Pramana – J. Phys. (2017) 89:28
DOI 10.1007/s12043-017-1429-5

Propagation characteristics of ion-acoustic double layer in multicomponent inhomogeneous auroral zone plasma

HARVINDER KAUR[1], TARSEM SINGH GILL[2] and PARVEEN BALA[3,*]

[1]Department of Physics, Khalsa College, Amritsar 143 002, India
[2]Department of Physics, Guru Nanak Dev University, Amritsar 143 005, India
[3]Department of Mathematics, Statistics and Physics, Punjab Agricultural University, Ludhiana 141 004, India
*Corresponding author. E-mail: pravi2506@gmail.com

MS received 17 June 2016; revised 23 February 2017; accepted 6 April 2017; published online 24 July 2017

Abstract. In the present investigation, ion-acoustic double layers in an inhomogeneous plasma consisting of Maxwellian and non-thermal distributions of electrons are studied. We have derived a modified Korteweg–de Vries (mKdV) equation for ion-acoustic double layers propagating in a collisionless inhomogeneous plasma. It is observed that the non-thermal parameters affect the amplitude and width of the double layer which further depend on the density.

Keywords. Ion-acoustic double layers; inhomogeneous plasma; auroral zone.

PACS Nos 52.35.Fp; 52.35.Sb; 94.05.Fg

1. Introduction

The double layers (DL) are localized asymmetric potential structures with a net potential drop. They can accelerate, decelerate or reflect plasma particles. Further, they have a tremendous role to play in space as well as laboratory plasmas. Indeed, the double layers are considered as appropriate candidates to interpret charged particle acceleration to high energies in plasmas, e.g. the auroral region of the ionosphere. Such localized structures were observed for the first time in S3-3 satellite experiment [1]. This was followed by the most striking proof of these structures in Viking satellite [2] observation. The American FAST spacecraft found strong double layers in auroral region [3].

Extensive research work on double layers has been carried out theoretically [4–7] and experimentally [8–15] due to their applications in plasma diagnostics, charged particle acceleration, understanding auroral phenomenon and to explain underlying basic physics of plasmas in laboratory as well as space environment. Theoretical models of double layers are invoked to explain the mechanism of solar flare [16], ion heating in linear turbulent device [17], plasma confined in tandem mirror device [18] and laser-produced plasma [19].

Several theoretical models have been used to explain double layers [20]. The modified Korteweg–de Vries (mKdV) equation with cubic nonlinearity has been used widely to describe the propagation characteristics of electrostatic double layers in different plasma systems [21–33]. It may further be noted that two-electron species are very common in laboratory [34–38] as well as space plasmas [1]. Ion-acoustic double layers (IADL) in the two-electron temperature plasma have been extensively studied both theoretically [25,26,30, 39] as well as experimentally [37,38]. The existence of IADL was confirmed in auroral and magnetosphere plasma, where two-electron temperature exists [1,2]. Goswami and Bujarbaura [40] presented two-electron Boltzmann model for the obliquely propagating DLs in magnetic field. This model was later on applied by Raddu and Chanteur [22]. In most of the above investigations, electron distributions considered were of Maxwellian-type, and such distributions occur in thermal plasmas. However, most of the space plasmas are not in ergodic equilibrium. Further, satellite-based observations confirm that plasmas usually deviated from the Maxwellian particle distribution. Several models of non-Maxwellian particle distributions have been proposed, e.g. non-thermal, kappa, vortex-type and Tsallis distributions. Non-thermal distribution, considered in the present investigation, was proposed by Cairns *et al* [41] to account for the density depletion observed in satellite observations. This model successfully explained the rarefactive solitons observed in Freja satellite observations. Subsequently, this distribution has been applied to a number of plasma systems. Satellite-based measurements on plasma density established the existence of DL in magnetosphere as well as auroral zone plasma [1,2]. However, it is worth mentioning that in general, the plasma distributions near a DL are strongly non-Maxwellian [36,42]. Recently, DLs with non-Maxwellian electrons/particle distributions have been the focus of research. Bandyopadhyay and Das [43] studied the effect of ion temperature and non-thermal electrons on small-amplitude IADLs and their stability. Gill *et al* [44] studied the ion-acoustic solitons and DLs in a collisionless plasma consisting of isothermal positive and negative ions with non-thermal electrons using reductive perturbation method. Existence of solitons and DLs are explored over a wide range of parameter space. It is noticed that beyond the critical values of non-thermal parameters, no DLs exist. Islam *et al* [45] studied small-amplitude IADL for plasma having electron species following vortex-like distribution and non-thermal distribution. Small-amplitude IADL in a plasma with superthermal electrons and thermal positrons were studied by Tribeche and Boubakour [46]. IADL in a magnetized positive–negative ion plasma with non-thermal electrons was investigated by El-Labany *et al* [47]. Their investigation reveals the existence of IADL in negative ion magnetoplasma with non-thermal electrons for (H^+,O_2^-) and (H^+,H^-) plasma. Rios and Galvano [48] have investigated the existence of obliquely propagating IADLs in magnetized two-electron plasma. The hot electron population is modelled via a kappa distribution function. It is found that only subsonic and rarefactive DLs exist for the entire range of parameters.

