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# Solitary wave solutions of two-dimensional nonlinear Kadomtsev–Petviashvili dynamic equation in dust-acoustic plasmas

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**Abstract.** Nonlinear two-dimensional Kadomtsev–Petviashvili (KP) equation governs the behaviour of nonlinear waves in dusty plasmas with variable dust charge and two temperature ions. By using the reductive perturbation method, the two-dimensional dust-acoustic solitary waves (DASWs) in unmagnetized cold plasma consisting of dust fluid, ions and electrons lead to a KP equation. We derived the solitary travelling wave solutions of the two-dimensional nonlinear KP equation by implementing sech–tanh, sinh–cosh, extended direct algebraic and fraction direct algebraic methods. We found the electrostatic field potential and electric field in the form travelling wave solutions for two-dimensional nonlinear KP equation. The solutions for the KP equation obtained by using these methods can be demonstrated precisely and efficiency. As an illustration, we used the readymade package of *Mathematica* program 10.1 to solve the original problem. These solutions are in good agreement with the analytical one.

**Keywords.** Magnetized plasma; dust-acoustic solitary waves; Kadomtsev–Petviashvili equation; mathematical methods.

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## 1. Introduction

The nonlinear propagation of such waves can give rise to the formation of solitons with negative or positive wave amplitudes. The solitons which has potential applications in astrophysical and space environments as well as in laboratory and technological studies [1]. Furthermore, electrostatic solitary waves have been observed in several regions, including the Earth's magnetotail, solar wind and polar magnetosphere [2]. Solitary waves and soliton solutions represent one of the most interesting and famous aspects of nonlinear phenomena in spatially extended systems. They appear as specific types of localized solutions of various nonlinear partial differential equations and possess several important properties [3].

Dusty plasmas are ideal medium for creating solitary waves and soliton solutions. Study of dusty plasmas is very attractive because of their theoretical features and applications. Due to the presence of charged dust grains in plasmas, different types of collective processes exist and very rich wave modes can be excited in dusty plasmas. One of these wave modes is the low-frequency dust-acoustic (DA) mode in an unmagnetized dusty plasma whose constituents are an inertial charge dust fluid and Boltzmann-distributed electrons and ions [4–7]. Shukla and Yu [8] have investigated finite-amplitude ion-acoustic solitary waves propagating obliquely to an external magnetic field in a plasma without dust particles. Yinhua and Yu [9] studied nonlinear ion-acoustic solitary waves propagating obliquely to the external magnetic field in a magnetized dusty plasma. Tagare [10] studied the dust-acoustic solitary waves (DASWs) in a dusty plasma consisting of cold dust particles and two-temperature isothermal ions by the reductive perturbation technique. Das *et al* [11] studied some different kinds of ion-acoustic solitary waves (IASWs) in plasma with trapped electrons by deriving the Korteweg–de Vries (KdV) equation [12].

The characteristics of DA solitary waves in a dusty plasma system consisting of dust fluid and superthermal electrons as well as ions are investigated both analytically and numerically. In most of the investigations, KdV equation or its variants for one-dimensional study [13], has been derived by using reductive perturbation method. Kadomstev and Petviashvili made an attempt to describe the solitons in two-dimensional systems by providing a model in terms of KP equation. Singh and Honzawa [14] investigated the KP equation of an ion-acoustic soliton in an unmagnetized two-dimensional weakly relativistic collisionless plasma with finite ion temperature. They studied the effects of ion temperature and relativistic factor on the width and amplitude of ion-acoustic solitons. The KP equation considered as two-dimensional extension of KdV equation has been derived for dust acoustic waves in hot dusty plasma [15]. Gill *et al* [16] reported the existence of compressive and rarefactive DA solitons from the solution of KP equation derived in two-dimensional dusty plasma in the presence of two-temperature ions. Pakzad [17] derived the KP equation for DASWs in a warm dusty plasma with variable dust charge, two-temperature ions and nonthermal electrons. El-Shewy *et al* [18] investigated solitary solutions and energy for the KP equation in an unmagnetized plasma consisting of hot and cold dust species with Boltzmann-distributed electrons and ions. The nonlinear dust-acoustic solitary waves in a dusty plasma with two nonthermal ion species at different temperatures were studied analytically [19–23]. The KP and modified KP (mKP) equations were derived using reductive perturbation method. It was observed that the formation of both rarefactive and compressive solitary waves were strongly influenced by the concentration and temperature of nonthermal ions [24–30].

This paper is organized as follows: An introduction is given in §1. In §2, the problem formulations of nonlinear two-dimensional DASWs in collisionless, unmagnetized cold plasma consisting of dust fluid, ions and electrons to derive the nonlinear two-dimensional KP equation have been formulated. In §3, the electric field potential and electric field in form travelling wave solutions of the KP equation have been obtained and analysed. Finally the conclusion is given in §4.

## 2. The dispersive basic model equations

We considered the two-dimensional DASWs in collisionless, unmagnetized cold plasma consisting of dust fluid, ions and electrons. The dynamics of DASWs is governed by the normalized fluid equations of nonlinear continuity (1), nonlinear motion of system (2) and (3) and linear Poisson equation (4) as

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{u}_d) = 0, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + v_d \frac{\partial u_d}{\partial y} = -s \frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial v_d}{\partial t} + u_d \frac{\partial v_d}{\partial x} + v_d \frac{\partial v_d}{\partial y} = -s \frac{\partial \phi}{\partial y}, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -s(n_d - 1) + c_1 \phi + c_2 \phi^2 + c_3 \phi^3, \tag{4}$$

where $\phi$ is the electrostatic potential, $\mathbf{u}_d$ is the fluid velocity, $u_d$ and $v_d$ are the velocities of the dust flow along $x$ and $y$ directions, $n_{j0}$ $(j = e, i, d)$ is the equilibrium value of the number density of the charge particles (electrons, ions and dust), $s = \pm 1$ for positive and negative dust, $T$ is the kinetic temperature; $K_{\mathrm{B}}$ is the Boltzmann constant, $m$ is the mass of the species, $Z_d$ is the variable number of dust grains, $T_e$ and $T_i$ are the electron and ion plasma temperature, $c_j$ $(j = e, i, d)$ is the equilibrium value of the acoustic speed of the particles (electrons, ions and dust), $m_j$ $(j = e, i, d)$ is the mass of the particles (electrons, ions and dust) and

$$c_1 = \frac{s}{\mu - 1}\left(c_e\left(K_e - \frac{1}{2}\right) + c_i\left(K_i - \frac{1}{2}\right)\right),$$

$$c_2 = \frac{s}{2(1-\mu)}\left(\mu c_e^2\left(K_e^2 - \frac{1}{4}\right) + c_i^2\left(K_i^2 - \frac{1}{4}\right)\right),$$

$$c_3 = \frac{s}{6(\mu - 1)}\left(\mu c_e^3\left(K_e^2 - \frac{1}{4}\right)\left(K_e + \frac{3}{2}\right) + c_i^3\left(K_i^2 - \frac{1}{4}\right)\left(K_i + \frac{3}{2}\right)\right),$$

$$\mu = \frac{n_{e0}}{n_{i0}},$$

where isotropic kappa $K$ is the distribution function to model the plasma superthermality, the real parameter $K$ measures the deviation from the Maxwell–Boltzmann equilibrium (which is recovered in the limit $K \to \infty$) and $K_{e,i}$ is the effect of superthermality of charged particles (parameter of electrons and ions). It is mentioned that spectral index must take sufficiently large values ($K > 3/2$) in order for a physically meaningful thermal speed to be defined. For $K_{e,i} \to \infty$, the Maxwellian limit is reproduced. The findings of the present investigation (in the limit $K_{e,i} \to \infty$) agree with the earlier investigations of two-dimensional DASWs in the absence of low-temperature ions, with constant dust charge and hot ions/electrons obeying Maxwellian distribution.

