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Dynamical regimes and intracavity propagation delay in external cavity semiconductor diode lasers

E JAYAPRASATH and S SIVAPRAKASAM*

Department of Physics, Pondicherry University, Pondicherry 605 014, India
*Corresponding author. E-mail: siva@iitk.ac.in

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Abstract. Intracavity propagation delay, a delay introduced by a semiconductor diode laser, is found to significantly influence synchronization of multiple semiconductor diode lasers, operated either in stable or in chaotic regime. Two diode lasers coupled in unidirectional scheme is considered in this numerical study. A diode laser subjected to an optical feedback, also called an external cavity diode laser, acts as the transmitter laser (TL). A solitary diode laser acts as the receiver laser (RL). The optical output of the TL is coupled to the RL and laser operating parameters are optimized to achieve synchronization in their output intensities. The time-of-flight between the TL and RL introduces an intercavity time delay in the dynamics of RL. In addition to this, an intracavity propagation delay arises as the TL's field propagated within the RL. This intracavity propagation delay is evaluated by cross-correlation analysis between the output intensities of the lasers. The intracavity propagation delay is found to increase as the external cavity feedback rate of TL is increased, while an increment in the injection rate between the two lasers resulted in a reduction of intracavity propagation delay.

Keywords. Semiconductor lasers; chaos synchronization; intracavity propagation delay.

PACS Nos **42.55.–f; 42.55.Px; 42.65.Sf**

1. Introduction

Chaos synchronization of semiconductor diode lasers has been a topic of interest for many years due to its direct application in secure optical communication [1–21]. Secure optical communication essentially involves encoding of a message (signal) in the chaotic output of a transmitter laser (TL) which is to be recovered at a receiver laser (RL) [3,6–9]. The encoded message can be decoded at the receiver by synchronizing chaotic output of the receiver with that of the transmitter. There have been extensive studies on synchronization of unidirectionally coupled chaotic semiconductor lasers [10–18]. Two major types of synchronization regimes had been identified: (i) complete/perfect (RL will synchronize with TL with a delay equal to the difference between time-of-flight (τ_f) between the transmitter and the receiver laser and external cavity round-trip time (τ_{ext})) [10–14] and (ii) generalized/injection-locked synchronization (RL will synchronize with TL with a delay equal to the time-of-flight (τ_f)) [15–18]. The cross-correlation analysis between the transmitter and the receiver, and obtaining maximum cross-correlation coefficient, allows the estimation of this time delay. The maximum correlation peak for complete and injection-locked synchronization should occur with a time delay equal to $\tau_f - \tau_{ext}$ and τ_f respectively for complete and generalized synchronization. Hence, this time delay should be corrected in the process of analysing synchronization. However, in the case of injection-locked synchronization, due to the intracavity propagation of the TL output within the cavity of the RL, the maximum correlation does not occur at τ_f or zero (if $\tau_f = 0$), but at an incrementally higher delay than zero [15] which is noted as intracavity propagation delay (τ_{PD}). The TL acquires this delay, as it travels within the cavity of the receiver [18].

In order to implement secure optical communication, it is essential to operate the lasers in chaotic regime. Such chaotic nature could inherently influence the quantum of intracavity propagation delay. Thus, it becomes essential to have a method of control and the same needs an investigation and is the focus of this paper. We have found that, the value of intracavity propagation delay depends on (i) external cavity feedback rate (τ_{ext}) of the TL and (ii) injection rate between the TL and the RL.