In auroral zone plasma and magnetosphere region, the measurements show that the plasma density is spatially non-uniform where DLs were observed. It is therefore, desirable to investigate this realistic situation by studying IADLs in an inhomogeneous plasma. The non-uniformity in the plasma plays a crucial role in characterizing the physics of the nonlinear waves [49–55]. Experimental and theoretical studies have been widely performed on ion-acoustic solitons [52,56–60]. As a consequence of inhomogeneity, the KdV equation describing the soliton behaviour is modified either by varying coefficients or by some additional terms and it is generally treated as modified KdV equation. Watanabe and Yajima [61] have numerically integrated KdV equation with the additional damping term for a varying medium to see the growth and disintegration or damping of a soliton due to the inhomogeneity of the system. Yadav *et al* [62] have studied IADL in inhomogeneous plasma in two-temperature electrons. However, they considered both electrons hot as well as cold, obeying Maxwellian distribution. In auroral zone plasma, the particle distribution is non-thermal and plasma is treated as non-uniform. So, in the present model, we have opted for a more suitable situation for investigating the double layers by considering inhomogeneous plasma consisting of cold electrons, where hot electrons obeying non-thermal distribution [42] and stationary ions. KdV equation is derived, with additional terms, that governs the propagation of double layers in the presence of density gradient in a plasma. The modified KdV equation is also solved for constant density gradient to obtain characteristics of the double layers. The organization of the paper is as follows: Section 2 deals with the derivation of modified KdV equation using reductive perturbation technique. In §3, the double-layer solution is studied and the last section is devoted to discussion and conclusion of the present investigation.

2. Modified KdV equation using reductive perturbation technique

Let us consider a collisionless unmagnetized inhomogeneous plasma consisting of cold electrons obeying Maxwellian distribution, hot electrons obeying a non-Maxwellian distribution, i.e. non-thermal and stationary ions. The dynamics of ion-acoustic waves is governed by

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\partial \phi}{\partial x} - \nu_1 \frac{v}{\eta} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_{eh} + n_{ec} - n \tag{3}$$

$$n_{ec} = n_{0c}\exp(\sigma_c \phi), \tag{4}$$

where $\sigma_c = T_h/T_c$.

The non-thermal distribution for hot electrons is given by Carins *et al* [41]

$$f_{0h}(v) = \frac{n_{0h}}{\sqrt{2\pi v_{\text{th}}^2}} \frac{1 + (\gamma v^4/v_{\text{th}}^4)}{1+\gamma} \exp\left(-\frac{v^2}{2v_{\text{th}}^2}\right), \tag{5}$$

where $n_{ec}(n_{eh})$ is the cold (hot) electron number density normalized by its equilibrium value n_0, v_{th} is the thermal speed of the hot electrons and γ is a parameter which determines the population of energetic non-thermal electrons. The distribution of electrons in the presence of non-zero potential can be found by replacing v^2/v_{th}^2 by $(v^2/v_{\text{th}}^2) - 2\phi$. Thus, integration over the resulting distribution function gives the following expression for the electron density:

$$n_{eh} = n_{0h}(1 - \beta\phi + \beta\phi^2), \tag{6}$$

where $\beta = 4\gamma/(1+3\gamma)$. In eqs (1)–(6) n and t^{-1} are normalized by a reference value of the unperturbed number density and the corresponding value of the ion-plasma frequency, respectively. The velocity v and ϕ are normalized by $(T_h/m)^{1/2}$ and T_h/e respectively.