The propagation of dust-acoustic waves in collisionless, unmagnetized dusty plasma consisting of electrons, two-temperature ions and highly negatively charged dust grains are considered. The current balance equation is [28]

$$\left(\frac{\partial}{\partial t} + \vec{v_d} \cdot \vec{\nabla}\right) q_d \approx o,$$

where $q_d = Z_d n_d$. Here, $Z_d$ is the number of the charge residing on a dust particle and $q_d$ is the dust charge variable. Total charge neutrality at equilibrium requires that

$$n_{0e} + n_{0d} Z_{0d} = n_{0il} + n_{0ih},$$

where $n_{0e}, n_{0d}, n_{0il}, n_{0ih}$, are the equilibrium values of the number densities of electrons, dust, lower temperature ions and higher temperature ions respectively. $Z_{0d}$ is the unperturbed number of charges on the dust particles.

By using the reductive perturbation methods, the stretching coordinates of the scale are given as

$$\xi = \epsilon(x - Vt), \qquad \eta = \epsilon^2 y, \qquad \tau = \epsilon^3 t, \tag{5}$$

where $V$ is the phase velocity of the wave along the $x$-direction and $\epsilon$ is a small expansion parameter proportional to the amplitude of the perturbation which characterizes the strength of nonlinearity of the system. To obtain two-dimensional nonlinear KP dynamical equation, expand the fluid velocity, density and electrical potential in power series of $\epsilon$ as

$$\begin{aligned}
n_d &= n_{d0} + \epsilon^2 n_{d1} + \epsilon^4 n_{d2} + \epsilon^6 n_{d3} + \cdots,\\
u_d &= u_{d0} + \epsilon^2 u_{d1} + \epsilon^4 u_{d2} + \epsilon^6 u_{d3} + \cdots,\\
v_d &= \epsilon^3 v_{d1} + \epsilon^5 v_{d2} + \epsilon^7 v_{d3} + \cdots,\\
Z_d &= 1 + \epsilon^2 Z_{d1} + \epsilon^4 Z_{d2},\\
\phi &= \phi_0 + \epsilon^2 \phi_1 + \epsilon^4 \phi_2 + \epsilon^6 \phi_3 + \cdots.
\end{aligned} \tag{6}$$

Consider small deviations from the equilibrium state for which $n_{d0} = 1, u_{d0} = 0, \phi_0 = 0$. By using the reduction perturbation method from eqs (1)–(4) and new scaling (5) and (6), the collecting coefficients of lowest order of $\epsilon$ are as follows:

$$\begin{aligned}
n_{d1} &= \frac{u_{d1}}{V}, \qquad u_{d1} = s\frac{\phi_1}{V},\\
c_1 &= \frac{1}{V^2}, \qquad \frac{\partial u_{d1}}{\partial \xi} = \frac{s}{V}\frac{\partial \phi_1}{\partial \eta}.
\end{aligned} \tag{7}$$

For higher order of $\epsilon$

$$-V\frac{\partial n_{d2}}{\partial \xi} + \frac{\partial n_{d1}}{\partial \tau} + \frac{\partial u_{d2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{d1}u_{d1}) + \frac{\partial v_{d1}}{\partial \eta} = 0, \tag{8}$$

$$-V\frac{\partial u_{d2}}{\partial \xi} + \frac{\partial u_{d1}}{\partial \tau} + u_{d1}\frac{\partial u_{d1}}{\partial \xi} = -s\frac{\partial \phi_2}{\partial \xi}, \tag{9}$$

$$-V\frac{\partial v_{d2}}{\partial \xi} + \frac{\partial v_{d1}}{\partial \tau} + u_{d1}\frac{\partial v_{d1}}{\partial \xi} = -s\frac{\partial \phi_2}{\partial \eta}, \tag{10}$$

$$\frac{\partial^2 \phi_1}{\partial x^2} + s n_{d2} = \frac{1}{V^2}\phi_2 + c_2\phi_1^2. \tag{11}$$

By eliminating second-order quantities from eqs (8)–(11), Saini *et al* obtained the two-dimensional nonlinear KP equation as

$$\frac{\partial^2 \phi_1}{\partial \tau \partial \xi} + A\left(\phi_1 \frac{\partial^2 \phi_1}{\partial \xi^2} + \left(\frac{\partial \phi_1}{\partial \xi}\right)^2\right) + B\frac{\partial^4 \phi_1}{\partial \xi^4} + C\frac{\partial^2 \phi_1}{\partial \eta^2} = 0, \tag{12}$$

where $A, B, C$ are given by

$$A = \frac{3s}{2V} - V^2 c_2, \quad B = \frac{V^3}{2}, \quad C = \frac{V}{2}.$$

We considered $\phi_1 \equiv \phi$ for mathematical simplicity. Consider the travelling wave solutions as

$$\phi_1(\xi, \eta, \tau) = \phi(\theta) \quad \text{and} \quad \theta = k\xi + \ell\eta + \omega\tau, \tag{13}$$

where $k$, $\ell$ and $\omega$ are wave numbers and frequency. Then eq. (12) becomes

$$(\omega k + C\ell^2)\phi'' + Ak^2(\phi\phi'' + (\phi')^2) + Bk^4\phi^{(4)} = 0. \tag{14}$$

## 3. Dust-acoustic solitary wave solutions

We obtained the dust-acoustic solitary wave solutions of the nonlinear two-dimensional KP equation by applying the extended and modified different kinds of function methods. The analytic solutions of eq. (12) give different values for the electrostatic potential $\phi$, which has the following cases:

