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Where is SUSY?

AMITAVA DATTA

Department of Physics, University of Calcutta, 92 APC Road, Kolkata 700 009, India
E-mail: adatta_ju@yahoo.co.in

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Abstract. The searches for supersymmetry at the Large Hadron Collider (LHC) have so far yielded only null results and have considerably tightened the bounds on the sparticle masses. This has generated some skepticism in the literature regarding the 'naturalness of SUSY' which qualitatively requires some sparticles to be relatively light. Re-examining some of the bounds from LHC searches, it is argued with specific examples that the above skepticism is a red herring because (i) a quantitative and universally accepted definition of 'naturalness' is not available and (ii) even if some conventional definitions of naturalness is accepted at their face values, the alleged tension with the apparently stringent LHC bounds wither away once the strong assumptions, by no means compelling, underlying such bounds are relaxed.

Keywords. Supersymmetry; Large Hadron Collider; naturalness; dark matter.

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1. Introduction

Supersymmetry (SUSY) is a novel idea that relates bosons and fermions by transformations in the Super space [1] (ref. [1] is an excellent text book containing references to the original papers). It was proposed in the early 1970s and research continued for several years purely out of quiet academic interest. No obvious application in particle physics or well-orchestrated propaganda was noticable. Potentially important multipronged applications gradually opened up in the late 1970s (see [2] and the thousand odd papers citing this pioneering work for references on early SUSY phenomenology). After that, the phenomenology of SUSY took off with a bang. As on 31.10.16, the number of entries in the INSPIRE HEP data base containing SUSY is 9830. The actual number may be much more as all papers on this subject do not explicitly have this magic word in the title.

The main prediction of SUSY is that corresponding to each fermion (boson) in the Standard Model (SM) there is a bosonic (fermionic) superpartner. This increases the spectrum of elementary particles by a factor greater than two. The formulation of the strategies for searching the superpartners – collectively called the sparticles – at the high-energy colliders began in the early eighties. As is well known, a hallmark of sparticle production is the observation of events with large missing transverse energy. In a paper published more than 30 years ago, the observation of such events was reported [3]. This was heralded by a phenomenological paper which worked out in details how the production of strongly interacting sparticles can lead to such signatures [4]. Unfortunately, it was subsequently realized that the above events were consistent with the SM background. However, the study of SUSY searches both phenomenologically and experimentally was set rolling and has been continuing with great enthusiasm ever since. Currently, SUSY search is a top-priority programme at the Large Hadron Collider (LHC) Run-II at 13 TeV. Since more than 30 years of painstaking searches have yielded only null results, the question 'Where is SUSY' is obviously legitimate. However, the main purpose of this review is to point out that the overtone of pesimissm or skepticism that often accompanies this question in the current literature is premature.

2. The vices and the virtues of the beautiful SUSY

- The large number sparticles predicted by SUSY is a warm invitation for the high-energy physicists to an exciting new particle search programme.
- It is obvious that SUSY must be a broken symmetry and each sparticle must be much heavier than the corresponding particle because it has not

been produced so far even at the LHC. However, if one writes the most general soft symmetry breaking terms in the renormalizable Lagrangian of the minimal supersymmetric Standard Model (MSSM) one would encounter more than 100 new unknown parameters (the so-called well-motivated models of SUSY breaking like the minimal supergravity model or the gauge-mediated supersymmetry breaking model [1] with smaller number of free parameters do not appear to be as compelling as they used to be in the pre-LHC era). The mass hierarchy among the sparticles determines the observability of a particular signal. The large number of free parameters is tantamount to many such hierarchies and possible signatures. This arbitrariness is by far the Achilles heel of SUSY. Fortunately, theoretical and phenomenological guidelines and several well-established experimental constraints, to be discussed below, provide useful guidelines for reducing the free parameters.

