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A novel technique for active vibration control, based on optimal tracking control

BEHROUZ KHEIRI SARABI*, MANU SHARMA and DAMANJEET KAUR

UIET, Panjab University, Chandigarh 160 014, India
*Corresponding author. E-mail: behrooz.kheiri@gmail.com
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Abstract. In the last few decades, researchers have proposed many control techniques to suppress unwanted vibrations in a structure. In this work, a novel and simple technique is proposed for the active vibration control. In this technique, an optimal tracking control is employed to suppress vibrations in a structure by simultaneously tracking zero references for modes of vibration. To illustrate the technique, a two-degrees of freedom spring-mass-damper system is considered as a test system. The mathematical model of the system is derived and then converted into a state-space model. A linear quadratic tracking control law is then used to make the disturbed system track zero references.

Keywords. Active vibration control; linear quadratic tracking; two-degrees of freedom system.

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1. Introduction

Demand for energy is ever increasing and researchers are trying hard to make lightweight automotive structures that have high fuel efficiency. This has led to the development of structures which are very light in weight and tend to vibrate easily. Lightweight structures tend to vibrate easily even under small excitations. Active vibration control (AVC) and passive vibration control (PVC) are well-known strategies to control vibrations. The PVC may not be appropriate where weight is a concern because it increases the overall mass of the structure. In AVC, vibration is suppressed using only an external source of energy and therefore there is no appreciable increase in weight of the system.

In the last few decades, many researches have been done for suggesting new control techniques for AVC. Control techniques which have been used so far are based on feedback and feedforward laws. Feedback control has been used by Chen *et al* [1] to control the vibrations of a composite beam. Constant gain and constant amplitude control algorithms were offered by Bailey and Hubbard [2] to control vibrations of a cantilever beam. Lam *et al* [3] proposed to use negative velocity feedback algorithm to actively control vibrations of a composite plate. In direct velocity feedback technique proposed by Balas [4], the feedback signal from velocity sensor multiplied with some gain is used for computing force to be applied by an actuator. Positive position feedback (PPF) control technique has been suggested by Goh and Caughey [5] as an alternative to direct velocity feedback control technique to control dynamic behaviour of structures. Sim and Lee [6] investigated acceleration feedback control technique with finite actuator dynamics for flexible structures to overcome instability. Stability based on the second direct method of Lyapunov feedback was used by Miller *et al* [7] to control the total mechanical energy of the system. Constant gain velocity feedback (CGVF) technique has been used frequently to control the dynamic response of structures [8]. Balas [9] and Meirovitch *et al* [10] introduced a modal control technique to control vibrations of structures. In this technique, mathematical model of the structure is reduced to the first few frequencies only using modal truncation. Thereafter, Meirovitch and Baruh [11] developed independent modal space control (IMSC), in which a modal filter estimates the modal states needed by the controller. Modified independent modal space control (MIMSC) scheme was used by Baz and Poh [12] to control the dynamic behaviour of systems. Singh and Zhang [13] proposed adaptive output feedback control, to control vibrations in a spacecraft. Optimal control also has been used frequently on structures to control vibrations. Linear quadratic

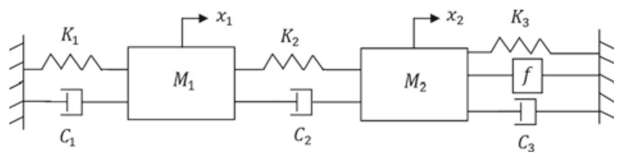

Figure 1. Schematic diagram of two-degrees of freedom spring-mass-damper system.

Gaussian (LQG) technique was employed by Xu and Koko [14] and later on applied on a plate structure by Dong *et al* [15] and Kumar *et al* [16], to control unwanted vibrations of plate structures. Linear quadratic regulator (LQR) has been found as a good technique to suppress the vibrations occurring in structures [17]. Advantages and disadvantages of some popular control techniques used for active vibration control are tabulated in table 1.

Control methods which have been proposed so far have some advantages and disadvantages. One has to choose the control law based on the complexity of structure, loading conditions, end application, computational power available etc. Optimal tracking control for active vibration control has the following advantages:

(1) This technique is robust to spillover problem (if mathematical model is not truncated).
(2) Full state measurement of the system is not required.
(3) It can be applied on both modal displacement and actual vibration displacement.
(4) Desired multiple transient responses can be simultaneously achieved.

In §2 the principle of active vibration control based on optimal tracking control has been demonstrated through an example of two degrees of freedom system, in §3 the results are shown and in §4 conclusions are drawn.

2. Illustration of principle of active vibration control

Consider a two-degree of freedom spring-mass-damper system as shown in figure 1.