*Case* I: *Sech–tanh method:* The nonlinear two-dimensional KP equation has solution in series of sech–tanh as

$$\phi(\theta) = a_0 + \sum_{i=1}^{n} \text{sech}^{i-1}\theta(a_i \,\text{sech}\,\theta + b_i \tanh\theta), \tag{15}$$

where $a_0, a_1, ..., a_n, b_1, ..., b_n$ are arbitrary constants. Balancing the highest-order nonlinear term and the highest-order linear partial derivative term in eq. (14) yields the value of $m = 2$. The solution of eq. (14) takes the form

$$\phi(\theta) = a_0 + a_1 \operatorname{sech}(\theta) + b_1 \tanh(\theta) + a_2 \operatorname{sech}^2(\theta) + b_2 \operatorname{sech}(\theta)\tanh(\theta). \quad (16)$$

Substituting from eq. (16) into eq. (14) and put the coefficients of $\operatorname{sech}(\theta)$, $\tanh(\theta)$, $\operatorname{sech}(\theta)\tanh(\theta)$, $\operatorname{sech}^2(\theta)$, $\tanh^2(\theta)$, then setting coefficients equal to zero, we obtain a system of algebraic equations. By solving this system, the parameters $k, \ell, \omega, a_0, a_1, a_2, b_1, b_2, A, B, C$ can be determined as

$$a_0 = -\frac{4(2k^4B + C\ell^2)}{3Ak^2}, \quad a_1 = b_1 = b_2 = 0,$$
$$a_2 = \frac{12k^2B}{A}, \quad \omega = \frac{C\ell^2 - 4Bk^4}{3k}, \quad (17)$$

$$a_0 = -\frac{2(k^4B + 2C\ell^2)}{3Ak^2}, \quad a_1 = b_1 = 0,$$
$$a_2 = \frac{6k^2B}{A}, \quad b_2 = \pm\frac{6ik^2B}{A}, \quad \omega = \frac{C\ell^2 - Bk^4}{3k}. \quad (18)$$

Substituting from eqs (17) and (18) into (16), the electrostatic potential of eq. (14) can be obtained as a dust-acoustic solitary wave solutions as

$$\phi_1(\eta,\xi,\tau) = \frac{4V^2}{3k^2(3s - 2c_2V^3)}(-2k^4V^2 - \ell^2 + 9k^4V^2 \operatorname{sech}(k\xi + \ell\eta + \omega\tau)), \quad (19)$$

$$\phi_2(\eta,\xi,\tau) = \frac{2V^2}{3k^2(3s - 2c_2V^3)}(-k^4V^2 - 2\ell^2 + 9k^4V^2 \operatorname{sech}(k\xi + \ell\eta + \omega\tau)(\operatorname{sech}(k\xi + \ell\eta + \omega\tau) + i\tanh(k\xi + \ell\eta + \omega\tau))) . \quad (20)$$

Because the electric field is irrotational, it is possible to express the electric field as the gradient of a scalar function, $\phi$, called the electrostatic potential (also known as the voltage). An electric field, $\vec{E}$, points from regions of high electric potential to regions of low electric potential, can be expressed mathematically as

$k = 1.6$, $\omega = 0.4$, $\ell = 0.3$, $\tau = 5$, $\zeta_0 = 0.4$ in the interval $[-4, 4]$ and $[-3, 3]$. According to the conditions of stability, the electrostatic potential is stable in the above interval.

*Case* II: *Sinh–cosh method:* Assume that the nonlinear two-dimensional KP equation (14) has the following formal solution:

$$\phi(\theta) = a_0 + \sum_{i=1}^{n} \sinh^{i-1}\theta(a_i \sinh\theta + b_i \cosh\theta), \quad (23)$$

where $a_0, a_1, ..., a_n, b_1, ..., b_n$ are arbitrary constants. Balancing the highest-order nonlinear term and the highest-order linear partial derivative term in eq. (14) yields the value of $m = 2$. The solution of eq. (14) is in the form

$$\phi(\theta) = a_0 + a_1 \sinh(\theta) + b_1 \cosh(\theta) + a_2 \sinh^2(\theta) + b_2 \sinh(\theta)\cosh(\theta). \quad (24)$$

Substituting from eq. (25) into eq. (14) and put the coefficients of $\sinh(\theta)$, $\cosh(\theta)$, $\sinh(\theta)\cosh(\theta)$, $\sinh^2(\theta)$, $\cosh^2(\theta)$, ..., then setting coefficients equal to zero, we obtain a system of algebraic equations. By solving this system, the parameters $k, \ell, \omega, a_0, a_1, a_2, b_1, b_2$ can be determined as

$$a_0 = -\frac{4k^4B + C\ell^2 + k\omega}{Ak^2}, \quad a_1 = b_1 = b_2 = 0,$$
$$a_2 = \frac{k^2B}{2A}, \quad (25)$$

$$a_0 = \pm\frac{k\sqrt{6A} - k^4B - C\ell^2 - k\omega}{Ak^2}, \quad b_1 = b_2 = 0,$$
$$a_1 = \mp\sqrt{\frac{2}{Ak^2} + \frac{Bk\sqrt{6}}{A\sqrt{A}}}, \quad a_2 = \mp\frac{1}{k\sqrt{6A}}, \quad (26)$$

$$a_0 = \frac{4Bk^2\sqrt{2}}{A}, \quad a_1 = b_1 = 0, \quad a_2 = \frac{8k^2B}{A},$$
$$b_2 = \pm\frac{8ik^2B}{A}, \quad c = \frac{k\omega - 4Bk^4\sqrt{2}}{\ell^2}. \quad (27)$$

$$\vec{E} = -\nabla\phi = -\frac{\partial\phi}{\partial\xi}\hat{e}_\xi - \frac{\partial\phi}{\partial\eta}\hat{e}_\eta$$

$$\vec{E}_1 = \frac{24k^2V^4}{2c_2V^3 - 3s}\operatorname{sech}^2(k\xi + \ell\eta + \omega\tau)\tanh(k\xi + \ell\eta + \omega\tau)(k\hat{e}_\xi + \ell\hat{e}_\eta), \quad (21)$$

$$\vec{E}_2 = \frac{6k^2V^4\cosh\left(\frac{1}{2}(k\xi + \ell\eta + \omega\tau)\right) + i\sinh\left(\frac{1}{2}(k\xi + \ell\eta + \omega\tau)\right)}{(3s - 2c_2V^3)\left(i\cosh\left(\frac{1}{2}(k\xi + \ell\eta + \omega\tau)\right) + \sinh\left(\frac{1}{2}(k\xi + \ell\eta + \omega\tau)\right)\right)^3}(k\hat{e}_\xi + \ell\hat{e}_\eta). \quad (22)$$

Dark solitary wave solution and contour plot for the electrostatic potential (19) and the electric field (21) are shown in figures 1a–1c when $s = 1$, $V = 2$, $c_2 = 0.7$,

Substituting eqs (26)–(28) into (25), the electrostatic potential of eq. (14) can be obtained as a dust-acoustic solitary wave solution as