- It is well known that the radiative correction to m_{H}^2 in the SM diverges quadratically, where m_{H} is the Higgs boson mass. Assuming that the SM is valid upto a scale Λ GeV, the radiative correction due to a SM particle can be schematically written as

$$\Delta m_{\mathrm{H}}^2 \sim \int_0^{\Lambda} \frac{q^2 \mathrm{d}q^2}{q^2 + m_{\mathrm{H}}^2} \sim C_1 \Lambda^2 + C_2 \ln \Lambda^2, \qquad (1)$$

where C_1 and C_2 are calculable coefficients. If the SM is embedded in a grant desert-type grand unified theory (GUT), then $\Lambda \sim 10^{16}$ GeV which is the typical magnitude of the GUT scale. To obtain the physical Higgs mass which is now known to be $\approx$125 GeV, the canonical renormalization prescription needs to cancel out the contribution $\propto M_{\mathrm{GUT}}^2$ ($\sim 10^{32}$ GeV^2) by adjusting the counter terms. This requires an uncanny extreme fine-tuning in the form of cancellation of 28 orders of magnitude in one part. In SUSY, the contribution of the corresponding sparticle cancels the above quadratic divergence in the limit of exact SUSY (i.e., any particle and its superpartner are degenerate). If SUSY is broken, the dangerous quadratic divergence still cancels but the log divergence remains. Substituting the computed value of C_2 one finds that the cancellation is not too fine-tuned provided the sparticles for which C_2 is large have masses $\sim$1 TeV (see [1] for a lucid exposition along with references to the original literature) (see [5] for important contributions to this field from India). This observation lead to the optimism that as the masses of some sparticles appear to be bounded from above thanks to naturalness, they are likely to be within the reach of the LHC searches. As we shall review below, some recent LHC searches have required some sparticle masses to be larger than a TeV or so. A section of the HEP community interpreted this as a tension between naturalness and SUSY.

  The above skepticism is by and large a red herring for two reasons. First, the upper bound on an sparticle mass obtained from the naturalness arguments is basically an order of magnitude estimate. For example, if an sparticle has a mass $\approx$3 TeV, say, it is likely to be beyond the kinematic reach of the LHC but it hardly makes the above cancellations seriously unnatural. Secondly, as elaborated in a subsequent section, the sparticle mass bounds from the LHC are based on certain simplifying assumptions which are by no means compelling. Relaxing them would weaken the bounds. Thus, it is fair to say that SUSY remains as attractive as it used to be in the pre-LHC era.
- It is well known for a long time that a large part of the Universe is made of dark matter (DM) which interacts only weakly and/or gravitationally. If this matter is made of elementary particles, they must also be cosmologically stable. More recent data collected by the Planck satellites have accurately measured the DM relic density of the Universe [6]. The measured value is

$$\Omega h^2 = 0.1186 \pm 0.0026, \qquad (2)$$

where h is the Hubble constant. The SM has no suitable DM candidate. In contrast, in R-parity conserving SUSY the stable lightest supersymmetric particle (LSP) is a popular DM candidate. In many models, the lightest neutralino ($\tilde{\chi}_1^0$) (see §3.2) is often chosen as the DM particle. Requiring consistency with the measured value of the DM relic density is an important constraint on SUSY models. This weakly interacting DM particle, if produced at the LHC, will escape detection. Thus, events containing it will show an apparent imbalance of energy–momentum. This provides a much needed link between collider physics and cosmology. Of course the observation of events with missing energy alone does not conclusively prove the existence of DM. Nevertheless, the observation of such events provides a tantalizing hint.
- The precisely measured muon anomalous magnetic moment ($a_\mu = \frac{1}{2}(g-2)_\mu$) is an important probe for new physics. More recent measurements have revealed a discrepancy between the SM predictions (including contributions from QED, electroweak and strong interactions). The measured discrepancy [7] is

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = (29.3 \pm 9.0) \times 10^{-10}. \quad (3)$$

A generic contribution to a_μ scales like m_μ^2/Λ^2 where Λ and m_μ refer to the scale of new physics and muon mass respectively. Thus, new physics can significantly contribute to a_μ provided its characteristic scale is not too high. For example, it has been pointed out that contributions from electroweak (EW) sparticles (see §3.2) can improve the agreement between the theory and the measured value [8].

3. Chasing the beautiful SUSY at the LHC

Although the search for SUSY has a history of more than 30 years, it should be borne in mind that the coveted TeV scale has been under the scanner only after the advent of the LHC [9]. All previous experiments, though relevant, probed sparticle masses in the ball park of a few hundred GeV at the most.

