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Symmetry energy of the nucleus in the relativistic Thomas–Fermi approach with density-dependent parameters

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Abstract. The symmetry energy of a nucleus is determined in a local density approximation and integrating over the entire density distribution of the nucleus, calculated utilizing the relativistic density-dependent Thomas-Fermi approach. The symmetry energy is found to decrease with increasing neutron excess in the nucleus. The isovector coupling channel reduces the symmetry energy, and this effect increases with increased neutron excess. The isovector coupling channel increases the symmetry energy integral in ^{40}Ca and reduces it in ^{48}Ca, and the interplay between the isovector and the isoscalar channels of the nuclear force explains this isotope effect.

Keywords. Symmetry energy of a nucleus; isovector coupling channel; relativistic Thomas–Fermi approximation; lead isotopes; isotones $N = 126$; ^{40}Ca; ^{48}Ca.

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1. Introduction

The interest for the isospin dependence of the nuclear force is growing rapidly in both nuclear physics and nuclear astrophysics, due to a new generation of radioactive beam facilities, such as the rare isotope accelerator in the United States of America, the SPIRAL2 at GANIL/France, and the GSI facility FAIR in Germany, which produce new data for neutron-rich nuclei. A key point is the interplay between the isoscalar and the isovector channels of the nuclear interaction as a function of the isospin asymmetry [1].

The nuclear symmetry energy plays an important role in a variety of nuclear phenomena. It is an ingredient determining the nuclear equation of state and the proton fraction inside a neutron star [2], the neutron skin thickness in heavy nuclei [3], and in analysing heavy-ion reactions [4]. The lack of experimental data pushes toward the use of reliable microscopic theoretical tools, based on nucleon–nucleon forces fitting nucleon–nucleon scattering data and deuteron properties, like the relativistic Brueckner–Hartree–Fock (RBHF) theory, one of the most reliable and feasible microscopic methods describing effective interactions in the nuclear medium. Theoretical and practical obstacles do not allow the direct application of the RBHF approach for calculating finite nuclei. The Brueckner scheme is an intermediate density approximation, losing its physical significance at low densities where the Brueckner-independent pair assumption becomes questionable [5].

The symmetry energy of a nucleus is determined in this work in a local density approximation (LDA), where the value of the nuclear matter symmetry energy is determined at the local density value inside the nucleus. The symmetry energy integral and the symmetry energy of the nucleus are then determined by integration over the entire density distribution of the nucleus, calculated by utilizing the relativistic density-dependent Thomas–Fermi (RDTF) approach, with density-dependent coupling parameters reproducing the nucleon self-energy resulting from RBHF theory [6].

This work studies the dependency of the symmetry energy of a nucleus on the number of neutrons and protons inside the nucleus. Based on the values determined for spherically symmetric even–even lead isotopes ($Z = 82$) and $N = 126$ isotones with known experimental masses, the density distributions of their nuclei have been determined in ref. [7]. In addition to the values determined for the calcium isotopes ^{40}Ca and ^{48}Ca, the density distributions of their nuclei have been determined in ref. [8].

The effect of isovector coupling channel of nuclear interaction on the dependency of the symmetry energy on the number of neutrons and protons in the nucleus is also analysed.

The method used for the determination of the symmetry energy integral and the symmetry energy of a nucleus is explained in §2. Section 3 reviews briefly the effective density-dependent interaction of the RDTF approach. Section 4 studies the dependency of the symmetry energy of a nucleus on the number of neutrons, and the effect of the isovector coupling channel on this dependency, in the case of spherically symmetric even–even lead isotopes ($Z = 82$) with known experimental masses, and §5 studies the dependency on the number of protons in the case of spherically symmetric even–even $N = 126$ isotones with known experimental masses. Section 6 considers the effect of isovector coupling channel on the symmetry energy integral of ^{40}Ca and ^{48}Ca. Section 7 summarizes the main conclusions.