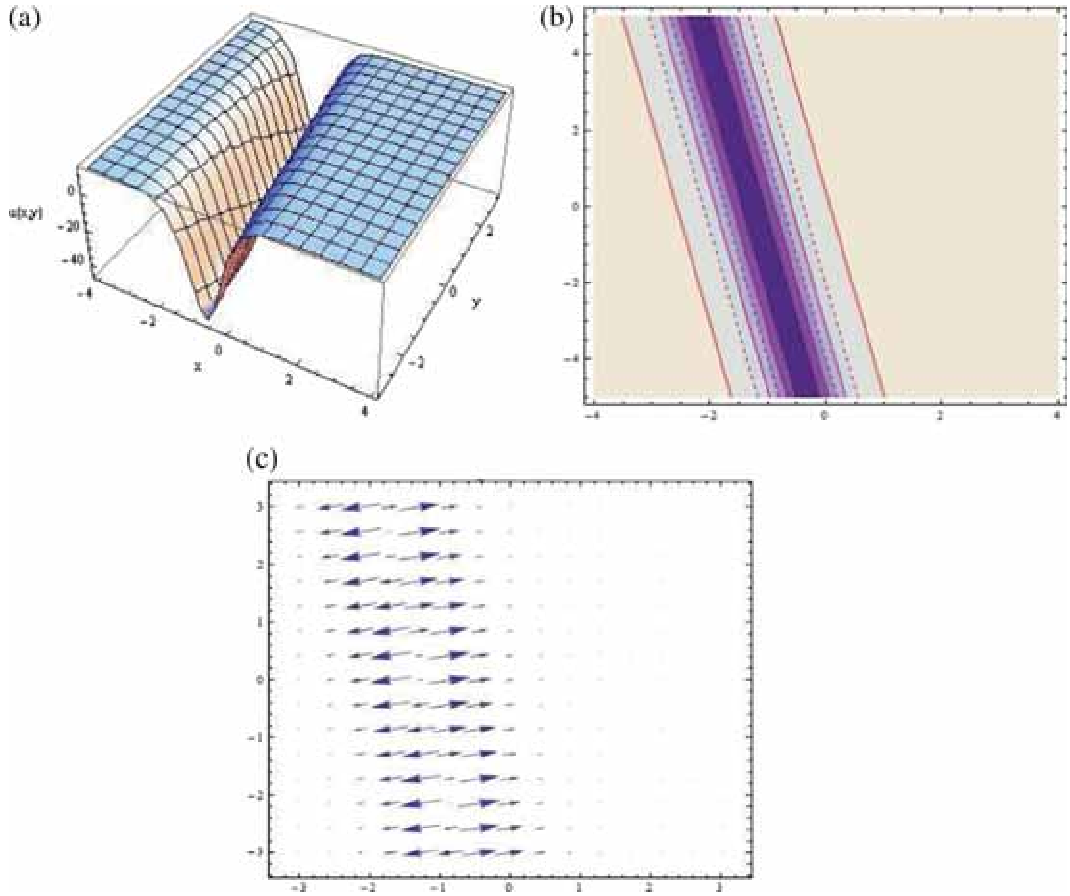


**Figure 1.** Electrostatic potential solution (19) with different shapes are plotted: (**a**) Dark solitary waves, (**b**) contour plot and (**c**) electric fields (21) in different directions are plotted in interval [3, −3].

$$\phi_1(\eta,\xi,\tau) = \frac{V(-2(4V^3k^4 + V\ell^2 + 2k\omega) + V^3k^4\sinh^2(k\xi + \ell\eta + \omega\tau))}{2k^2(3s - 2c_2V^3)}, \tag{28}$$

$$\phi_2(\eta,\xi,\tau) = \frac{4V^4k^2(\sqrt{2} - 1 + \cosh(2(k\xi + \ell\eta + \omega\tau)) + i\sinh(2(k\xi + \ell\eta + \omega\tau)))}{3s - 2c_2V^3}, \tag{29}$$

$$\phi_3(\eta,\xi,\tau) = \frac{V}{6k^2(2c_2V^3 - 3s)}\left(6k^4V^3 - 13k\sqrt{\frac{9s - 6c_2V^3}{V}} + 12V\ell^2 + 12k\omega + k\sqrt{\frac{3s-2c_2V^3}{V}}\right.$$
$$\left.\left(\sqrt{3}\cosh(2(k\xi+\ell\eta+\omega\tau)) - 6k\sqrt{\frac{4}{k^2} + \frac{2kV^3\sqrt{3V}}{\sqrt{3s-2c_2V^3}}}\sinh(k\xi + \ell\eta + \omega\tau)\right)\right). \tag{30}$$

An electric field, $\vec{E}$, points from regions of high electric potential to regions of low electric potential, is expressed mathematically as

$$\vec{E}_1 = \frac{k^2V^4}{6s - 4c_2V^3}\sinh(2(k\xi + \ell\eta + \omega\tau)) \times (k\hat{e}_\xi + \ell\hat{e}_\eta). \tag{31}$$

$$\vec{E}_2 = \frac{8k^2V^4}{3s - 2c_2V^3}\ (\sinh(2(k\xi + \ell\eta + \omega\tau)) - i\cosh(2(k\xi + \ell\eta + \omega\tau)))\,(k\hat{e}_\xi + \ell\hat{e}_\eta). \tag{32}$$

$$\vec{E}_3 = \frac{\sqrt{3sV - 2c_2V^4}}{k(9s - 6c_2V^3)}\cosh(k\xi + \ell\eta + \omega\tau) \times\left(\sqrt{\frac{4}{k^2} + \frac{2kV^3\sqrt{3V}}{\sqrt{3s - 2c_2V^3}}} - 2\sqrt{3}\sinh(k\xi + \ell\eta + \omega\tau)\right)(k\hat{e}_\xi + \ell\hat{e}_\eta). \tag{33}$$

Bistable solitary wave solution and contour plot for the electrostatic potential (28) and the electric field (31)

