of much research [24–32]. We pose the question, is quantum eraser possible in multislit interference experiments too, and are there any subtleties involved? We begin by looking at the issue of quantum erasing in a 3-slit interference experiment, which is the subject of this investigation.

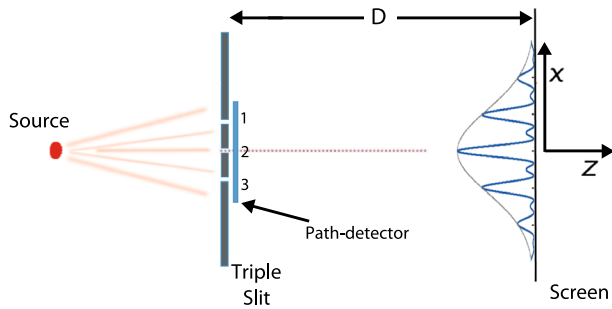


Figure 1. Schematic diagram of a triple-slit interference experiment. A quantum path detector could be added to the set-up, which is capable of obtaining information on which slit the particle passed through.

2. Three-slit interference and wave-particle duality

Three-slit interference is a bit more complex than its two-slit counterpart, because it involves superposition of three parts (see figure 1). A particle emerging from a triple-slit may be assumed to have the form

$$|\psi\rangle = \frac{1}{\sqrt{3}}|\psi_1\rangle + \frac{1}{\sqrt{3}}|\psi_2\rangle + \frac{1}{\sqrt{3}}|\psi_3\rangle, \quad (2)$$

where $|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3\rangle$ represent the state of the particle corresponding to it coming out of slit 1, 2 and 3, respectively. Sharpest interference is obtained when the amplitudes for the three possibilities are equal, which is $1/\sqrt{3}$ in our case. By virtue of the Born rule, the three-slit interference can be thought of as arising from three two-slit interferences [33].

If one were to introduce a ‘quantum’ path detector which can tell which of the three slits the particle has gone through, the state of the particle and the path detector may be written as

$$|\psi\rangle = \frac{1}{\sqrt{3}}|\psi_1\rangle|+\rangle + \frac{1}{\sqrt{3}}|\psi_2\rangle|0\rangle + \frac{1}{\sqrt{3}}|\psi_3\rangle|-\rangle, \quad (3)$$

where $|+\rangle$, $|0\rangle$, $|-\rangle$ represent three orthonormal states of the path detector. If the path detector is capable of gaining information on which slit the particle went through, by virtue of von Neumann’s idea of a quantum measurement [34], the combined state has to have the above form.

If the state of a particle, emerging from the triple slit, is given by (2), the state of the particle at a time t when it reaches the screen, would be different because it would have undergone a time evolution. Using quantum wave-packets is a useful way to study the dynamics of quantum particles (see e.g. [35]). In order to do an accurate analysis of the interference, we carry out a wave-packet analysis of the particle. We assume that the states emerging from the three slits are Gaussian wave-packets, centred at the respective slits, with a width

which is related to the width of the slits. Assume that the particle travels along the z -axis, and the three slits are parallel to the y -axis, located at $x = d$, $x = 0$, and $x = -d$. In order to keep the analysis simple, we ignore the dynamics along the z - and y -axes. The dynamics along the z -axis serves only to transport the particle from the slits to the screen. We assume that the particle travels in the z -direction with an average momentum p_0 , and a de Broglie wavelength $\lambda = h/p_0$ is associated with it. The particle travels a distance D in a time t , and $D = p_0 t/m$. This leads us to

$$\lambda D = ht/m. \quad (4)$$

The more interesting dynamics is in the x -direction where the wave-packets are expected to expand and overlap, giving rise to interference. One can go beyond this approximation by treating the dynamics along the y - and z -directions more rigorously [36]. However, for our purpose, this approximate treatment suffices.

The states of the particle in front of the three slits, just after it emerges, are assumed to be

$$\begin{aligned} \langle x|\psi_1\rangle &= C \exp\left(\frac{-(x-d)^2}{\epsilon^2}\right) \\ \langle x|\psi_2\rangle &= C \exp\left(\frac{-x^2}{\epsilon^2}\right) \\ \langle x|\psi_3\rangle &= C \exp\left(\frac{-(x+d)^2}{\epsilon^2}\right), \end{aligned} \quad (5)$$

where $C = (2/\pi\epsilon^2)^{1/4}$. The state (2) can then be represented as

$$\langle x|\psi\rangle = \frac{C}{\sqrt{3}}(e^{-(x-d)^2/\epsilon^2} + e^{-x^2/\epsilon^2} + e^{-(x+d)^2/\epsilon^2}). \quad (6)$$