2. Symmetry energy of the nucleus

The nuclear equation of state (EoS) gives the nucleon energy e as a function of the density ρ and the asymmetry parameter β:

$$e = e(\rho, \beta), \tag{1}$$

where the density ρ is the sum of the neutron and proton densities, ρ_n and ρ_p:

$$\rho = \rho_n + \rho_p, \tag{2}$$

and the asymmetry parameter β is defined by

$$\beta = \frac{\rho_n - \rho_p}{\rho}. \tag{3}$$

The nuclear symmetry energy $e_{sym}(\rho)$ is the quantity characterizing the isospin dependence of the EoS, and is obtained by expanding the EoS in terms of the asymmetry parameter β [9]:

$$e(\rho, \beta) = e(\rho, 0) + e_{sym}(\rho)\beta^2 + O(\beta^4), \tag{4}$$

where $O(\beta^4)$ stands for fourth and higher orders terms of β. Therefore, the nuclear symmetry energy is given by

$$e_{sym}(\rho) = \frac{1}{2}\left.\frac{\partial^2 e(\rho, \beta)}{\partial \beta^2}\right|_{\beta=0}. \tag{5}$$

The atomic nucleus consists of N number of neutrons and Z number of protons, and $A = N + Z$ is called the mass number. The asymmetry parameter B of the nucleus is defined as

$$B = \frac{N - Z}{A}. \tag{6}$$

The symmetry energy of a nucleus is calculated by applying a local density approximation (LDA), where the nuclear matter symmetry energy $e_{sym}(\rho)$ is determined at the local density value in the nucleus $\rho(r)$ at distance r from the centre of the nucleus. The symmetry energy $C_{sym}(A, B)$ of the nucleus is then determined by integration over the entire density distribution function of the nucleus [10]:

$$C_{sym}(A, B)B^2 = \frac{1}{A}\int d^3r\ \rho(r) e_{sym}(\rho(r))(\beta(r))^2, \tag{7}$$

where $\beta(r)$ is the local asymmetry parameter inside the nucleus:

$$\beta(r) = \frac{\rho_n(r) - \rho_p(r)}{\rho(r)}. \tag{8}$$

Equation (7) shows that the symmetry energy of the nucleus $C_{sym}(A, B)$ is undefined for symmetric nuclei, i.e., nuclei with the number of neutrons N equals the number of protons Z, since $B = 0$ for these nuclei. Therefore, the symmetry energy integral (SEI) is used when studying a symmetric nucleus, and comparing it with an asymmetric nucleus, with SEI describing the right-hand side of eq (7):

$$\text{SEI} = \frac{1}{A}\int d^3r\ \rho(r)\ e_{sym}(\rho(r))\ (\beta(r))^2, \tag{9}$$

as, for example, when studying ^{40}Ca and ^{48}Ca.

3. Effective density-dependent interaction

The effective nucleon–nucleon interaction is described in the relativistic density-dependent Thomas–Fermi approach (RDTF) by the electromagnetic field between protons and the exchange of four mesons: the isoscalar scalar meson σ, the isoscalar vector meson ω, the isovector scalar meson δ, and the isovector vector meson ρ. Density-dependent coupling parameters for the isoscalar mesons are introduced by

$$\frac{g_i(\rho)}{g_i(\rho_0)} - 1 = a_i\left(\exp\left[b_i\left(1 - \left(\frac{\rho}{\rho_0}\right)^{1/3}\right)\right] - 1\right), \quad i = \sigma, \omega, \tag{10}$$

where ρ_0 is the nuclear matter saturation density and a_i, b_i, and $g_i(\rho_0)$ are the coefficients of the density-dependent function $g_i(\rho)$. Density-dependent coupling parameters for the isovector mesons are introduced by

$$g_i(\rho) = g_i(\rho_0)\exp\left[b_i\left(1 - \frac{\rho}{\rho_0}\right)\right], \quad i = \delta, \rho, \tag{11}$$

where b_i and $g_i(\rho_0)$ are the coefficients of the density-dependent function $g_i(\rho)$. The coefficients a_i, b_i, and $g_i(\rho_0)$ ($i = \sigma, \omega$) and b_i and $g_i(\rho_0)$ ($i = \delta, \rho$) are adjusted to the outcome of the RBHF calculations of the nucleon self-energy in nuclear matter of ref. [11] according to