The concept of naturalness also evolved over the years from the initial intuitive argument summarized in the last section (for a recent review, see for example ref. [10]). However, a universally accepted definition of fine tuning does not exist. As a rough guideline, we shall follow [11] which has listed the following criteria using a reasonable definition of the fine-tuning parameter.

- From the stability of the EW scale, one requires the Higgsino (see below) mass to be <700 GeV.
- The most stringent naturalness bound, from the phenomenological point of view, is on the gluino (the superpartner of the gluon) mass: $m_{\tilde{g}} < 1.5$ TeV. Because this strongly interacting sparticle is produced with a large at the LHC, the experimental lower bounds on its mass are rather stringent.
- The naturalness bound on the stop (the superpartner of t) mass, considered to be a key parameter by many analyses, is the least stringent. In fact, $m_{\tilde{t}}$ could be beyond the LHC reach by allowing a small fine-tuning.

In the following subsections we shall examine the first two naturalness criteria in the light of LHC data.

3.1 *Limits from gluino searches*

We first present an example of LHC Run-II data which illustrates what the experiments look for. Generically a new physics signal consists of m-leptons + n-jets + $\not{E}_{\rm T}$ where m and n are integers. Table 1 of [12] reproduced as figure 1 shows four signal regions (SRs), each characterized by a set of criteria for selecting the leptons, jets and $\not{E}_{\rm T}$ in the desired signal. They are introduced to reject the SM background. As expected, all regions select events with high $\not{E}_{\rm T}$ only. If this requirement is somewhat relaxed for a SR, a relatively stronger cut on another observable is imposed. A very important development in the LHC experiments is the determination of the SM background from the data. In order to achieve this, events are also recorded in several control regions where the signal events are expected to be small. Using extrapolation techniques, the number of background events in each signal region is then determined. If the observed number of events is consistent with the backgrounds, a model-independent upper limit on the number of beyond Standard Model (BSM) events ($N_{\rm BSM}^{\rm obs}$) for each signal region can be obtained by standard statistical analysis (see table 6 of [12] reproduced as figure 2).

Table 1: Summary of the event selection criteria for the signal regions (see text for details).

Signal region	$N_{\rm lept}^{\rm signal}$	$N_{b-\rm jets}^{20}$	$N_{\rm jets}^{50}$	$E_{\rm T}^{\rm miss}$ [GeV]	$m_{\rm eff}$ [GeV]
SR0b3j	≥3	=0	≥3	>200	>550
SR0b5j	≥2	=0	≥5	>125	>650
SR1b	≥2	≥1	≥4	>150	>550
SR3b	≥2	≥3	-	>125	>650

Figure 1. An example of ATLAS selection criteria for new physics search at LHC Run-II.

Table 6: Signal model-independent upper limits on the number of BSM events ($N_{\rm BSM}$) and the visible signal cross-section ($\sigma_{\rm vis}$) in the four SRs. The numbers (in parentheses) give the observed (expected under the SM hypothesis) 95% CL upper limits. Calculations are performed with pseudo-experiments. The $\pm 1\sigma$ variations on the expected limit due to the statistical and systematic uncertainties in the background prediction are also shown.

	SR0b3j	SR0b5j	SR1b	SR3b
$N_{\rm BSM}^{\rm obs}$ ($N_{\rm BSM}^{\rm exp}$)	5.9 ($4.1^{+1.6}_{-0.8}$)	6.4 ($3.6^{+1.2}_{-1.1}$)	8.8 ($6.0^{+2.6}_{-1.6}$)	3.8 ($3.7^{+1.1}_{-0.5}$)
$\sigma_{\rm vis}^{\rm obs}$ [fb]	1.8	2.0	2.8	1.2

Figure 2. An example of ATLAS limits on new physics events from LHC Run-II.

In principle, any BSM scenario sensitive to the selection criteria presented in figure 1 can be constrained by the data in figure 2. In practice, however, the number of free parameters in a typical BSM model does not permit any unambiguous interpretation. The SUSY models are constrained by following one of the two approaches given below:

Most of the limits on sparticle masses have been obtained by the LHC Collaborations in terms of simplified models. A simplified model consists of a minimal set of sparticles which leads to a particular signature while all others are assumed to be decoupled. Optimum BRs of sparticle decays enhancing a particular signal are also assumed. Suitable relaxation of these assumptions results in much weaker bounds. Moreover, consistency of the model under study with the measured DM relic density or the measured a_μ cannot be tested unless other parameters are specified.