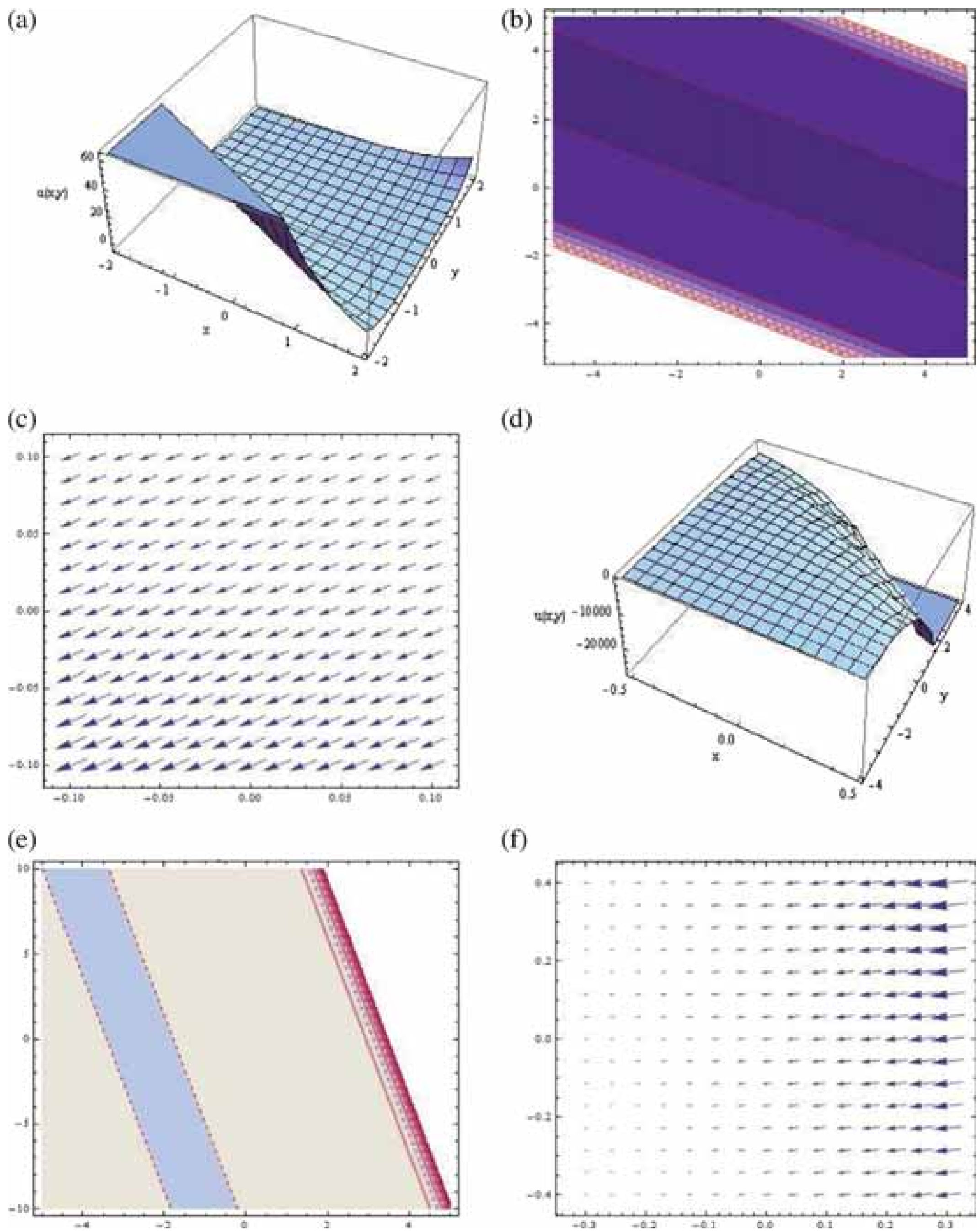


**Figure 2.** Electrostatic potential solution (28) with various shapes are plotted: (**a**) bistable solitary waves, (**b**) contour plot, (**c**) electric field (31) in one direction shapes, (**d**) electrostatic potential (30) represented by the bistable solitary wave, (**e**) contour plot representing electrostatic potential and (**f**) electric field (33) in one direction shapes.

are shown in figures 2a–2c when $s = -1$, $V = 2$, $c_2 = -0.7, k = 0.6$, $\omega = -0.4$, $\ell = 1.3$, $\tau = 3$, $\zeta_0 = 0.4$ in the interval $[-2, 2]$. Figures 2d–2f represent the electrostatic potential (30) and the electric field (33) as bistable solitary wave solution and contour plot when $s = 1$, $V = 0.2$, $c_2 = 0.8$, $k = 1.9$, $\omega = 1.4$, $\ell = 0.3, \tau = 4$ in the intervals $[-0.5, 0.5]$ and $[-4, 4]$. According to the conditions of stability, the electrostatic potential is stable in the above interval.

*Case* III: *Extended direct algebraic method:* Suppose that the nonlinear two-dimensional KP eq. (14) has the following travelling wave solution:

$$\phi(\theta) = \sum_{i=0}^{m} a_i \varphi^i(\theta) \quad \text{and} \quad (\varphi')^2 = \alpha\varphi^2 + \beta\varphi^3 + \gamma\varphi^4, \tag{34}$$

where $\alpha$, $\beta$, $\gamma$ are arbitrary constants. Balancing the nonlinear term and the highest-order derivative in eq.

(14) gives $m = 2$. The solution of eq. (14) is in the form

$$\phi(\theta) = a_0 + a_1\varphi + a_2\varphi^2. \tag{35}$$

By substituting (35) and (36) into eq. (14) yields a set of algebraic equations for $a_0, a_1, a_2, k, \ell, \omega, \alpha, \beta, \gamma$. The solution for the system of equations can be found as

$$a_0 = -\frac{Bk^4\beta^2 + 4C\gamma\ell^2 + 4k\gamma\omega}{4A\gamma k^2},$$

$$a_1 = -\frac{6\beta Bk^2}{A}, \quad a_2 = -\frac{12\gamma Bk^2}{A}, \quad \alpha = \frac{\beta^2}{4\gamma}. \tag{36}$$

Substituting eq. (37) into (36), the electric field potential of eq. (14) can be obtained as an electromagnetic solitary wave solution

$$\phi_1(\xi,\eta,\tau) = \frac{V}{4\gamma k^2(2c_2V^3 - 3s)}\left(-2k^4V^3\beta^2 + 4V\gamma\ell^2 + 8k\gamma\omega + 3k^4V^3\beta^2\epsilon^2\tanh^2\left(\frac{\beta}{4\sqrt{\gamma}}(k\xi + \ell\eta + \omega\tau)\right)\right). \tag{37}$$

$$\phi_2(\xi,\eta,\tau) = \frac{V}{4k^2(3s - 2c_2V^3)}\left(\frac{2k^4V^3\beta^2}{\gamma} - 4V\ell^2 - 8k\omega + \frac{3k^4V^3\beta^2\epsilon^2\sinh^2\left(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau)\right)}{\gamma\left(\delta + \cosh\left(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau)\right)\right)^2}\right). \tag{38}$$

$$\phi_3(\xi,\eta,\tau) = \frac{V}{4\gamma k^2(2c_2V^3 - 3s)}\left(4V\gamma\ell^2 + 8k\gamma\omega + k^4V^3\beta^2\left(\frac{3(1+p^2)\delta^2\epsilon^2}{\left(p + \sinh\left(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau)\right)\right)^2} - 2\right)\right.$$
$$\left. + \frac{3k^4V^3\beta^2\epsilon^2\cosh\left(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau)\right)\left(2\delta\sqrt{1+p^2}\cosh\left(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau)\right)\right)}{\left(p + \sinh\left(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau)\right)\right)^2}\right). \tag{39}$$