$$\Sigma_{\rm sp}(\rho) + \Sigma_{\rm sn}(\rho) = -2\,\frac{g_\sigma^2(\rho)}{m_\sigma^2}(\rho_{\rm sp} + \rho_{\rm sn}), \tag{12}$$

$$\Sigma_{\rm sp}(\rho) - \Sigma_{\rm sn}(\rho) = -2\,\frac{g_\delta^2(\rho)}{m_\delta^2}(\rho_{\rm sp} - \rho_{\rm sn}), \tag{13}$$

$$\Sigma_{\rm 0p}(\rho) + \Sigma_{\rm 0n}(\rho) = 2\,\frac{g_\omega^2(\rho)}{m_\omega^2}(\rho_{\rm p} + \rho_{\rm n}), \tag{14}$$

$$\Sigma_{\rm 0p}(\rho) - \Sigma_{\rm 0n}(\rho) = 2\,\frac{g_\rho^2(\rho)}{m_\rho^2}(\rho_{\rm p} - \rho_{\rm n}), \tag{15}$$

where $\Sigma_{\rm sp}(\rho)$ denotes the RBHF result for the scalar component of the proton self-energy at density ρ, $\Sigma_{\rm sn}(\rho)$ is the RBHF result for the scalar component of the neutron self-energy at density ρ, $\Sigma_{\rm 0p}(\rho)$ is the RBHF result for the vector component of the proton self-energy at density ρ, and $\Sigma_{\rm 0n}(\rho)$ is the RBHF result for the vector component of the neutron self-energy at density ρ. m_σ, m_ω, m_δ, and m_ρ are the masses of the σ, ω, δ, and ρ mesons. $\rho_{\rm sp}$ and $\rho_{\rm sn}$ are the scalar proton and neutron densities determined by

$$\rho_{\rm sp,sn}=\frac{1}{2\pi^2}\left(M_{\rm p,n}p_{\rm p,n}\varepsilon_{\rm p,n} - M_{\rm p,n}^3\ \ln\frac{p_{\rm p,n}+\varepsilon_{\rm p,n}}{M_{\rm p,n}}\right), \tag{16}$$

where

$$M_{\rm p,n} = m_{\rm N} + \Sigma_{\rm sp,sn}, \tag{17}$$

$$p_{\rm p,n}^3 = 3\pi^2\rho_{\rm p,n}, \tag{18}$$

$$\varepsilon_{\rm p,n}^2 = p_{\rm p,n}^2 + M_{\rm p,n}^2. \tag{19}$$

The coefficients of the resulting density-dependent parametrization of the RBHF potential Bonn A [5] are given in table 1. The masses $m_{\rm N}$, m_σ, m_ω, m_δ, and m_ρ and the saturation density ρ_0 are those of the Bonn A potential. See ref. [6], for instance, for a detailed description of the RDTF approach.

The isoscalar coupling channel of the nuclear interaction is represented by the isoscalar scalar meson σ and the isoscalar vector meson ω, while the isovector coupling channel is represented by the isovector scalar meson δ and the isovector vector meson ρ. In order to analyse the role played by the isovector coupling channel, the results obtained using the parameterization of table 1 with all mesons ($\sigma\omega\delta\rho$) included are compared with the results when only isoscalar mesons ($\sigma\omega$) are included.

Table 1. The density-dependent parameter set. m_i is the mass of the ith meson. a_i, b_i, and $g_i(\rho_0)$ are the coefficients of the parametrization of the density-dependent coupling parameters ($i = \sigma, \omega, \delta, \rho$). $m_{\rm N} = 938.926$ MeV is the average nucleon mass used in ref. [5], and $\rho_0 = 0.185$ fm^{-3} is the saturation density resulting from the RBHF potential Bonn A [5].