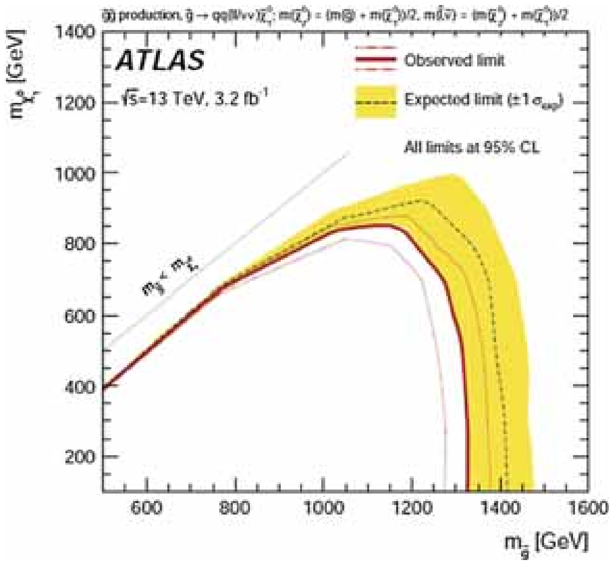

Figure 3. An ATLAS exclusion contour in the $m_{\tilde{g}}$–$m_{\tilde{\chi}_1^0}$ plane [12].

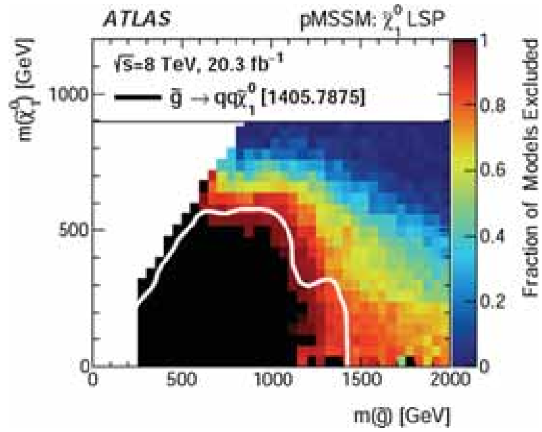

Figure 4. An example of ATLAS exclusion of randomly generated pMSSM models [13].

An example of this approach is illustrated in figure 3 [12]. The signal arises from $\tilde{g}$ pair production where each gluino decays via the channel shown in figure 3. All squarks are assumed to be heavy.

The red exclusion contour represents the limits in the $m_{\tilde{g}}$–$m_{\tilde{\chi}_1^0}$ plane, where $\tilde{\chi}_1^0$ refers to the lightest sparticle – the DM candidate. In drawing this exclusion contour, several assumptions which optimize the signal have been made (see [12] for the details). In spite of these favourable assumptions, there is no limit on $m_{\tilde{g}}$ for $m_{\tilde{\chi}_1^0} > 850$ GeV. For such relatively heavy LSPs, gluinos are produced with reasonably large cross-section only if $m_{\tilde{g}}$ and $m_{\tilde{\chi}_1^0}$ are close together. This results in degraded limits due to the small energy release in $\tilde{g}$ decays. Generically, such compressed sparticle mass spectra yield poor limits. It goes without saying that the important issue of naturalness vs. SUSY cannot be settled on the basis of these limits riddled with so many caveats.

The second popular approach for interpreting the LHC data has been within the framework of the phenomenological MSSM (pMSSM). This model has 19 free parameters. The large number of free parameters in the MSSM as noted above have been reduced to 19 by invoking some reasonable assumptions like there is no flavour changing neutral current involving the first two generations of quarks and leptons, there is no CP-violating phase other than the one in the SM etc.