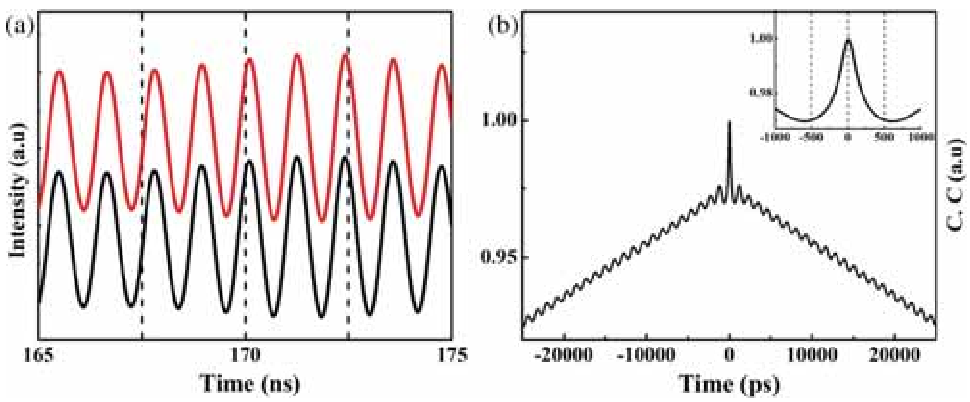

Figure 1. (**a**) Illustration of periodic oscillation ($\kappa = 0.7\ \text{ns}^{-1}$) time series of the TL and the RL. The lower (upper) traces correspond to TL (RL) output respectively. (**b**) The corresponding cross-correlation (CC) coefficient between the TL and the RL. The inset figure shows the expanded version near zero in time-scale.

2. Theoretical model

The system under investigation consists of two semiconductor diode lasers acting as the transmitter and the receiver. Transmitter is subjected to optical feedback from an external cavity mirror and its optical output is unidirectionally coupled to the receiver laser, which is a solitary diode laser. The numerical studies of the system under consideration are carried out by solving the rate equations for the complex fields $E(t)$ and carrier densities $N(t)$ of the lasers. These are Lang–Kobayashi equations [2] adapted for the considered system and are:

$$\frac{\mathrm{d}E_{\mathrm{T}}(t)}{\mathrm{d}t} = \frac{1}{2}(1+i\alpha)\left[G_{\mathrm{T}}(t) - \frac{1}{\tau_{\mathrm{p}}}\right]E_{\mathrm{T}}(t) + \kappa E_{\mathrm{T}}(t-\tau_{\mathrm{ext}})\exp(-i\omega_{\mathrm{T}}\tau_{\mathrm{ext}}) + \sqrt{2\beta_{\mathrm{T}}N_{\mathrm{T}}(t)}\xi_{\mathrm{T}}(t), \tag{1}$$

$$\frac{\mathrm{d}E_{\mathrm{R}}(t)}{\mathrm{d}t} = \frac{1}{2}(1+i\alpha)\left[G_{\mathrm{R}}(t) - \frac{1}{\tau_{\mathrm{p}}}\right]E_{\mathrm{R}}(t) + \eta E_{\mathrm{T}}(t-\tau_{\mathrm{f}})\exp[-i(\omega_{\mathrm{T}}\tau_{\mathrm{f}} + \Delta\omega t)] + \sqrt{2\beta_{\mathrm{R}}N_{\mathrm{R}}(t)}\xi_{\mathrm{R}}(t), \tag{2}$$

$$\frac{\mathrm{d}N_{\mathrm{T,R}}(t)}{\mathrm{d}t} = \frac{J_{\mathrm{T,R}}}{e} - \frac{N_{\mathrm{T,R}}(t)}{\tau_n} - G_{\mathrm{T,R}}(t)\left|E_{\mathrm{T,R}}(t)\right|^2. \tag{3}$$

