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Hierarchy problem and BSM physics

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Abstract. The ‘hierarchy problem’ plagues the Standard Model of particle physics. The source of this problem is our inability to answer the following question: Why is the Higgs mass so much below the GUT or Planck scale? A brief description about how ‘supersymmetry’ and ‘composite Higgs’ address this problem is given here.

Keywords. Higgs boson; supersymmetry; composite Higgs.

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1. Introduction

In quantum field theory, the mass of a spin-zero elementary particle is not ‘protected’ from receiving large quantum corrections. If the Higgs-like object, discovered by the CMS and ATLAS Collaborations of LHC at CERN, indeed turns out to be the Higgs boson of the Standard Model (SM), then why is its mass around 125 GeV, and not close to the maximum possible energy scale of the theory, e.g. grand unification scale (10^{16} GeV) or the Planck scale (10^{19} GeV)? This question constitutes the so-called ‘hierarchy problem’ [1–4]. Does it mean that the SM is just a good effective theory at weak scale, which needs to be supplemented by a more fundamental beyond the Standard Model (BSM) theory at higher scale? Which kinds of BSM theory elegantly address the hierarchy problem? In this talk, I shall describe what the hierarchy problem is all about, and very briefly describe how ‘supersymmetry’ [5] and ‘composite Higgs’ [6], as two broad scenarios, address this problem. I shall avoid details, and only highlight the essential points relevant in the present context.

2. Hierarchy problem

One-loop quantum corrections to the Higgs boson mass from fermion loops go like $\Delta m_h^2(f) = -(y_f^2 2\Lambda^2)/16\pi^2$ and from the bosonic (e.g. scalar) loops as $\Delta m_h^2(s) = +(\lambda_S^2\Lambda^2)/16\pi^2$, where Λ is the highest scale of the theory. These are not merely calculation of divergences, as they can be removed through regularizations. Here we are talking about large finite corrections arising from Λ-scale states (fermions and bosons) which couple to the Higgs. Even though Λ turns out to be very large, one can exploit the sign difference between fermion and boson loops, and tune the magnitude of the couplings, to keep the net contribution to the Higgs mass in 100 GeV range. But even if couplings are adjusted in a given order in perturbation theory, the adjustment is offset in the next order, and so tuning has to be done order by order in perturbation theory. This is what is called the hierarchy problem. Importantly, as Higgs mass is not protected, the electroweak vacuum expectation value (vev) v is also not protected ($m_h \sim \lambda v$, where λ is the quartic coupling), and hence the entire SM is plagued by the hierarchy problem. To sum up, we are facing a situation where we have to accept a miraculous cancellation of one part in 10^{28} in getting $m_h^2 = \mathcal{O}$ (100 GeV)^2 – which is really a big hierarchy. In QED, electron mass is protected by chiral symmetry. Similarly, the photon always remains massless thanks to gauge symmetry. Can we associate the Higgs boson with a similar symmetry that can protect its mass? In the SM, there is no such symmetry. Can this be a guiding principle behind looking for BSM physics?

3. Supersymmetry

Supersymmetry relates matter particles with force particles, i.e. it relates fermions with bosons, in the sense that for every known fermion there is a bosonic partner and vice versa. No superparticle has been found so far, which means that supersymmetry is very badly broken (in masses, not in couplings). But even then, the quadratically growing large contributions to the Higgs mass, leading to the big hierarchy, get cancelled between loops with virtual particles carrying opposite spins (particles vs. sparticles). This happens because the couplings in the fermion and boson loops are related. Minimal supersymmetric model has two Higgs doublets and therefore five physical scalar degrees of freedom – three neutral (two CP-even and one CP-odd) and one pair of charged scalars. Without going into the details, I shall mention two generic but crucial features that capture the tension in supersymmetric model building:

(1) In supersymmetry, the quartic couplings are related to gauge couplings. This is why at tree level the Higgs mass is at most the Z boson mass. However, significantly large loop corrections are arising mainly from the large top mass. At one loop, the approximate estimate of the lighter CP-even Higgs mass is given by

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + 3m_t^4 \ln(m_{\tilde{t}}^2/m_t^2)/(\sqrt{2}\pi^2 v^2). \quad (1)$$

Here $\tan\beta = v_2/v_1$, the ratio of the two vevs. If $\tan\beta > 3$ or so, the tree-level contribution maximizes but it is limited by $M_Z \simeq 91$ GeV. So, to reach up to 125 GeV, one needs a large supersymmetry breaking soft mass, more specifically a really heavy stop ($m_{\tilde{t}} \sim 10$ TeV). Even though we have not explicitly displayed it, there is a prominent role of trilinear scalar coupling A_t in determining the Higgs mass. Large value of A_t ($\sim$ TeV) can generate sizable mixing between the left- and right-type stop squarks, and this mixing can bring down the desired value of the average stop mass necessary to reproduce $m_h = 125$ GeV. Higher loop computation of the Higgs mass further brings down the required stop mass (e.g. the conclusion from leading 3-loop computation is that a 3 GeV stop can reproduce $m_h = 125$ GeV even without the necessity of large left–right scalar mixing. Nevertheless, the overall message is that we need rather heavy stop to reproduce 125 GeV for the Higgs mass.