The ATLAS Collaboration has confronted models belonging to this class with 22 data sets obtained from different search channels – each having a structure similar to the one in figure 1 in principle – from LHC Run- I at 8 TeV. They obtained thousands of pMSSM scenarios by randomly generating the 19 free parameters and testing them in the light of the above sets of data. The results are presented in figure 4 [13]. In the black region in the $m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$ plane corresponding to relatively small values of the masses, all randomly generated models are in conflict with the data set. On the other hand, in the indigo region, all the models are consistent with the data sets. In the intermediate region, various fractions of the generated models as denoted by the colour code shown are allowed. The main point to be noted is that in a large number of models $m_{\tilde{g}} < 1.0$ TeV is very much allowed by the data. Moreover, this multichannel analysis excludes parameter spaces with $m_{\tilde{g}} \approx m_{\tilde{\chi}_1^0}$ more efficiently than an analysis based on a single gluino decay channel as shown in figure 4 (see the white contour). This improvement is due to the inclusion of the monojet + $\not{E}_T$ data in the multichannel analysis [13]. The last analysis is designed to search for signal events having very little visible energy and are tagged by a high p_T jet due to initial state radiation. More or less similar bounds follow from other gluino searches except for the case where the squarks and the gluino are assumed to be approximately mass degenerate. Here, the lower limit on their common mass tends to violate the naturalness bound $m_{\tilde{g}} < 1.5$ TeV. It is, therefore, fair to conclude that there is no threat to the idea of naturalness as formulated in [11] from the LHC data as yet.

3.2 *Why the electroweak sparticles are important?*

The EW sparticles are important for the following reasons:

- Majority of the DM producing mechanisms depend mainly on EW sparticles: the electroweakinos and the sleptons.
- Contributions of light EW sparticles to anomalous magnetic moment of the muon improves the agreement between theoretical prediction and data [8].
- As already discussed, the mass of the Higgsino is crucial for testing the naturalness of a SUSY model.
- In case the superpartners of the strongly interacting sparticles – the squarks and the gluino – expected to be produced with large cross-sections are beyond the reach of the LHC, the discovery of SUSY hinges on the properties of the EW sparticles. This does not appear to be an unlikely scenario in view of the steadily increasing mass lower bound on the strong sparticles from the LHC searches.

The spectrum of the EW sparticles in the pMSSM contains the following. For theoretical consistency, the pMSSM requires two doublets of Higgs bosons. After the Higgs mechanism, the spectrum contains four physical spin-zero particles h, A, H^0 and $H^{\pm}$ (in the following we shall assume that the strongly interacting sparticles and the heavier Higgs bosons A, H^0 and $H^{\pm}$ are decoupled, i.e., beyond the reach of the LHC). They have three spin-half superpartners $\tilde{H}^{\pm}$, $\tilde{H}_1^0$ and $\tilde{H}_2^0$ collectively called the charged and neutral higgsinos. The charged gaugino (the superpartner of the charged W) and the charged higgsino mix to produce two physical states $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^{\pm}$ called the charginos. The two neutral gauginos (the superpartners of the neutral component of the W multiplet and the $U(1)$ gauge boson) and the above neutral higgsinos mix among themselves leading to four physical states $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$ and $\tilde{\chi}_4^0$ (arranged in the order of ascending masses) known as the neutralinos. Collectively, the charginos and the neutralinos are called the electroweakinos (ewikinos). In the MSSM the masses and the compositions of the electroweakinos are determined by four independent parameters: the $U(1)$ gaugino mass parameter M_1, the $SU(2)$ gaugino mass parameter M_2, the higgsino mass parameter μ and $\tan\beta$, the ratio of the vacuum expectation values of the two neutral Higgs bosons. Throughout this paper we take $\tan\beta = 30$ which usually gives a better agreement with the a_μ data. In addition, each spin-half chiral fermion in the SM has a spin-zero superpartner. The electron, for example, has two spin-0 superpartners called L- and R-type selectrons ($\tilde{e}_{\mathrm{L}}$ or $\tilde{e}_{\mathrm{R}}$). The three neutrinos have L-type scalar superpartners only. The LHC searches are based on the assumption that $\tilde{\chi}_2^{\pm}$, $\tilde{\chi}_3^0$ and $\tilde{\chi}_4^0$ are decoupled.