We assume the particle travels freely, and has a mass m , and hence its evolution is governed by the operator

$$U(t) = \exp(-ip^2 t/2m\hbar), \quad (7)$$

where p represents the operator for the x -component of the momentum of the particle. Using the time evolution governed by the above, the state of the particle, when it reaches the screen after a time t , is given by

$$\begin{aligned} \psi(x, t) &= \frac{C_t}{\sqrt{3}} \left(e^{-(x-d)^2/(\epsilon^2+ia)} + e^{-x^2/(\epsilon^2+ia)} \right. \\ &\quad \left. + e^{-(x+d)^2/(\epsilon^2+ia)} \right), \end{aligned} \quad (8)$$

where $a = 2\hbar t/m = \lambda D/\pi$ and $C_t = [2/\pi(\epsilon + ia/\epsilon)]^{1/4}$. The probability of the particle, hitting the screen at a position x , is given by

$$\begin{aligned} |\psi(x, t)|^2 &= \frac{|C_t|^2}{3} e^{-2x^2/\Omega} \left(1 + e^{-2d^2/\Omega} 2 \cosh(4xd/\Omega) \right) \end{aligned}$$

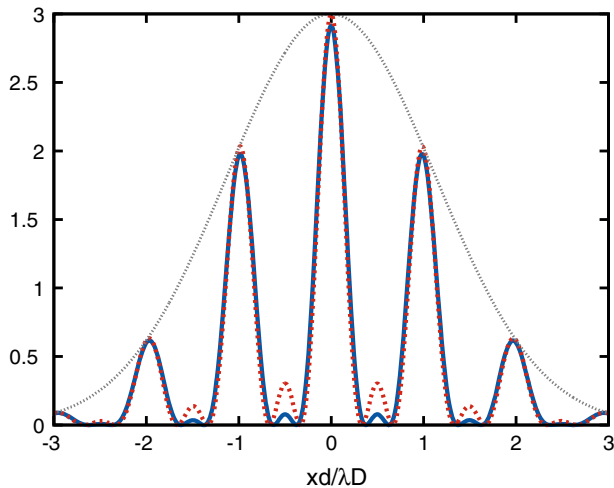


Figure 2. Recovered interference pattern, given by $|\psi_{\uparrow}(x, t)|^2$ (solid line) and the original 3-slit interference pattern, given by (10) (dashed line). The two are clearly different. The dotted line represents the lost interference in the presence of which-way information, given by (12).

$$\begin{aligned} &+ e^{-(d^2-2xd)/\Omega} 2 \cos(2xd/a - d^2/a) \\ &+ e^{-(d^2+2xd)/\Omega} 2 \cos(2xd/a + d^2/a) \\ &+ e^{-2d^2/\Omega} 2 \cos(4xd/a) \Big), \end{aligned} \quad (9)$$

where $\Omega = \epsilon^2 + \lambda^2 D^2 / \pi^2 \epsilon^2$, and is related to the expanded width of the wave-packets. In the limit where the slits are very narrow and the wave-packets spread much much wider than d and overlap with each other strongly, i.e. $\Omega \gg d^2$, the above can be approximated by

$$\begin{aligned} |\psi(x, t)|^2 \approx & \frac{|C_t|^2}{3} e^{-2x^2/\Omega} (1 + 2 \cosh(4xd/\Omega) \\ & + 4 \cosh(2xd/\Omega) \cos(2xd/a) \\ & + 2 \cos(4xd/a)). \end{aligned} \quad (10)$$

The above expression represents a 3-slit interference pattern (see figure 2).

In the presence of a path detector, the state after the particle emerges from the triple slit, is given by

$$\begin{aligned} \langle x | \psi \rangle = & \frac{C}{\sqrt{3}} (e^{-(x-d)^2/\epsilon^2} |+\rangle \\ & + e^{-x^2/\epsilon^2} |0\rangle + e^{-(x+d)^2/\epsilon^2} |-\rangle), \end{aligned} \quad (11)$$

which is just the position representation of (3). After travelling to the screen, the probability of the particle hitting it at a position x is given by

$$\begin{aligned} |\psi(x, t)|^2 = & \frac{|C_t|^2}{3} e^{-2x^2/\Omega} \\ & \times (1 + e^{-2d^2/\Omega} 2 \cosh(4xd/\Omega)). \end{aligned} \quad (12)$$

One can see, that the cosine terms, which represented interference in (9), are missing in the above expression. Basically they are killed due to the orthogonality of $|+\rangle$, $|0\rangle$, $|-\rangle$. The states $|+\rangle$, $|0\rangle$, $|-\rangle$ carry information about which slit the particle passed through. So, it emerges that the storing of which-way information is enough to destroy interference. Wave-particle duality in a 3-slit experiment has recently been quantitatively stated by a new duality relation [28]. However, here our focus is to explore the possibility of quantum erasing in such an experiment, which is dealt with in the following discussion.