On a conductor, a surface charge will experience a force in the presence of an electric field. This force is the average of the discontinuous electric field at the surface charge. This average in terms of the field just outside the surface amounts to electrostatic pressure as: $P = (\varepsilon_0/2)E^2$. This pressure tends to draw the conductor into the field, regardless of the sign of the surface charge.

$$P_1 = \frac{9\varepsilon_0\epsilon^4k^4\beta^6V^8(k^2+\ell^2)}{2\gamma^3(2c_2V^3 - 3s)^2}$$
$$\times \operatorname{csch}^6\left(\frac{\beta}{4\sqrt{\gamma}}(k\xi + \ell\eta + \omega\tau)\right)$$
$$\times \sinh^8\left(\frac{\beta}{4\sqrt{\gamma}}(k\xi + \ell\eta + \omega\tau)\right). \tag{43}$$

An electric field $\vec{E}$ can be deduced mathematically as

$$\vec{E}_1 = \frac{3\epsilon^2k^2\beta^2V^4}{8\gamma^{3/2}(2c_2V^3 - 3s)}\tanh\left(\frac{\beta}{4\sqrt{\gamma}}(k\xi + \ell\eta + \omega\tau)\right)\operatorname{sech}^2\left(\frac{\beta}{4\sqrt{\gamma}}(k\xi + \ell\eta + \omega\tau)\right)(k\hat{e}_\xi + \ell\hat{e}_\eta). \tag{40}$$

$$\vec{E}_2 = \frac{3\epsilon^2k^2\beta^2V^4\sqrt{\alpha}\left(1 + \delta\cosh(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau))\right)\sinh(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau))}{2\gamma(2c_2V^3 - 3s)\left(\delta + \cosh(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau))\right)^3}(k\hat{e}_\xi + \ell\hat{e}_\eta). \tag{41}$$

$$\vec{E}_3 = \frac{3\epsilon^2k^2\beta^2V^4\sqrt{\alpha}(k\hat{e}_\xi + \ell\hat{e}_\eta)}{4\gamma(3s - 2c_2V^3)\left(p + \sinh(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau))\right)^3}\left(\delta\sqrt{1+p^2}\left(3 + \cosh(2\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau))\right.\right.$$
$$\left. - 2p\sinh(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau))\right) + 2\cosh(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau))$$
$$\left. \times \left(1 + (1+p^2)\delta^2 - p\sinh(\sqrt{\alpha}(k\xi + \ell\eta + \omega\tau))\right)\right). \tag{42}$$

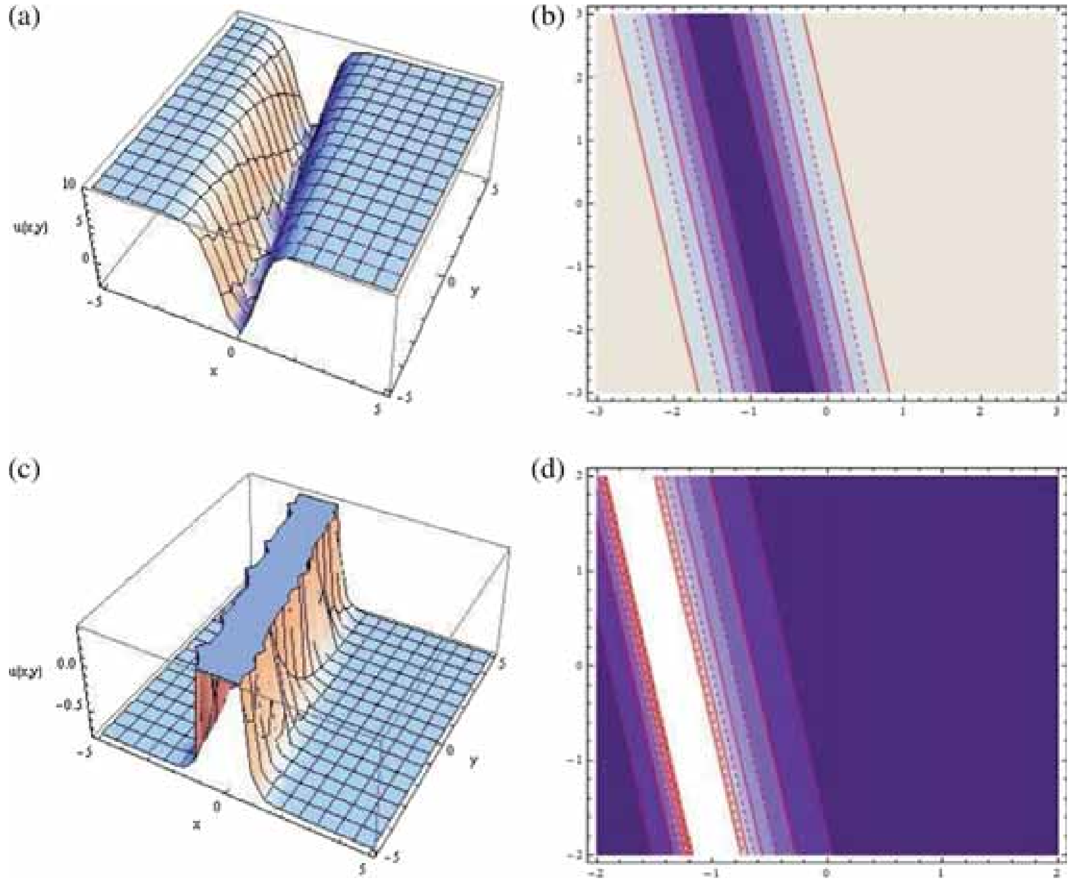


**Figure 3.** Travelling waves solutions (38) with various shapes are plotted: (**a**) dark solitary waves, (**b**) contour plot. Electrostatic potential solutions (39) with various shapes are plotted: (**c**) bright solitary waves and (**d**) contour plot.

$$P_2 = \frac{9\varepsilon_0\alpha\epsilon^4k^4\beta^4V^8(k^2+\ell^2)\left(1+\delta\cosh(\sqrt{\alpha}(k\xi+\ell\eta+\omega\tau))\right)^2\,\sinh^2(\sqrt{\alpha}(k\xi+\ell\eta+\omega\tau))}{4\gamma^2(2c_2V^3-3s)^2\left(\delta+\cosh(\sqrt{\alpha}(k\xi+\ell\eta+\omega\tau))\right)^6}. \qquad (44)$$

$$\begin{aligned} P_3 &= \frac{9\varepsilon_0\alpha\epsilon^4k^4\beta^4V^8(k^2+\ell^2)}{16\gamma^2(3s-2c_2V^3)^2\left(p+\sinh(\sqrt{\alpha}(k\xi+\ell\eta+\omega\tau))\right)^6} \\ &\quad\times\Bigg(\delta\sqrt{1+p^2}\left(3+\cosh(2\sqrt{\alpha}(k\xi+\ell\eta+\omega\tau))\right. \\ &\quad\left.-2p\sinh(\sqrt{\alpha}(k\xi+\ell\eta+\omega\tau))\right) \\ &\quad+2\cosh(\sqrt{\alpha}(k\xi+\ell\eta+\omega\tau))\left(1+(1+p^2)\delta^2\right. \\ &\quad\left.-p\sinh(\sqrt{\alpha}(k\xi+\ell\eta+\omega\tau))\right)\Bigg)^2. \end{aligned} \qquad (45)$$

Bistable dark solitary wave solution and contour plot for the electrostatic potential (38) are shown in figures 3a and 3b when $s=1,\ V=2,\ c_2=0.7,\ k=1.6,\ \omega=-0.4,\ \ell=0.3,\ \tau=4,\ \alpha=4,\ \beta=1.6,\ \gamma=1.9,\ \epsilon=1.6,\ \delta=2$ in the interval $[-5,5]$. Figures 3c and 3d represent the electrostatic potential (39) as bistable bright solitary wave solution.