3.3 *The limits from electroweakino searches*

The electroweakino searches at the LHC mainly considers the $3l + \not{E}_{\mathrm{T}}$ channel from $\tilde{\chi}_1^{\pm} - \tilde{\chi}_2^0$ pair production. The $\tilde{\chi}_1^{\pm}$ decays into a lepton (only the e and the μ are considered) and $\not{E}_{\mathrm{T}}$ is carried by the neutrinos and the $\tilde{\chi}_1^0$. Similarly, the $\tilde{\chi}_2^0$ decays into two leptons and $\not{E}_{\mathrm{T}}$. The $3l$ yield depends crucially on the compositions and relative masses of the electroweakinos.

The ATLAS and the CMS Collaborations interpreted their trilepton search results [14,15] in terms of a simplified model based on the assumptions that $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$ are wino-dominated and mass degenerate while $\tilde{\chi}_1^0$ is bino-dominated (the discussions in this subsection are based on LHC Run-I data at 8 TeV). Moreover, the L-type sleptons are light so that they can mediate the electroweakino decays into leptons with large BRs. The black exclusion contour obtained by the ATLAS Collaboration reproduced in figure 5 represents the mass limits in this simplified model. In [16,17] the impact of the LHC constraints on pMSSM models based on different assumptions were studied. The pink contour in figure 5 is obtained in a pMSSM by our simulation. This contour captures the basic features of the simplified model contour obtained by ATLAS. This validates our simulation [16]. As this pMSSM model had additional parameters, we could study the compatibility of this model with the measured DM relic density of the Universe and the anomalous magnetic moment of the muon. The narrow red dotted strips represent the parameter space compatible with the relic density data. The dark blue, green and brown regions are compatible with the measured a_μ at the 1σ, 2σ and 3σ levels respectively. The first two cases certainly give better agreement than the SM. The black short-dashed contour is ruled out from LHC bounds on

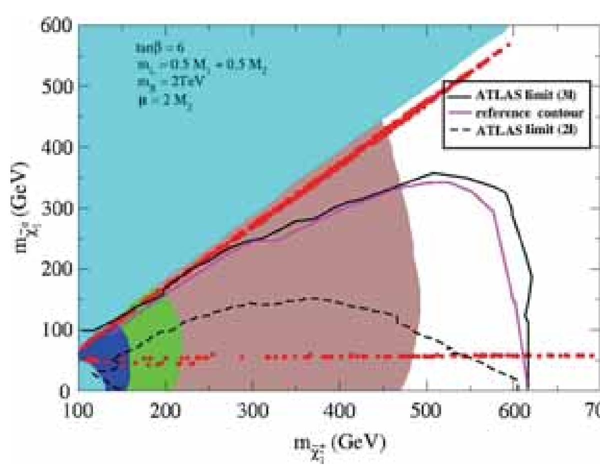

Figure 5. Constraints from LHC, the DM relic density and a_μ on a pMSSM model with wino-like $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$.

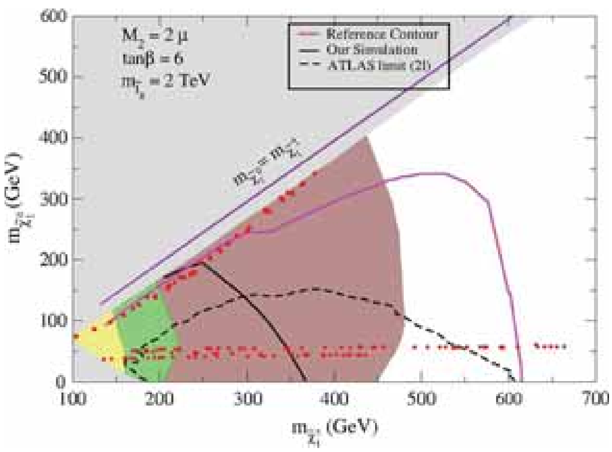

Figure 6. The same as in figure 5 except that $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$ are higgsino-like.

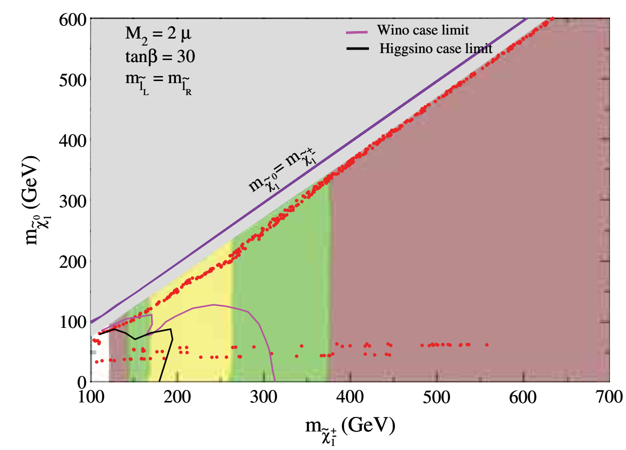

Figure 7. The same as in figure 6 except that all sleptons are heavier than $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$.