The total electron density $n_e = n_{eh} + n_{ec}$ given by eqs (4) and (6) is

$$n_e = n_0\left(1 + \alpha_h\phi + \frac{\Delta}{2}\phi^2 + \frac{\delta}{6}\phi^3 + \cdots\right), \tag{7}$$

where

$$\Delta = n_h + \sigma_c^2 n_c,$$
$$\delta = (3\beta + 1)n_h + \sigma_c^3 nc,$$
$$\mu_e = n_{0h}/n_{0c}$$
$$\alpha_h = T_h/T_{\text{eff}}$$

and

$$T_{\text{eff}} = n_0 T_h T_c/(n_{0h}T_c(1-\beta) + n_{0c}T_h).$$

We have assumed that the plasma is quasineutral in the equilibrium state. The density ratio of the two-electron species μ_e may be changed due to the inhomogeneity of the plasma. In the present analysis, we assume that scale length of the variation of μ_e is much greater than the scale length of n_0. So we assume μ_e as constant, and so we shall discuss only the effect of equilibrium plasma density n_0 on the characteristics of the double layer.

We carry out a reductive perturbation analysis of eqs (1) and (7) to obtain the mKdV equation which governs the behaviour of the one-dimensional small-amplitude double layer in the collisionless plasma. To determine the soliton behaviour, we carry out a perturbation expansion based on the assumption that double layer width is small compared to the scale length of the plasma inhomogeneity. Under such conditions, the soliton retains its identity and further its amplitude, width and speed are slowly varying functions of the position. In this analysis, we use a set of stretched coordinates, which is appropriate for spatially inhomogeneous plasma, along with the zeroth-order fluid velocities. For the inhomogeneous plasma, suitable choices of stretched coordinates are as follows [52,63,64]:

$$\xi = \epsilon\left(\int \frac{\mathrm{d}x'}{\lambda_0(x')} - t\right), \quad \eta = \epsilon^3 x, \tag{8}$$

where ϵ is a small ($0 < \epsilon < 1$) expansion parameter characterizing the strength of inhomogeneity and λ_0 is the phase velocity of the wave. In the uniform plasma, λ_0 is a constant. However, in a non-uniform plasma, λ_0 is a function of the slow variable η. Since the basis for the perturbation expansion is that the scale length is sufficiently large, it follows that ϵ can be taken as the formal expansion parameter. The condition $\epsilon \ll 1$ implies that the plasma dimension must be much larger than the Debye length, which is satisfied in most cases.

The dependent variables are expanded in powers of ϵ for the reductive perturbation as follows:

$$n = n_0 + \epsilon n_1 + \epsilon^2 n_2 + \cdots$$
$$v = v_0 + \epsilon v_1 + \epsilon^2 v_2 + \cdots$$
$$\phi = \phi_0 + \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots$$
$$\nu_1 = \epsilon^3\nu. \tag{9}$$

Since n_0 and λ_0 are to be independent of t, we have

$$\frac{\partial\lambda_0}{\partial\xi} = \frac{\partial n_0}{\partial\xi} = 0. \tag{10}$$

Substituting eqs (7), (8) and (9) in the system of eqs (1)–(3) and equating coefficients of same powers of ϵ, we get the following set of equations:

$$\frac{\partial v_0}{\partial\xi} = 0, \quad \frac{\partial\phi_0}{\partial\xi} = 0, \quad \phi_0 = 0 \tag{11}$$
$$n_1 = \alpha_h n_0\phi_1 \tag{12}$$
$$v_1 = (\lambda_0 - v_0)\alpha_h\phi_1 \tag{13}$$
$$(\lambda_0 - v_0) = \frac{1}{\sqrt{\alpha_h}}. \tag{14}$$