*Case* IV: The fractional travelling wave solution should be considered as

$$\phi(\theta)=\sum_{i=0}^{n}a_i\varphi^i(\theta)$$

and

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\theta}=\frac{\alpha-\beta\varphi^3(\theta)+\delta\varphi^6(\theta)}{\varphi^2(\theta)}, \qquad (46)$$

where $\alpha,\ \delta,\ \beta$ are arbitrary constants. Balancing the highest-order derivative term with the nonlinear term in eq. (14) gives $m=6$. The fractional solution of eq. (14) is in the form

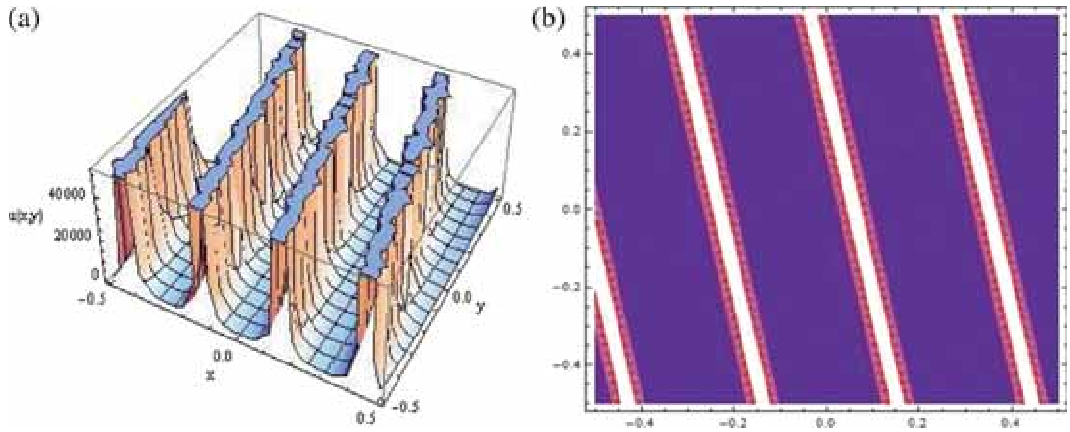


**Figure 4.** Electrostatic potential solution (49) with different shapes are plotted: (**a**) bistable periodic solitary waves and (**b**) contour plot.

$$\phi(\xi) = \sum_{i=0}^{6} a_i \varphi^i(\theta),$$

$$\varphi(\theta) = \left(\frac{\beta + \sqrt{4\alpha\delta - \beta^2}\tanh\left(\frac{3}{2}\sqrt{4\alpha\delta - \beta^2}(\theta + \theta_0)\right)}{2\delta}\right)^{1/3}. \tag{47}$$

Substituting (46) and (47) into eq. (14) yields a set of algebraic equations for $a_0,\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ k,\ \omega,\ \alpha,\ \beta,\ \delta,\ C,\ A,\ B$. The system of equations can be solved as

$$a_0 = \frac{k\omega + C\ell^2 - 27Bk^3\beta^2}{Ak^2}, \quad a_1 = 0, \quad a_2 = 0,$$
$$a_3 = \frac{108Bk\beta\delta}{A}, \quad a_4 = 0, \quad a_5 = 0,$$
$$a_6 = -\frac{108Bk\delta^2}{A}, \quad \alpha = \frac{9Bk^3\beta^2 - k\omega - C\ell^2}{36A\delta k^3}. \tag{48}$$

Substituting eq. (48) into (47), the electrostatic potential of eq. (14) can be obtained as a solitary wave solution

$$\phi(\xi,\eta,\tau) = \frac{V}{k^2(3s - 2c_2V^3)}\left(V\ell^2 + 2k\omega + 27k^3V^3(\beta^2 - 4\alpha\delta) \times \tan^2\left(\frac{3}{2}\sqrt{4\alpha\delta - \beta^2}(k\xi + \ell\eta + \omega\tau + \theta_0)\right)\right). \tag{49}$$

Then the electric field can be obtained as

$$\vec{E} = \frac{81kV^4\left(\sqrt{4\alpha\delta - \beta^2}\right)^3}{2c_2V^3 - 3s}\tan\left(\frac{3}{2}\sqrt{4\alpha\delta - \beta^2}(k\xi + \ell\eta + \omega\tau + \theta_0)\right) \times \sec^2\left(\frac{3}{2}\sqrt{4\alpha\delta - \beta^2}(k\xi + \ell\eta + \omega\tau + \theta_0)\right) \times (k\hat{e}_\xi + \ell\hat{e}_\eta). \tag{50}$$

Comparing the pressure and velocity expressions yields the plane-wave impedance relation, the fluid pressure, modelled by some convenient equation of state

$$P = \frac{6561\varepsilon_0 k^2 V^8(k^2 + \ell^2)\left(4\alpha\delta - \beta^2\right)^3}{(2c_2V^3 - 3s)^2} \times \tan^2\left(\frac{3}{2}\sqrt{4\alpha\delta - \beta^2}(k\xi + \ell\eta + \omega\tau + \theta_0)\right) \times \sec^4\left(\frac{3}{2}\sqrt{4\alpha\delta - \beta^2}(k\xi + \ell\eta + \omega\tau + \theta_0)\right). \tag{51}$$

The electrostatic potentials and pressures defined in the pervious cases are a Hamiltonian system for which the momentum is given by

$$M = \lim_{s\to\infty} \frac{1}{2}\int_0^s \phi_i^2\, \mathrm{d}\theta, \tag{52}$$

where $i = 1, 2, 3$. The sufficient condition for the stability of the dust acoustic solitary wave solutions is

$$\frac{\partial M}{\partial \omega} > 0. \tag{53}$$

Bistable periodic solitary wave solution and contour plot for the electrostatic potential (49) are represented in figures 4a and 4b when $s = 1,\ V = 2,\ c_2 = 0.7,\ k = 1.6,\ \omega = 0.4,\ \ell = 0.3, \tau = 4,\ \alpha = 3,\ \beta = 1.6,\ \epsilon = 0.6,\ \delta = 1.9$ in the interval $[-5, 5]$. Figures 5a and 5b represent the numerical solution of eq. (14) in the interval $[-10, 10]$ and $[-5, 5]$. Electrostatic potential (pulse) variation for the same values are represented as solitary wave solutions in a plane (figures 5c–5i).