Meson i	σ	ω	δ	ρ
m_i (MeV)	550	782.6	983	769
g_i ($\rho0$)	9.297	11.269	4.701	2.370
a_i	0.2941	0.3451		
b_i	2.217	2.113	1.223	1.634

4. Symmetry energy in lead isotopes $Z = 82$

Figure 1 displays the results for the symmetry energy in spherically symmetric even–even lead isotopes with known experimental masses for both cases of ($\sigma\omega\delta\rho$) and ($\sigma\omega$). The symmetry energy $C_{\rm sym}$ of the ^{208}Pb nucleus has the value of 17.22 MeV in the case ($\sigma\omega\delta\rho$) and $C_{\rm sym} = 24.63$ MeV in the case ($\sigma\omega$). These values agree with the values given in ref. [10], where $C_{\rm sym} \approx 25$ MeV for the ^{150}Sm nucleus and $C_{\rm sym} \approx 20$ MeV for the ^{40}S nucleus. Samaddar *et al* [10] used Thomas–Fermi approach with the non-relativistic modified Seyler–Blanchard (SBM) effective interaction.

Isoscaling studies of the reactions of a ^{64}Ni beam (25 MeV/nucleon) with a ^{208}Pb target, performed with the superconducting solenoid line (BigSol line) at the Cyclotron Institute of the Texas A&M University, with the use of the Fermi gas model for temperature estimation, give the value $C_{\rm sym} = 18.9 \pm 3.2$ MeV [12].

Two remarks concerning the validity and the limitations of the comparison between experimental and calculated values should be added. First, isoscaling approach attempts to isolate the effects of nuclear symmetry energy in the fragment yields, thus allowing a direct study of the role of this term of the nuclear binding energy in the formation of hot fragments, but primary fragments can only approximately have the same temperature, and average fitted values are used. And second, a value of 2.9 MeV for the average excitation energy per nucleon of the primary quasiprojectiles was extracted in ref. [12], while calculated values are ground-state values, but such an excitation energy is far lower than the multifragmentation region. Despite these limitations, the calculated values of 17.22 MeV in the ($\sigma\omega\delta\rho$) case lies within the experimental range of 18.9 ± 3.2 MeV, while the ($\sigma\omega$) value of 24.63 MeV is larger than the experimental range.

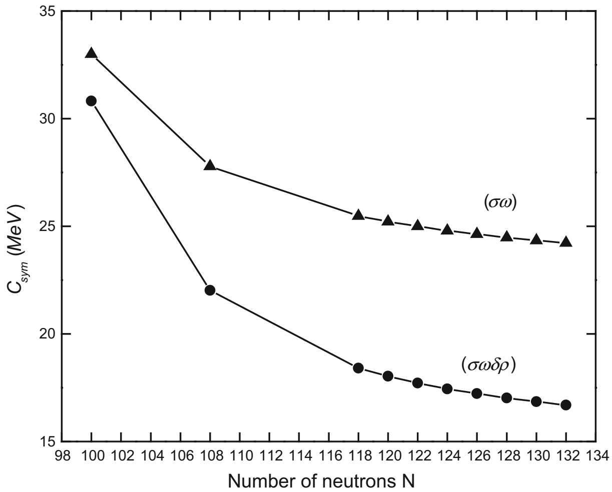

Figure 1. Symmetry energy in spherically symmetric even–even lead isotopes with known experimental masses. The figure compares the results for the cases $(\sigma\omega\delta\rho)$ and $(\sigma\omega)$.

Figure 1 shows that the symmetry energy of a nucleus decreases with increasing number of neutrons for both cases $(\sigma\omega\delta\rho)$ and $(\sigma\omega)$. Two facts need to be considered to explain this result, which seems to contradict the meaning of a symmetry energy. The first one is that the nuclear symmetry energy $e_{\text{sym}}(\rho)$ in eqs (4) and (5) decreases with decreasing density ρ, as has been found, for instance, in ref. [13] for relativistic hadronic models like the one used in this work. The second fact is that heavy nuclei have large proton numbers, and thus large Coulomb repulsive forces that push the protons to larger radii, leading to a depression in the central charge density. The study of the charge density distributions in lead isotopes has shown that the central depression of the density increases with increasing number of neutrons in these isotopes [14].

It can be inferred from figure 1 that the decrease of nuclear symmetry energy $e_{\text{sym}}(\rho)$ in the central region of the nucleus, as a result of increased central depression of the density with increasing number of neutrons, leads to the decrease of the symmetry energy of the nucleus calculated using eq. (7) with increasing number of neutrons.