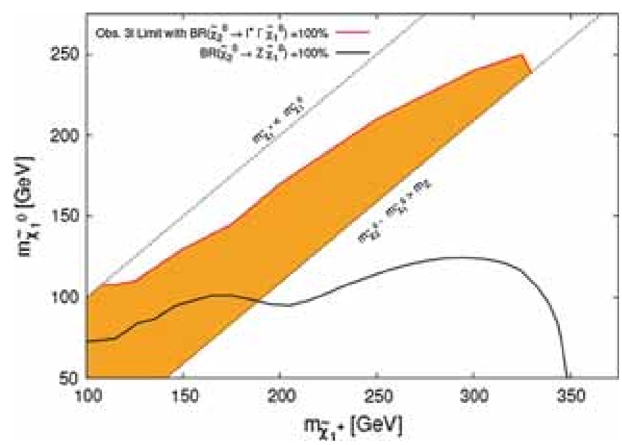

Figure 8. The same as in figure 6 except that all sleptons are heavier than $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$.

slepton mass from direct slepton searches. It should be noted that a rather small parameter space is allowed if all constraints are imposed even in a pMSSM.

The situation changes significantly if $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$ are higgsino-like while the LSP is either a bino or a bino–higgsino admixture [17]. All other features are the same as the ones considered in figure 5 [16]. The first point to be noted is that the sparticle mass bounds in this case represented by the black exclusion contour are much weaker than that obtained in figure 5. It may be noted that the strongest bound on $m_{\tilde{\chi}_1^{\pm}}$ for negligible $m_{\tilde{\chi}_1^0}$ is about 370 GeV which is well within the naturalness limit on higgsino mass mentioned above. The colour codes are the same as in figure 5 except that in the yellow region, the a_μ constraint is satisfied at the 1σ level (figure 6).

We next consider a pMSSM where both $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$ are lighter than both L- and R-type sleptons. In this scenario, the above sparticles cannot decay into leptons via on-shell sleptons. As a result, their leptonic decays occur via on-shell W and Z bosons. This results in smaller leptonic branching ratios for both the above electroweakinos. This leads to the weakest mass limits as shown by the black exclusion contour in figure 7. For ready reference we have also presented in figure 7 the pink exclusion contour which is obtained by the ATLAS Collaboration in a very similar simplified model with wino-like $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$. However, if the sleptons are heavier than the electroweakinos but not decoupled and wino-like $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$ can decay into nearly on-shell sleptons [18] relatively stronger bounds are obtained as shown by the yellow region in figure 8 [18]. The black contour represents the exclusion contour when the sleptons are so heavy that their contribution to the electroweakino decays are almost negligible (the black contour is based on more recent data compared to the pink contour in figure 7).

4. Conclusion

- SUSY had its genesis in a fundamental query – can a symmetry relating the bosons and fermions be formulated? No practical application was envisaged.
- In course of time, many non-trivial predictions were found which are now being tested by the LHC searches for the sparticles, the accurate measurement of a_μ, the precise measurement of the DM relic density of the Universe etc.
- In order to ameliorate the naturalness problem of the SM, some of the sparticles should have relavely low masses (~1 TeV). Although the naturalness argument has an intuitive appeal, a quantitative measure of naturalness is not available.
- No SUSY signal has been seen by the LHC Run-I experiments at 7/8 TeV or the preliminary Run-II experiments at 13 TeV. This has resulted in lower bounds on some sparticle masses of 1 TeV or so.

However, none of the bounds are without caveats. Even if the bounds are taken at their face values, they hardly violate the naturalness conditions. This has been illustrated with several examples. The skepticism regarding the naturalness of SUSY sometimes found in the literature is, therefore, premature.
- SUSY remains as attractive today as it was in the 1970s.

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