In these equations, the indices T and R indicate the transmitter and receiver lasers respectively. Here, J is the injection current, $J_{\mathrm{T}} = 1.10 I_{\mathrm{th}}$ and $J_{\mathrm{R}} = 1.08 I_{\mathrm{th}}$, $e = 1.602 \times 10^{-19}$ C is the electron charge. $G(t) = g_0[N(t) - N_{\mathrm{th}}]/(1+\varepsilon|E(t)|^2)$ is the optical gain, where carrier number at the threshold $N_{\mathrm{th}} = 1.5 \times 10^8$, linear gain coefficient $g_0 = 12 \times 10^{-8}\ \text{ps}^{-1}$, gain saturation coefficient $\varepsilon = 5 \times 10^{-7}$, linewidth enhancement factor $\alpha = 3.8$, the external cavity round-trip time $\tau_{\mathrm{ext}} = 10$ ns, carrier lifetime $\tau_n = 2$ ns, photon lifetime $\tau_{\mathrm{p}} = 2$ ps. The lasers are operated at wavelength $\lambda = 830$ nm, and hence detuning between the laser is zero ($\Delta\omega = 0$), $\tau_{\mathrm{in}} = 8$ ps is the internal cavity round-trip time. $r_2 = 0.58$ is the facet reflectivity of the laser. $\beta = 10^{-5}\ \text{ns}^{-1}$ is the spontaneous emission rate. In the simulations, except the bias levels, the internal laser parameters are assumed to be the same for both TL and RL. The Gaussian noise sources $\xi_{\mathrm{T,R}}$, are with zero mean and unity variance [9]. The time-of-flight between the TL and RL is kept zero throughout the investigation. κ is the feedback rate of the transmitter laser and η is the injection rate between the TL and the RL.

3. Dynamics of the laser and intracavity propagation delay

Our focus of this paper is to study the character of intracavity propagation delay while synchronizing the TL and RL operated in different dynamical scenario. In order to obtain various dynamics, we appropriately chose to adjust the feedback rate (κ) of the TL, and as κ increases, the corresponding dynamics vary from periodic to multiperiodic and chaotic dynamical output at the TL.

Initially, we have kept the feedback rate of the TL at $0.7\ \text{ns}^{-1}$ rendering the TL output of undamped relaxation oscillation. The injection rate (η) between the TL and the RL is kept at $15\ \text{ns}^{-1}$. The chaotic output of the TL is unidirectionally injected to the RL and the time traces are shown in figure 1a, in which upper (lower) traces correspond to RL (TL) respectively. The time trace of RL is shifted vertically for clarity. The intracavity propagation delay (τ_{PD}) is estimated by performing cross-correlation (CC) analysis between the outputs of the transmitter and the receiver, and identifying the maximum correlation coefficient peak position in the CC

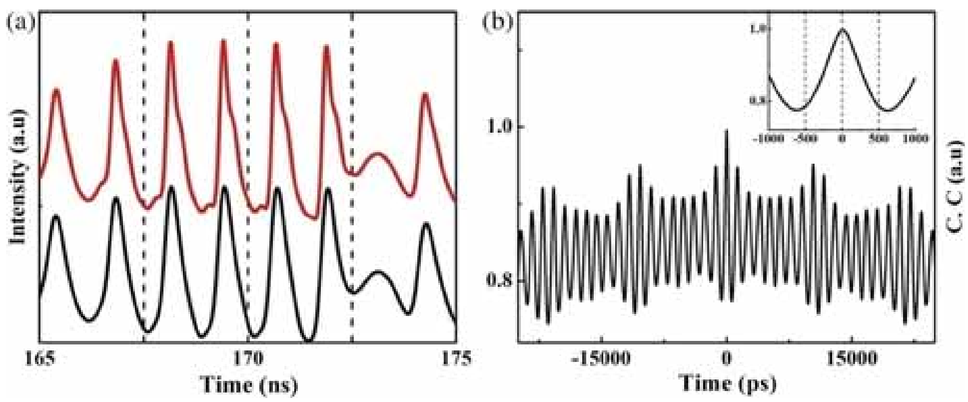

Figure 2. (**a**) Multiperiodic oscillatory ($\kappa = 1.5$ ns^{-1}) time series of the TL and the RL and (**b**) the corresponding cross–correlation (CC) coefficient between the TL and the RL. The inset figure shows the expanded version near zero in time-scale.