Various equations of higher order of ϵ in the perturbation technique are summarized below.

$$\frac{\partial}{\partial\eta}(n_0 v_0) = \nu \tag{15}$$
$$\frac{\partial}{\partial\eta}\left(\frac{v_0^2}{2} + \phi_0\right) = -\nu\frac{v_0}{n_0}. \tag{16}$$

Using eqs (11), (15) and (16) we get

$$n_0 = \frac{(\nu\eta + A)^2}{B}, \tag{17}$$
$$v_0 = \frac{B}{(\nu\eta + A)}, \tag{18}$$

where A and B are constants of integration to be determined using boundary conditions.

To derive the mKdV equation, various equations of higher order of ϵ in the perturbation technique are summarized below.

$$-\frac{\partial n_2}{\partial\xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial\xi}(n_0 v_2 + n_1 v_1 + n_2 v_0) = 0 \tag{19}$$
$$-n_0\frac{\partial v_2}{\partial\xi} - n_1\frac{\partial v_1}{\partial\xi}$$
$$+\frac{1}{\lambda_0}\left(n_0 v_0\frac{\partial v_2}{\partial\xi} + n_0 v_1\frac{\partial v_1}{\partial\xi} + n_1 v_0\frac{\partial v_1}{\partial\xi}\right)$$

$$+n_0 v_0 \frac{\partial v_0}{\partial \eta} + \frac{n_0}{\lambda_0}\frac{\partial \phi_2}{\partial \xi} + \frac{n_1}{\lambda_0}\frac{\partial \phi_1}{\partial \xi} + n_0 \frac{\partial \phi_0}{\partial \eta} = 0 \quad (20)$$

$$-n_0 \alpha_h \phi_2 - \frac{n_0}{2}\Delta \phi_1^2 + n_2 = 0. \quad (21)$$

With algebraic manipulations of eqs (19)–(21), we have

$$n_0 a \phi_1^2 = 0, \quad (22)$$

where $a = (3\alpha_h^2 - \Delta)/2$. n_0 and ϕ_1 are not zero, and therefore, to satisfy eq. (22), a should be at least of the order of ϵ. So, $n_0 a \phi_1^2$ becomes of the order of ϵ^3, and therefore, it should be included in the next order of Poisson's equation [62].

In the next order equations, using the first- and second-order equations, we get

$$\frac{\partial \phi}{\partial \eta} + \frac{a}{\lambda_0^2}\phi_1 \frac{\partial \phi}{\partial \xi} + \frac{b}{2\lambda_0^2}\phi_1^2 \frac{\partial \phi}{\partial \xi} + \frac{\nu}{2n_0\lambda_0 v_0}(2 + v_0(1 - v_0))\phi_1 + \frac{1}{2n_0\lambda_0^4}\frac{\partial^3 \phi_1}{\partial \xi^3} = 0. \quad (23)$$

Here, $b = (15\alpha_h^3 - \delta)/2$. The fourth term in the mKdV eq. (23) is due to ionization effects. In order to obtain the solitary wave solution, we have simplified the equation to a standard form. For this, we use a new transformation [65]

$$\phi_1 = g(\eta)\Phi_1, \quad (24)$$

where

$$g(\eta) = \exp\left[-\nu \int \frac{2 + v_0(1 - v_0)}{2v_0\lambda_0 v_0} \mathrm{d}\eta\right].$$

Using this transformation in (23), we get the well-known form of mKdV equation and nonlinear coefficient has g which depends upon the density gradient

$$\frac{\partial \Phi_1}{\partial \eta} + a_1(n_0)\Phi_1 \frac{\partial \Phi_1}{\partial \xi} + a_1(n_0)\Phi_1^2 \frac{\partial \Phi}{\partial \xi} + \alpha_1(n_0)\frac{\partial^3 \Phi_1}{\partial \xi^3} = 0, \quad (25)$$

where $a_1(n_0) = ag/\alpha_h^{3/2}\lambda_0^2$, $b_1(n_0) = bg^2/2\alpha_h^{3/2}\lambda_0^2$ and $\alpha_1(n_0) = 1/2n_0\alpha_h^{3/2}\lambda_0^4$. These are functions of slow space variables. When $\beta \to 0$, the mKdV eq. (25) is the same as that obtained by Yadav *et al* [62].