Figure 1 also shows that the inclusion of the isovector coupling channel generally leads to a decrease of the value of the symmetry energy of a nucleus, and that this decrease grows with increasing number of neutrons. The net effect of the isovector coupling channel is a weaker binding between protons and a stronger binding between neutrons in the case of neutron number exceeding proton number in the nucleus, and this effect sharpens with growing excess of the number of neutrons over the number of protons.

5. Symmetry energy in $N = 126$ isotones

Figure 2 displays the results for the symmetry energy in spherically symmetric even–even $N = 126$ isotones with known experimental masses, i.e., the nuclei ^{206}Hg, ^{208}Pb, ^{210}Po, ^{212}Rn, and ^{214}Ra, for both cases of $(\sigma\omega\delta\rho)$ and $(\sigma\omega)$.

Figure 2 shows that the symmetry energy of a nucleus increases with increasing number of protons for both cases $(\sigma\omega\delta\rho)$ and $(\sigma\omega)$, and that the inclusion of the isovector coupling channel generally leads to a decrease of the value of the symmetry energy of a nucleus, and that this decrease declines with increasing number of protons.

These results agree with the results of the previous section, because the increased number of protons in isotonic nuclei means a decline of the neutron excess in these nuclei. Therefore, the symmetry energy of a nucleus is found to increase with decreasing neutron excess, and the effect of the isovector coupling channel is found to decline with decreasing neutron excess.

6. Symmetry energy integral in ^{40}Ca and ^{48}Ca

Table 2 lists the results for the symmetry energy integral (SEI) in ^{40}Ca and ^{48}Ca, for both cases $(\sigma\omega\delta\rho)$ and $(\sigma\omega)$. Table 2 shows that the isovector coupling channel increases the symmetry energy integral in ^{40}Ca and reduces it in ^{48}Ca. This isotope effect might be considered similar to the isotope effect in the charge densities of ^{40}Ca and ^{48}Ca, which has been studied in ref. [8],

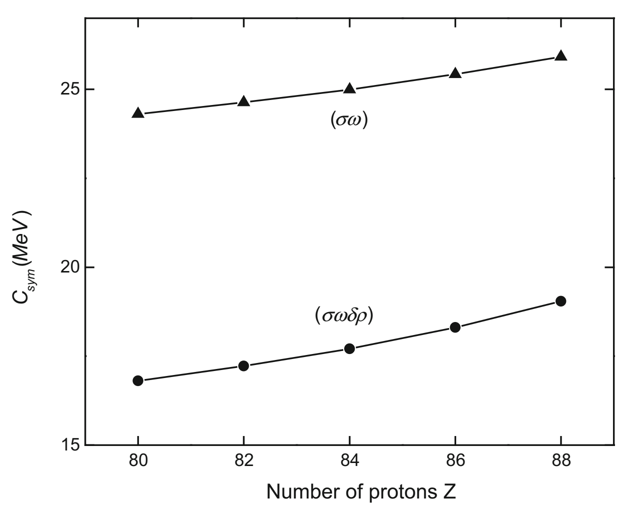

Figure 2. Symmetry energy in spherically symmetric even–even N = 126 isotones with known experimental masses. The figure compares the results for the cases $(\sigma\omega\delta\rho)$ and $(\sigma\omega)$.

Table 2. Symmetry energy integral of ^{40}Ca and ^{48}Ca in the units of MeV. Table compares the results for the cases $(\sigma\omega\delta\rho)$ and $(\sigma\omega)$.