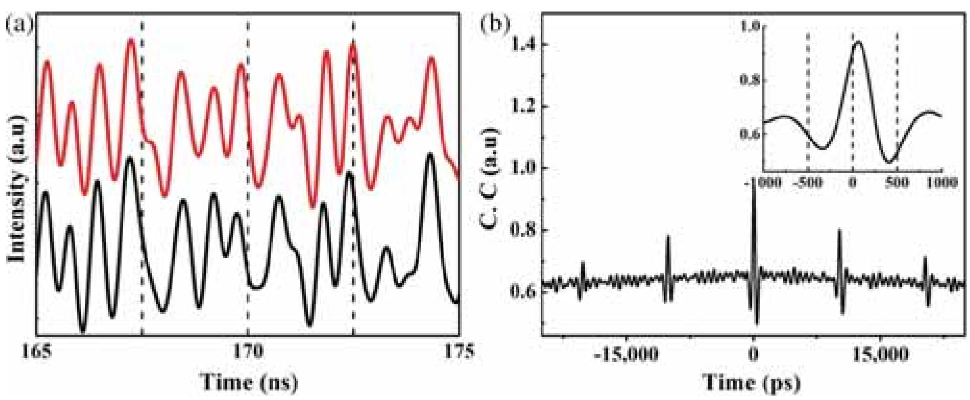

Figure 3. (**a**) Time series of the transmitter–receiver lasers for TL feedback rate $\kappa = 9$ ns^{-1}: The lower (upper) traces correspond to TL (RL) output respectively. (**b**) Cross-correlation coefficient between the TL and the RL. The inset shows the expanded version near zero in time-scale.

plot. The CC plot is shown in figure 1b and the maximum correlation peak occurs at 32.5 ps. It is evident from this plot that, though the time-of-flight between the TL and the RL is zero, there exists a small delay between the TL and the RL, i.e., the receiver lags behind the transmitter laser for an amount of 32.5 ps, which is introduced by the laser cavity of the RL and is called intracavity propagation delay (τ_{PD}).

Next the feedback rate (κ) of the TL is increased to 1.5 ns^{-1}. The transmitter exhibits multiperiodicity output. The output of the TL is unidirectionally injected to the RL and the corresponding time traces and associated CC plots are shown in figure 2 for injection rate $\sigma = 15$ ns^{-1}. The time traces of the TL (upper traces) and the RL (lower traces) are shown in figure 2a (the waveforms of the RL are vertically shifted for clarity) and the corresponding CC plots are shown in figure 2b. It is evident from the time traces that the lasers are synchronized. From figure 2b, the maximum correlation coefficient is found to occur at 43.5 ps which is the τ_{PD}. and is relatively larger than that of the value of undamped relaxation oscillation (figure 1b).

The details that have been discussed in figures 1 and 2 are the existence and character of intracavity propagation delay (τ_{PD}) in a non-chaotic regime of operation. Now, as we further increase the feedback rate of the TL to 9 ns^{-1}, the transmitter laser output exhibits chaotic dynamics. The injection rate (η) between the TL and the RL is kept at 15 ns^{-1} and the time traces are shown in figure 3a. The upper (lower) traces correspond to the RL (TL) respectively. The time trace of the RL is shifted vertically for clarity. From the performance of CC analysis between the TL and the RL, the intracavity propagation delay (τ_{PD}) is found to be 62.5 ps. The CC plot is shown in figure 3b. This conveys that, if the dynamics of the system varies from regular behaviour (periodic) to chaotic behaviour, the corresponding intracavity propagation delay increases.

To evaluate the influence of intracavity propagation delay on synchronization, the synchronization plot between the transmitter and the receiver is plotted prior and after the correction of τ_{PD} in the RL output and is shown in figure 4. For the quantification of the synchronization quality, we have fitted the synchronization

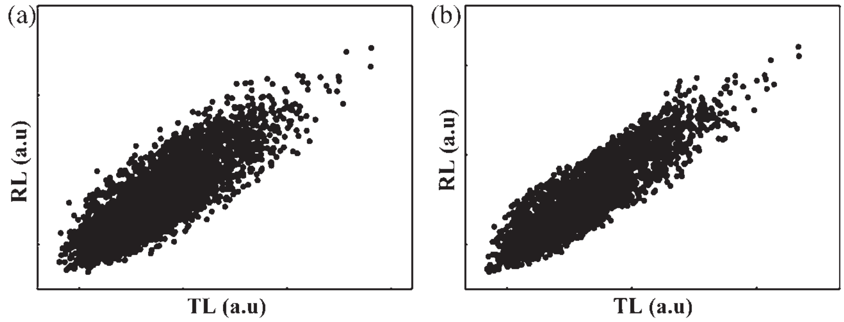

Figure 4. (**a**) Synchronization plot between the TL and the RL. (**b**) Synchronization plot between the TL and the RL after the intracavity propagation delay correction in RL.