3. Double layer solution

To study the double layer characteristics, we employ a perturbation expansion based on the assumption that the width of the double layer is small in comparison to the scale length of the plasma inhomogeneity. Under this condition, the double layer retains its identity, and its amplitude, width and velocity are slowly varying functions of space. Such type of plasma exists in the auroral and magnetospheric regions, where the scale length of plasma density inhomogeneity, of the order of 1000 km, is much larger compared to the typical double layer thickness of approximately 200 km [1].

The coefficients a_1, b_1 and α_1 in eq. (25) are assumed to be functions of a slow space variable $\chi = \epsilon\eta$ with $\epsilon \ll 1$. Following Johnson [60] and Ko and Kuehl [66,67], we have an asymptotic solution of the form

$$\Phi_1 = \Phi_1^{(0)}(\zeta, \chi) + \epsilon\Phi_1^{(1)}(\zeta, \chi) + \epsilon^2\Phi_1^{(2)}(\zeta, \chi) + \cdots, \quad (26)$$

where ζ a fast variable defined by the transformation

$$\zeta = \xi - \int^{\eta} \frac{\mathrm{d}\eta'}{u(\eta')}, \quad (27)$$

where u the velocity, which may be spatially varying, is to be determined.

Substituting eqs (26) and (27) in eq. (25) and integrating, we have

$$\frac{1}{2}\left(\frac{\mathrm{d}\Phi}{\mathrm{d}\zeta}\right)^2 + V(\Phi) = 0, \quad (28)$$

where

$$V(\Phi) = n_0\lambda_0^2\Phi^2\left(-\lambda_0^2 u^{-1} + \frac{ag}{3}\Phi + \frac{bg^2}{12}\Phi^2\right). \quad (29)$$

We have used $\Phi_1^{(0)} = \Phi$ for convenience and the boundary conditions as $\Phi \to 0$, $\mathrm{d}\Phi/\mathrm{d}\zeta$ and $\mathrm{d}^2\Phi/\mathrm{d}\zeta^2 \to 0$.

For double layer solution, the Sagdeev potential $V(\Phi)$ should satisfy the following conditions:

$$V(\Phi) = 0 \quad \text{at} \quad \Phi = 0, \Phi_m \quad (30)$$

$$V'(\Phi) = 0 \quad \text{at} \quad \Phi = 0, \Phi_m \quad (31)$$

$$V''(\Phi) = 0 \quad \text{at} \quad \Phi = 0, \Phi_m, \quad (32)$$

where Φ_m is the extremum value of Φ.

Using conditions (30) and (31) in eq. (29), we obtain

$$\Phi_m = -\frac{2a}{bg} \quad (33)$$

$$\frac{\lambda_0^2}{u} = -\frac{bg^2\Phi_m^2}{12\alpha_h^{3/2}}. \quad (34)$$

Since λ_0, b, g are o(1) and $\Phi_m \ll 1$, u is very large. Transformation (27) together with (8) implies that the Mach number of the double layer M, is given by

$$\frac{1}{M} = \frac{1}{\lambda_0} - \frac{1}{u}. \quad (35)$$

u is very large and therefore, from eq. (35), the velocity of the double layer in the linear wave frame, u_1, is given by

$$u_1 \simeq \frac{\lambda_0^2}{u}. \tag{36}$$

Therefore, from eqs (24), (34) and (36), we obtain

$$u_1 = -\frac{b\phi_m^2}{12\alpha_h^{3/2}}. \tag{37}$$

It implies that the velocity of the double layer in the linear wave frame u_1 is directly proportional to the square of the amplitude of the double layer and inversely proportional to the temperature of hot electrons.