(2) One elegant feature of supersymmetry is that one of the (soft) mass-squared parameters (more precisely, $m_{H_u}^2$) starting from a positive value at high scale can become negative near the weak scale under the influence of large top Yukawa coupling. This negative mass-square triggers electroweak breaking. From the electroweak symmetry breaking conditions, it follows that

$$0.5M_Z^2 \simeq -|\mu|^2 - M_{H_u}^2 \simeq -|\mu^2| + \mathcal{O}(1)\, m_{\tilde{t}}^2. \quad (2)$$

The first equality in the above equation is actually a near-equality because we have assumed $\tan\beta > 3$, a choice that substantially reduces the dependence on m_{H_d}. Note that $m_{H_u}^2$ has flipped sign in the second near-equality. It receives large corrections from the stop mass at one loop, but the loop suppression factor is quite offset by a large log (of ratio of high scale to weak scale, where the high scale can be the GUT scale). The mass parameter μ is a supersymmetry preserving coefficient of $H_u H_d$ term in the superpotential. It implies a cancellation between μ and the supersymmetry breaking parameter (namely, the stop mass) to eventually generate $M_Z = 91$ GeV. As the stop mass has been pushed to TeV range by the requirement $m_h = 125$ GeV, we face a cancellation between $m_{\tilde{t}}$ and μ to the level of one part in 10^3. This is what we call the 'little hierarchy' problem. Although the big hierarchy has been solved by the very structure of supersymmetry, this little hierarchy continues to haunt us. This is basically the tension! It arises not so much from the non-observation of the superparticles in colliders, but from the requirement of reproducing 125 GeV mass of the Higgs. Supersymmetric model building is all about addressing this problem.

4. Composite Higgs

Suppose we ask the following question: Can the Higgs be a pseudo-Goldstone boson of some sort, e.g. like a pion? Here the big hierarchy problem is not there because the Higgs would be a composite object, and beyond the compositeness scale (which is not large) the Higgs would dissolve. This is precisely why we were never bothered about the Planck scale correction to the pion mass! Indeed, we could employ the same mechanism of chiral symmetry breaking responsible for pion mass generation in the case of electroweak breaking. The problem would be that the W-mass would be $gf_\pi/2 = 29$ MeV – a phenomenological disaster!

A way out is to employ not the standard QCD group but a QCD-like group, which was called 'technicolour', whose slow (logarithmic) running creates TeV scale. Then $f_\pi \to f \sim 1$ TeV, which helps to reproduce the correct W-mass. But technicolour models faced many troubles, e.g. unacceptably large flavour changing neutral currents. But the real killer was the oblique electroweak S-parameter, namely, the coefficient of $W_3 - B$ quantum mixing. It turns out that $S \sim v^2/f^2$, where $v = 246$ GeV is the electroweak vev. In technicolour, the strongly interacting sector 'directly' participates in electroweak breaking. This is why $v \sim f$, and hence,

$S \sim 1$. But LEP electroweak precision observables put a limit, $S < 0.01$. This was a big blow for technicolour.

But we can still employ the technicolour idea by asking the following question: Can v and f be treated as independent parameters? Suppose we do not allow the strongly interacting sector directly to participate in electroweak breaking. We rather employ the strong sector to give us a set of pseudo-Nambu–Goldstone bosons (pNGBs), and in the second step, use those pNGBs as the Higgs-like scalar degrees of freedom to trigger electroweak symmetry breaking. And, this is precisely what the 'composite Higgs' models are all about.

A concrete example is the following. We assume that the strongly interacting sector has a global symmetry $G = SO(5)$. Suppose strong dynamics spontaneously breaks this to $H = SO(4)$. Simple counting of generators tells us that there would be $10 - 6 = 4$ broken generators corresponding to G/H coset. These four broken generators are associated with four Goldstone bosons. As G is also 'explicitly' broken by gauge and Yukawa interactions, those Goldstone bosons would have a potential, i.e. they would actually be pseudo-Goldstone bosons. They would form the four degrees of freedom of the Higgs doublet. The Higgs state thus originates from the strongly interacting sector. It cannot 'directly' couple to the SM elementary fermions and gauge bosons. The Higgs couples to exotic or composite fermions and gauge bosons which mix with the SM particles. Thus, the Yukawa and gauge couplings are more like form-factors. Without getting into the details of the model, let us write down the effective Lagrangian involving the Higgs and the gauge bosons (form-factors can be expressed as modified couplings in the infrared limit):

$$L_{\text{eff}} = \frac{g^2 v^2}{4}\left(|W|^2 + \frac{1}{2\cos\theta_W^2} Z^2\right) \times \left[2\sqrt{1-\xi}\frac{h}{v} + (1-2\xi)\frac{h^2}{v^2} + \cdots\right],$$

where $\xi = v^2/f^2$. The dynamics dictates that the Higgs remembers its Goldstone origin and its coupling involves the global symmetry breaking scale f. The Higgs couplings to SM gauge bosons are thus modified from their SM values. The limit $\xi \to 0$ takes us to the SM, while $\xi = 1$ corresponds to technicolour. The hZZ or hWW coupling thus interpolates between the SM and technicolour limits.

The elementary fermions may couple linearly with the strong sector composite operators – a framework known as 'partial fermion compositeness'. Thus, the Higgs can sense the elementary fermions through composite intermediate states. This is why the Yukawa couplings get modified from their SM values (similar to what we saw for gauge boson couplings). More precise measurements of Higgs branching ratios would be necessary to favour (or disfavour) these models.

5. Conclusion

Seeking a solution to the hierarchy problem guides us in our endeavour to explore more fundamental underlying theory. Supersymmetry and composite Higgs are two broad classes of models that encompass the features of many other appealing BSM scenarios. We need more data to decipher the underlying dynamics, be it supersymmetry or composite Higgs or something completely different.

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