3. Quantum eraser

The three-state quantum path detector may be thought of as a pseudo-spin-1, whose z -component S_z has the eigenstate $|+\rangle$, $|0\rangle$, $|-\rangle$. These states can also be written in terms of the eigenstates of the x -component of this pseudospin S_x , which we represent as $|\uparrow\rangle$, $|\rightarrow\rangle$, $|\downarrow\rangle$. In this representation, the eigenstates of S_z look like

$$\begin{aligned} |+\rangle &= \frac{1}{2} |\uparrow\rangle + \frac{1}{\sqrt{2}} |\rightarrow\rangle + \frac{1}{2} |\downarrow\rangle \\ |0\rangle &= \frac{1}{\sqrt{2}} |\uparrow\rangle - \frac{1}{\sqrt{2}} |\downarrow\rangle \\ |-\rangle &= \frac{1}{2} |\uparrow\rangle - \frac{1}{\sqrt{2}} |\rightarrow\rangle + \frac{1}{2} |\downarrow\rangle. \end{aligned} \quad (13)$$

Using this, the state (3) can be written as

$$\begin{aligned} |\psi\rangle = & \frac{1}{\sqrt{3}} \left[\left(\frac{|\psi_1\rangle}{2} + \frac{|\psi_2\rangle}{\sqrt{2}} + \frac{|\psi_3\rangle}{2} \right) |\uparrow\rangle \right. \\ & + \left(\frac{|\psi_1\rangle}{\sqrt{2}} - \frac{|\psi_3\rangle}{\sqrt{2}} \right) |\rightarrow\rangle \\ & \left. + \left(\frac{|\psi_1\rangle}{2} - \frac{|\psi_2\rangle}{\sqrt{2}} + \frac{|\psi_3\rangle}{2} \right) |\downarrow\rangle \right]. \end{aligned} \quad (14)$$

It is quite obvious that the above state will not lead to an interference pattern, because it is the same as (3). However, if one were to measure the x -component of the pseudospin, and detect the particle in correlation with the three results of the pseudospin measurement, the following three situations will arise:

$$\begin{aligned} \langle \uparrow | \psi(t) \rangle &= \frac{1}{\sqrt{3}} U(t) \left(\frac{|\psi_1\rangle}{2} + \frac{|\psi_2\rangle}{\sqrt{2}} + \frac{|\psi_3\rangle}{2} \right) \\ \langle \downarrow | \psi(t) \rangle &= \frac{1}{\sqrt{3}} U(t) \left(\frac{|\psi_1\rangle}{2} - \frac{|\psi_2\rangle}{\sqrt{2}} + \frac{|\psi_3\rangle}{2} \right) \\ \langle \rightarrow | \psi(t) \rangle &= \frac{1}{\sqrt{3}} U(t) \left(\frac{|\psi_1\rangle}{\sqrt{2}} - \frac{|\psi_3\rangle}{\sqrt{2}} \right). \end{aligned} \quad (15)$$

The state $\langle \uparrow | \psi(t) \rangle$, for example, represents the state of the particle hitting the screen, provided that the pseudospin S_x has been found in the state $|\uparrow\rangle$. The same can be represented in the position basis as follows:

$$\begin{aligned} |\psi_{\uparrow}(x, t)\rangle &= \frac{1}{\sqrt{3}} \left(\frac{\psi_1(x, t)}{2} + \frac{\psi_2(x, t)}{\sqrt{2}} + \frac{\psi_3(x, t)}{2} \right) \\ |\psi_{\downarrow}(x, t)\rangle &= \frac{1}{\sqrt{3}} \left(\frac{\psi_1(x, t)}{2} - \frac{\psi_2(x, t)}{\sqrt{2}} + \frac{\psi_3(x, t)}{2} \right) \\ |\psi_{\rightarrow}(x, t)\rangle &= \frac{1}{\sqrt{3}} \left(\frac{\psi_1(x, t)}{\sqrt{2}} - \frac{\psi_3(x, t)}{\sqrt{2}} \right). \end{aligned} \quad (16)$$