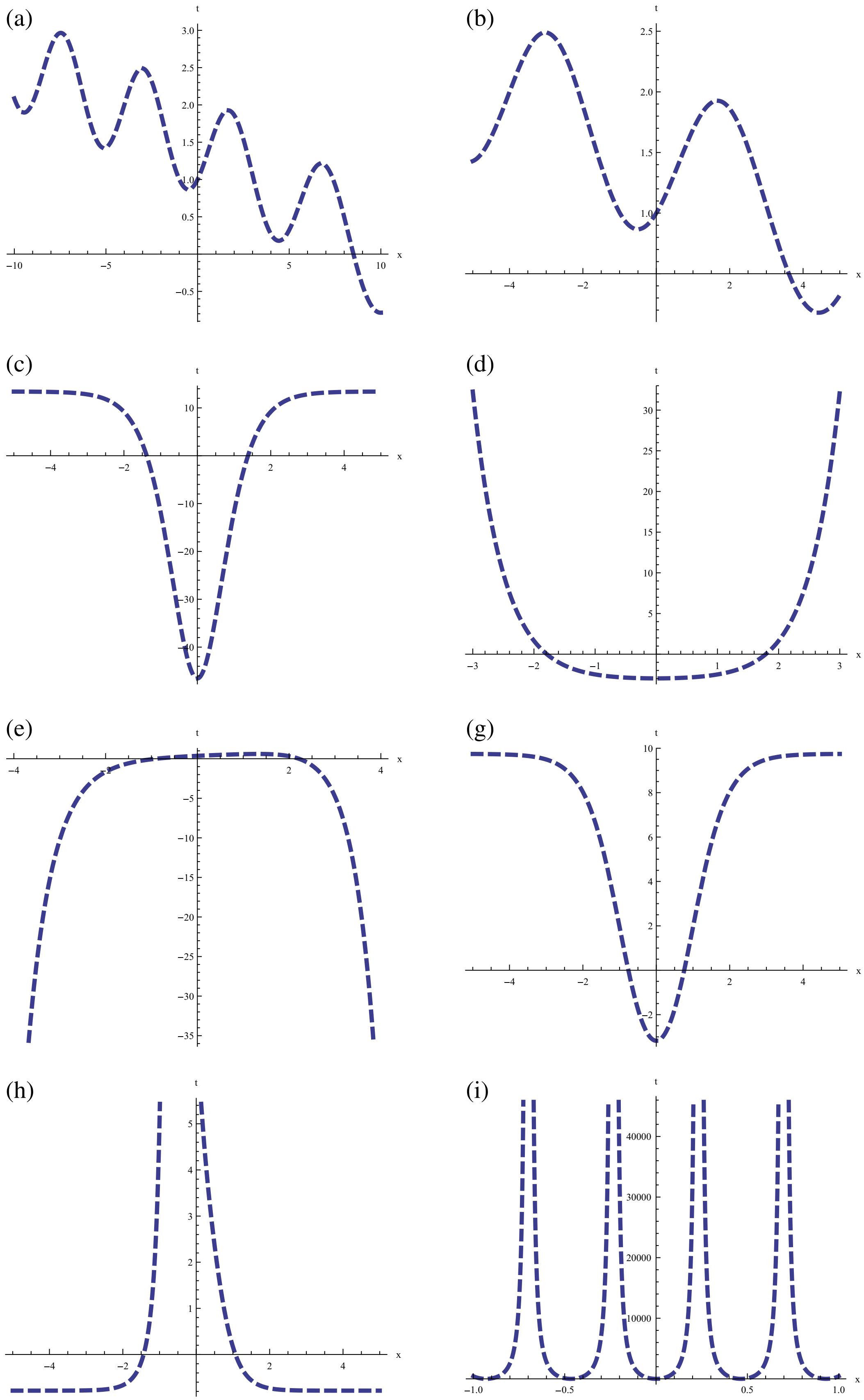


**Figure 5.** (**a–b**) Numerical solution of eq. (14) in interval [−10, 10] and [−5, 5], (**c–d**) one-dimensional soliton solutions (19) and (28) in the interval [−5, 5] and [−3, 3], (**e–g**) one-dimensional soliton solutions (30) and (38) in the interval [−4, 4] and (**h–i**) one-dimensional soliton solutions (39) and (49) in the interval [−4, 4] and [−1, 1].

## 4. Conclusion

An unmagnetized collisionless electron–ion plasma is considered, featuring a superthermal (non-Maxwellian) electron distribution, which is modelled by a (kappa) distribution function. In plasmas with higher proportions of superthermal electrons (that is, plasmas with a lower $K$ value), the nonlinearity and width are affected, producing solitons with decreased amplitude and width with more localized electric fields. Propagation of nonlinear waves in dusty plasmas with variable dust charge and two-temperature ions is analysed. We discussed the problem formulations of two-dimensional DASWs in unmagnetized cold plasma. The nonlinear two-dimensional KP equation is derived by using the reductive perturbation theory. We obtained dust-acoustic solitary wave solutions of the two-dimensional nonlinear KP equation by implementing sech–tanh, sinh–cosh, extended direct algebraic and fraction direct algebraic methods. We found the electrostatic potential, electric field and pressure in the form of travelling wave solutions for two-dimensional nonlinear KP equation. The solutions for the KP equation are obtained precisely and efficiency of the method can be demonstrated. The stability analysis for the electric field potentials, electric fields and magnetic fields are discussed with respect to the sufficient condition for soliton stability. As an illustration, we used the readymade package *Mathematica* program 10.1 to solve the original problem. These solutions are in good agreement with the analytical one.

The sech–tanh, sinh–cosh, extended direct algebraic and fraction direct algebraic methods are always valid no matter whether there exist small physical parameters or not. It provides a convenient way to guarantee the convergence of approximation series; and provides great freedom to choose the equation type of linear subproblems and the base functions of the solutions. We obtained dust-acoustic solitary wave solutions of two-dimensional nonlinear KP equation in dusty plasmas and its stability. The dust-acoustic solitary wave solutions represented the electrostatic potential. This potential is a scalar, and hence no direction is involved. Potential being a scalar quantity, it is easier to deal with, than electric field. Hence, when we need to determine an electric field, it is easy to find the potential first and find the field from it. The electrostatic potential appeared in the nonlinear equation of motion (3) and linear Poisson equation (4). Furthermore, we obtained the electrostatic field potential, electric field and fluid pressure in the form of travelling wave solutions for two-dimensional nonlinear KP equation.

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