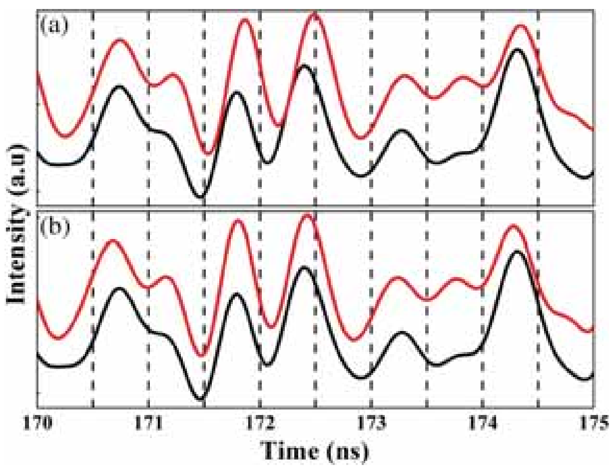

Figure 5. Time series output for the TL feedback rate $\kappa = 9\ \mathrm{ns}^{-1}$ and injection rate between the TL and the RL $\eta = 15\ \mathrm{ns}^{-1}$. (**a**) TL (lower trace), RL (upper trace), without the intracavity propagation delay correction. (**b**) With the $\tau_{\mathrm{PD}}(= 62.5\ \mathrm{ps})$ correction.

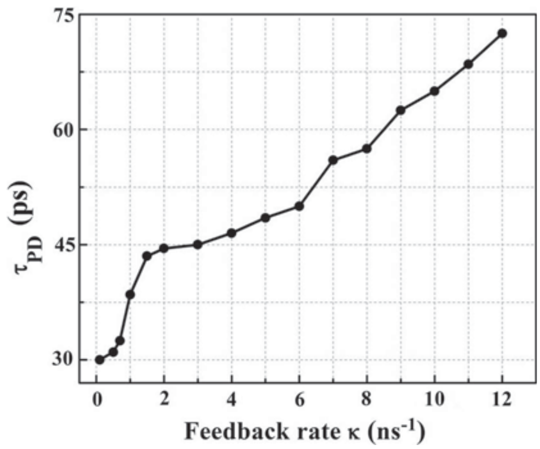

Figure 6. Dependence of intracavity propagation delay on feedback rate.

plots to a straight line and the deviations (Δm) of slope (m) [14] are evaluated. The deviation values are 11.10×10^{-4} and 8.89×10^{-4} corresponding to the correction without (figure 4a) and with (figure 4b) τ_{PD}. Thus, it is evident that the deviation does decrease if the intracavity propagation delay is taken into account.

In order to visualize the intracavity propagation delay in more detail, the time traces of figure 3a is further expanded and is shown in figure 5. The original time evolution is shown in figure 5a, whereas, in figure 5b the necessary intracavity propagation delay correction has been incorporated in receiver laser output. As is evident from figure 5a, there is an inherent time delay between the TL and the RL outputs. These observations reveal that τ_{PD} is an important factor to be taken into account in achieving synchronization between the chaotic diode lasers.