Substituting eqs (33) and (34) in eq. (29), we obtain

$$V(\Phi) = \frac{bg^2 n_0 \lambda_0^2}{12\alpha_h^{3/2}} \Phi^2 (\Phi_m - \Phi)^2. \tag{38}$$

The boundary conditions (30)–(32) is satisfied only when $b < 0$. $b < 0$ implies that $u_1 > 0$, and therefore the double layer is supersonic.

Substituting the value of $V(\Phi)$ from eq. (38) in eq. (28) and integrating, we obtain

$$\Phi = \frac{\Phi_m}{2}\left[1 - \tanh\left(-\frac{bg^2 n_0 \lambda_0^2}{24\alpha_h^{3/2}}\right)\Phi_m \zeta\right]. \tag{39}$$

The width d of the double layer is approximately given by

$$d \simeq 2\sqrt{-\frac{6b\alpha_h^{3/2}}{a^2 n_0}}. \tag{40}$$

From eqs (24) and (33), the amplitude of the double layer, ϕ_m, is given as

$$\phi_m = -\frac{2a}{b}. \tag{41}$$

4. Discussion and conclusion

As obvious from eq. (41), the maximum amplitude of the double layer is independent of the plasma density. On the contrary, the width is inversely proportional to the square root of plasma density. It implies that the width decreases as double layer moves towards the higher density region. The effect of non-thermal parameter on amplitude and width is shown in figures 1 and 2.

In figure 1, it is noticed that increase in non-thermal parameter leads to decrease in amplitude. In figure 2, we have displayed the width as a function of density for four different values of non-thermal parameters β. Two important observations are noticed from the study. Firstly, the width decreases linearly with density as observed from straight lines curves. Secondly, there is a significant decrease of width with increase in non-thermal parameters.

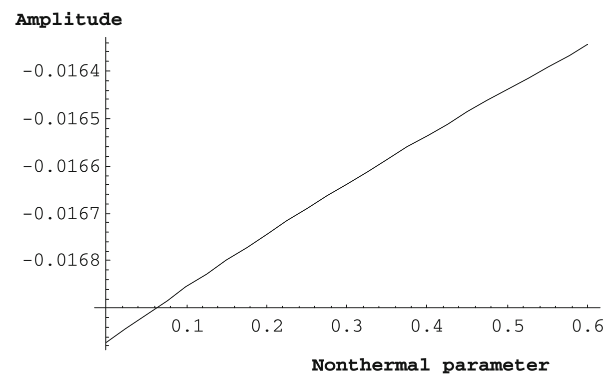

Figure 1. The variation of amplitude with β. Other parameters are $n_c = 0.2$, $n_h = 0.8$, $\sigma_h = 0.008$.

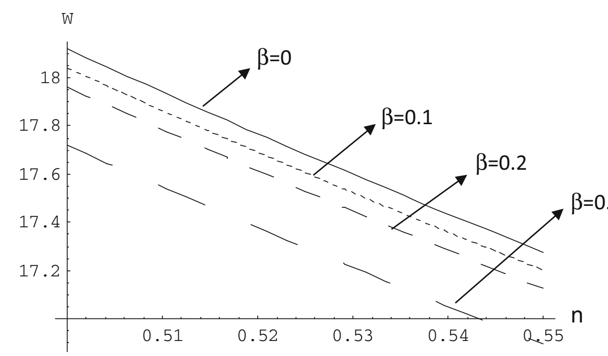

Figure 2. The variation of width with n for different values of β. Other parameters are $n_c = 0.2$, $n_h = 0.8$, $\sigma_h = 0.008$.

In the present study, we have investigated double layers in the inhomogeneous plasma containing non-thermal electrons. We have derived a modified KdV equation and solved it for parameters relevant to auroral zone plasma [68]. Functional dependence of amplitude and width is studied in relation to density and non-thermal parameters. The absolute value of the potential comes out to be nearly 4000 mV, which is one order higher than that observed in satellite measurements.

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