	$(\sigma\omega\delta\rho)$	$(\sigma\omega)$
^{40}Ca	0.046	0.024
^{48}Ca	0.352	0.536

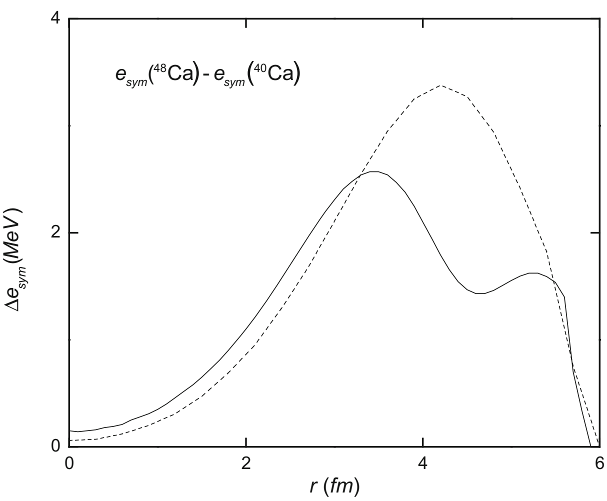

Figure 3. The difference between the ^{48}Ca and the ^{40}Ca local symmetry energies. The solid curve represents the results in the case $(\sigma\omega\delta\rho)$ and the dashed curve in the case $(\sigma\omega)$.

where the interplay between the isoscalar and the isovector channels of the nuclear force was presented as a possible explanation for this isotope effect.

Figure 3 shows the difference between the nuclear symmetry energy inside the ^{48}Ca nucleus and inside the ^{40}Ca nucleus as a function of the distance r from the centre of the nucleus:

$$\Delta e_{\mathrm{sym}}(r) = e_{\mathrm{sym}}(\rho(r))(^{48}\mathrm{Ca}) - e_{\mathrm{sym}}(\rho(r))(^{40}\mathrm{Ca}), \tag{20}$$

for both cases $(\sigma\omega\delta\rho)$ and $(\sigma\omega)$.

A double-humped nature of the nuclear symmetry energy difference can be inferred from figure 3 for the $(\sigma\omega\delta\rho)$ results, but not for the $(\sigma\omega)$ results. This is similar to the double-humped nature obtained in ref. [8] in the study of the isotope effect in the charge densities of ^{40}Ca and ^{48}Ca for the $(\sigma\omega\delta\rho)$ case, but not for the $(\sigma\omega)$ case, which has been explained by the interplay between the isoscalar and the isovector channels of the nuclear force. The double-hump observed in figure 3 in the $(\sigma\omega\delta\rho)$ case leads to a decrease of the value of SEI in the ^{48}Ca nucleus and an increase in the value in the ^{40}Ca nucleus when compared to the case $(\sigma\omega)$, which has one hump of a higher value compared to the values of the two humps for the case $(\sigma\omega\delta\rho)$.

7. Summary

The symmetry energy of a nucleus is determined by applying a local density approximation, where the nuclear matter symmetry energy is determined at the local density value inside the nucleus. The symmetry energy integral and the symmetry energy of the nucleus are then determined by integrating over the entire density distribution function of the nucleus, calculated by utilizing the relativistic density-dependent Thomas–Fermi approach, with density-dependent coupling parameters reproducing the nucleon self-energy resulting from the relativistic Brueckner–Hartree-Fock theory.

The symmetry energy of the nucleus decreases with increasing neutron excess in the nucleus, due to the decrease of the nuclear symmetry energy in the central region of the nucleus, resulting from the decrease of the density in this region with increased neutron excess.

The isovector coupling channel generally leads to a decrease of the symmetry energy of a nucleus, and this decrease grows with increased neutron excess in the nucleus. The net effect of the isovector coupling channel is a weaker binding between protons and a stronger binding between neutrons when neutron number exceeds the proton number in the nucleus, and this effect sharpens with growing excess of the number of neutrons over the number of protons.

The isovector coupling channel increases the symmetry energy integral in ^{40}Ca and reduces it in ^{48}Ca. This isotope effect is similar to the isotope effect in the charge densities of ^{40}Ca and ^{48}Ca, and is explained by the double-humped nature of the nuclear symmetry energy difference between the ^{48}Ca nucleus and the ^{40}Ca nucleus as a function of the distance r from the centre of the nucleus, observed in the results with the isovector coupling channel included, while only one hump of a higher value compared to the values of the two humps of the previous case is observed in the results ignoring the isovector coupling channel. The interplay between the isoscalar and the isovector channels of the nuclear force explains this double-humped nature.

Acknowledgements

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