We repeated this study for multiple values of feedback rate (κ) ranging from $0.1\ \mathrm{ns}^{-1}$ to $12\ \mathrm{ns}^{-1}$ and obtained different kinds of dynamical output at TL and the output is injected to RL. For each value of feedback rate, the corresponding intracavity propagation delay (τ_{PD}) has been calculated by performing CC analysis between the TL and the RL, and the result is shown in figure 6. From the figure it is observed that, as the feedback rate increases, the value of τ_{PD} also increases and varies from 30 ps to 72.5 ps for the system considered. Hence, it can be emphasized that as the complexity of the laser dynamics increases the respective intracavity propagation delay also increases. Also, this intracavity propagation delay value (τ_{PD}) will vary with operating parameters of the laser, and so τ_{PD} should be calculated at all circumstances of the system that are being considered.

4. Injection rate dependence of intracavity propagation delay

The effects of injection strength on intracavity propagation delay for different dynamical regimes have been studied in this section. Figure 7 summarizes the effect of injection rate on τ_{PD} in these three regimes of operation. Irrespective of the regime of operation, the τ_{PD} value decreases as the injection strength increases. It can be

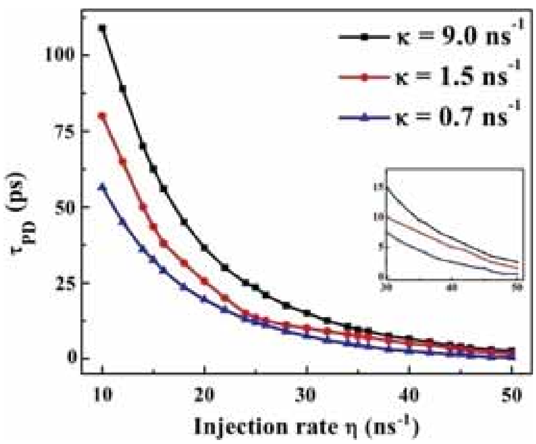

Figure 7. Dependence of intracavity propagation delay for three regimes on injection rate. Bottom trace (triangles) corresponds to periodic regime ($\kappa = 0.7\ \text{ns}^{-1}$), middle trace (circles) corresponds to multiperiodic regime ($\kappa = 1.5\ \text{ns}^{-1}$) and top trace (squares) corresponds to chaotic regime ($\kappa = 9\ \text{ns}^{-1}$). The inset shows the expanded version near the higher value of injection rate.

argued that, at stronger injection rate the RL field is able to couple relatively sooner than the TL field, resulting in a lesser contribution by a laser (RL) to intracavity propagation delay. In figure 7, the bottom trace (triangles) shows the dependency of τ_{PD} for the periodic regime of operation. As the injection rate is increased, the τ_{PD} value is found to decrease to 0.5 ps from 57 ps. The middle trace (circles) corresponds to τ_{PD} behaviour for the multiperiodic regime. It is found that the value of τ_{PD} decreases from 80 ps to 1.5 ps. The top trace (squares) corresponds to the chaotic regime, wherein the value of τ_{PD} is found to reduce to 2.5 ps from 109 ps. It is thus emphatic to state that as the laser becomes chaotic in its operation a correction of the intracavity propagation delay becomes significant in achieving synchronization between them and thus implementing secure optical communication.

5. Conclusions

In summary, we have numerically investigated the dependence of intracavity propagation delay on the feedback rate and injection rate, while achieving synchronization in a unidirectionally coupled semiconductor lasers. It is found that, the intracavity propagation delay is relatively larger as the dynamics varies from regular to chaotic behaviour implying that intracavity propagation delay is a parameter that depends on the dynamics of the lasers. Thus, we conclude and state that the complexity of the system dynamics influences the intracavity propagation delay. The influence of injection rate on intracavity propagation delay shows that at larger injection rates, τ_{PD} is bound to decrease. Injection rate is thus a controlling parameter on the intracavity propagation delay irrespective of the dynamical regime of operation. In secure optical communication, it is essential to operate the laser in chaotic domain and thus the value of intracavity propagation delay (τ_{PD}) is bound to play a significant role. To conclude, we have shown that the intracavity propagation delay is a parameter of importance in chaotic semiconductor lasers and needs to be characterized and corrected while being utilized in implementing secure optical communication. An experimental verification of these results, though beyond the scope at this stage, would be attempted to compliment and further this research in the near future.

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