Since $\psi_1(x, t)$, $\psi_2(x, t)$, $\psi_3(x, t)$ have already been worked out in the preceding calculation, it is straightforward to show that

$$\begin{aligned} |\psi_{\uparrow}(x, t)|^2 &= \frac{|C_t|^2}{3} e^{-2x^2/\Omega} \left(\frac{1}{2} + \frac{1}{2} \cosh(4xd/\Omega) \right. \\ &\quad \left. + \sqrt{2} \cosh(2xd/\Omega) \cos(2xd/a) \right. \\ &\quad \left. + \frac{1}{2} \cos(4xd/a) \right) \\ |\psi_{\downarrow}(x, t)|^2 &= \frac{|C_t|^2}{3} e^{-2x^2/\Omega} \left(\frac{1}{2} + \frac{1}{2} \cosh(4xd/\Omega) \right. \\ &\quad \left. - \sqrt{2} \cosh(2xd/\Omega) \cos(2xd/a) \right. \\ &\quad \left. + \frac{1}{2} \cos(4xd/a) \right) \\ |\psi_{\rightarrow}(x, t)|^2 &= \frac{|C_t|^2}{3} e^{-2x^2/\Omega} (1 - \cos(4xd/a)). \end{aligned} \quad (17)$$

Clearly, $|\psi_{\uparrow}(x, t)|^2$ in the above expression does represent a 3-slit interference. Thus, we see that by detecting the particles in coincidence with the pseudospin state $|\uparrow\rangle$ erases the which-path information and brings back the interference. Quantum erasing is possible in 3-slit interference as well. A careful look reveals that the interference given by $|\psi_{\uparrow}(x, t)|^2$ is different from the true 3-slit interference given by (10), as can also be seen by plotting the two (see figure 2).

The question that arises now is, in what basis should one measure the pseudospin, so that detecting particles on the screen, in coincidence with one of the states of the basis, would yield true 3-slit interference? Let us assume that the basis states are $|\alpha\rangle$, $|\beta\rangle$, $|\gamma\rangle$, and that coincident detection of particles with $|\alpha\rangle$ yields true ‘unshifted’ 3-slit interference. This can happen only if

$$\langle \alpha | + \rangle = \langle \alpha | 0 \rangle = \langle \alpha | - \rangle \equiv c. \quad (18)$$

By normalization, $|c| = 1/\sqrt{3}$, and the state $|\alpha\rangle$ can be

$$|\alpha\rangle = \frac{1}{\sqrt{3}} (|+\rangle + |0\rangle + |-\rangle). \quad (19)$$

That normalized states $|\beta\rangle$, $|\gamma\rangle$ should be chosen such that they are unbiased for the states $|+\rangle$, $|0\rangle$, $|-\rangle$, thus satisfying the condition

$$\begin{aligned} |\langle \beta | + \rangle|^2 &= |\langle \beta | 0 \rangle|^2 = |\langle \beta | - \rangle|^2 = \frac{1}{3} \\ |\langle \gamma | + \rangle|^2 &= |\langle \gamma | 0 \rangle|^2 = |\langle \gamma | - \rangle|^2 = \frac{1}{3}. \end{aligned} \quad (20)$$

Cube roots of unity come in handy here, and the two states can be chosen as

$$\begin{aligned} |\beta\rangle &= \frac{1}{\sqrt{3}} (e^{i\pi/3} |+\rangle - |0\rangle + e^{-i\pi/3} |-\rangle) \\ |\gamma\rangle &= \frac{1}{\sqrt{3}} (e^{-i\pi/3} |+\rangle - |0\rangle + e^{i\pi/3} |-\rangle). \end{aligned} \quad (21)$$

Using this, the state (3) can be written as

$$\begin{aligned} |\psi\rangle &= \frac{1}{3} [(|\psi_1\rangle + |\psi_2\rangle + |\psi_3\rangle) |\alpha\rangle \\ &\quad + (e^{i\pi/3} |\psi_1\rangle - |\psi_2\rangle + e^{-i\pi/3} |\psi_3\rangle) |\beta\rangle \\ &\quad + (e^{-i\pi/3} |\psi_1\rangle - |\psi_2\rangle + e^{i\pi/3} |\psi_3\rangle) |\gamma\rangle]. \end{aligned} \quad (22)$$

Particles hitting the screen can be divided into three subensembles, depending on the state of the pseudospin obtained. The states of the particle, correlated with $|\alpha\rangle$, $|\beta\rangle$, $|\gamma\rangle$, are $\psi_{\alpha}(x, t)$, $\psi_{\beta}(x, t)$, $\psi_{\gamma}(x, t)$, respectively. They can be represented as follows:

$$\begin{aligned} \psi_{\alpha}(x, t) &= \frac{1}{3} (\psi_1(x, t) + \psi_2(x, t) + \psi_3(x, t)) \\ \psi_{\beta}(x, t) &= \frac{1}{\sqrt{3}} \left(e^{i\pi/3} \psi_1(x, t) - \psi_2(x, t) \right. \\ &\quad \left. + e^{-i\pi/3} \psi_3(x, t) \right) \\ \psi_{\gamma}(x, t) &= \frac{1}{\sqrt{3}} \left(e^{-i\pi/3} \psi_1(x, t) - \psi_2(x, t) \right. \\ &\quad \left. + e^{i\pi/3} \psi_3(x, t) \right). \end{aligned} \quad (23)$$

The three complementary interference patterns can then be shown to have the forms (figure 3)

$$\begin{aligned} |\psi_{\alpha}(x, t)|^2 &= \frac{|C_t|^2}{3} e^{-2x^2/\Omega} \frac{1}{3} [1 + 2 \cosh(4xd/\Omega) \\ &\quad + 4 \cosh(2xd/\Omega) \cos(2xd/a) \\ &\quad + 2 \cos(4xd/a)] \\ |\psi_{\beta}(x, t)|^2 &= \frac{|C_t|^2}{3} e^{-2x^2/\Omega} \frac{1}{3} [1 + 2 \cosh(4xd/\Omega) \\ &\quad + 4 \cosh(2xd/\Omega) \cos(2xd/a + \pi/3) \\ &\quad + 2 \cos(4xd/a + 2\pi/3)] \\ |\psi_{\gamma}(x, t)|^2 &= \frac{|C_t|^2}{3} e^{-2x^2/\Omega} \frac{1}{3} [1 + 2 \cosh(4xd/\Omega) \\ &\quad + 4 \cosh(2xd/\Omega) \cos(2xd/a - \pi/3) \\ &\quad + 2 \cos(4xd/a - 2\pi/3)]. \end{aligned} \quad (24)$$

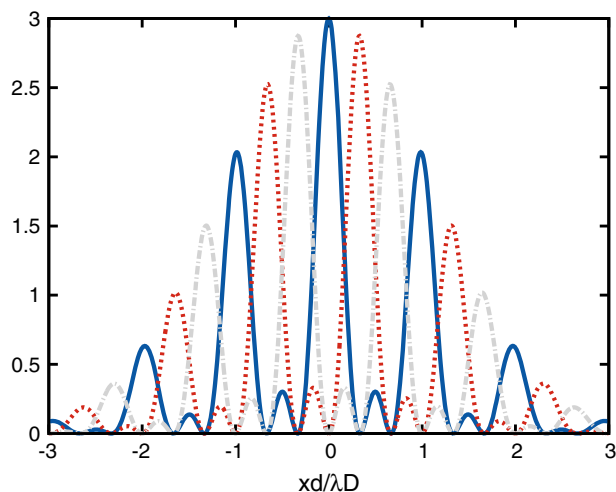


Figure 3. Recovered interference pattern (unnormalized), given by $|\psi_\alpha(x, t)|^2$ (solid line), given by $|\psi_\beta(x, t)|^2$ (dashed line) and given by $|\psi_\gamma(x, t)|^2$ (dot-dashed line).

Notice that all the three cases in (24) produce true, but mutually shifted, 3-slit interference patterns. It implies that detecting particles in coincidence with the state $|\alpha\rangle$, $|\beta\rangle$ or $|\gamma\rangle$ constitutes quantum eraser for the 3-slit interference experiment. The interference, which was lost because of the path-detector states carrying which-way information, is recovered after the path information is erased by reading the path detector in one of the states $|\alpha\rangle$, $|\beta\rangle$, $|\gamma\rangle$.

4. Discussion

In the preceding analysis, we have shown that the concept of quantum eraser can be extended to the case of 3-slit interference. If the which-path information of the interfering particle is stored in a quantum path detector the interference is lost. By erasing the path information stored in the quantum path detector, the lost interference can be made to come back.

In a typical implementation of quantum eraser for a 2-slit interference experiment, the particles are detected in coincidence with two orthogonal states of the path detector. For example, in a particular implementation with photons passing through a double slit, they were detected in coincidence with two orthogonal polarization states of the photon [21]. Both these states lead to the recovery of true 2-slit interference. We have shown that in the case of 3-slit interference too, three complementary interference patterns can be recovered.

A quantum eraser for 3-slit interference will definitely be harder to implement, than 2-slit interference. However, we feel that an earlier proposal for quantum eraser for 2-slit interference using a modified Stern-Gerlach

set-up [37], may be a good candidate for extending to 3-slit interference.

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