Mass M_1 is connected to a boundary through a spring of stiffness K_2 and a damper with damping coefficient C_1. Mass M_2 is connected to a boundary through a spring of stiffness K_3 and a damper with damping coefficient C_3. Actuator f is capable of exerting force f on mass M_2. Two masses are connected to each other by a spring of stiffness K_2 and a damper of damping coefficient C_2. Free body diagrams of the two masses are drawn in figure 2.

Equations of motion are written from free body diagrams as

$$M_1\ddot{x}_1 + K_2(x_1 - x_2) + C_2(\dot{x}_1 - \dot{x}_2) + K_1x_1 + C_1\dot{x}_1 = 0 \quad (1)$$

$$M_2\ddot{x}_2 + C_3\dot{x}_2 + K_3x_2 + f + K_2(x_2 - x_1) + C_2(\dot{x}_2 - \dot{x}_1) = 0. \quad (2)$$

So we have a system of two second-order ordinary differential equations which are coupled with each other. These equations can be converted into a state-space format by taking

$$x_3 = \dot{x}_1 \quad \text{and} \quad x_4 = \dot{x}_2.$$

Now eqs (1) and (2) can be rewritten as

$$M_1\dot{x}_3 + K_2(x_1 - x_2) + C_2(x_3 - x_4) + K_1x_1 + C_1x_3 = 0 \quad (3)$$

$$M_2\dot{x}_2 + C_3x_4 + K_3x_2 + f + K_2(x_2 - x_1) + C_2(x_3 - x_4) = 0. \quad (4)$$

These equations can be expressed as matrix equation of motion as

$$\{\dot{x}\}_{4\times1} = [A]_{4\times4}\{x\}_{4\times1} + [B]_{4\times1}\{f\}_{1\times1}, \quad (5)$$

where

$$\{x\} = \{x_1 \quad x_2 \quad x_3 \quad x_4\}^T$$

$$[A] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-K_1-K_2}{M_1} & \frac{K_2}{M_1} & \frac{-C_1-C_2}{M_1} & \frac{-C_2}{M_1} \\ \frac{-K_2}{M_2} & \frac{-K_2-K_3}{M_2} & \frac{C_2}{M_2} & \frac{-C_2-C_3}{M_2} \end{bmatrix}$$

$$[B] = \{0 \quad 0 \quad 0 \quad -1\}^T.$$

Continuous state-space equations can be converted to discrete form as

$$x(k+1) = Fx(k) + Gu(k), \quad (6)$$

where F and G are discretized forms of A and B matrices, discretized using a sampling time interval. The performance index of the following form is employed so as to derive optimal control gains:

$$J = \frac{1}{2}[Cx(k_f) - z(k_f)]^T S[Cx(k_f) - z(k_f)] + \frac{1}{2}\sum_{k=k_0}^{k_f-1}[Cx(k) - z(k)]^T Q[Cx(k) - z(k)] + u^T(k)Ru(k) \quad (7)$$

with boundary condition $x(k_0) = x_0$. Here z is an n-dimensional reference vector, C, S and Q are $m \times m$ symmetric, positive semidefinite weight matrices, R is

Table 1. Advantages and disadvantages of some popular control techniques used for active vibration control.

Control technique	Advantages	Disadvantages
Velocity feedback [4]	It is unconditionally stable and robust to spillover	Requires either measurement or estimation of the states. It requires truly collocated actuators and sensors
Positive position feedback (PPF) [18]	It is effective in suppressing specific vibration mode by maximizing the damping in the targeted mode without destabilizing other modes	The efficiency of PPF control is highly dependent on the choice of frequency of the PPF compensator. It is a collocated control method. Requires either measurement or estimation of the states
Acceleration feedback [19]	Acceleration is often easier to measure than displacement or velocity. A system with acceleration feedback control is unconditionally stable, regardless of damping and natural frequencies of the structure	Requires either measurement or estimation of the states
Modal space control [20]	Some specific modes can be individually or simultaneously controlled	Prone to spillover effects. Requires either measurement or estimation of the states
Independent modal space control [21]	It is easy to design and can be applied on linear and nonlinear systems, shows stable characteristics, less computational effort is required and is relatively insensitive to locations of the actuators	It needs a modal filter to estimate the modal states needed by the control law. It is not unconditionally stable because of spillover problem. It requires number of actuators to be equal to the number of controlled modes
Modified independent modal space control (MIMSC) [20]	Several modes can be controlled simultaneously	Prone to spillover effects. Requires either measurement or estimation of the states
Adaptive output feedback control [22]	Robust to uncertainty in the system	Stability is not guaranteed, poor control results in large variations in the plant
Linear quadratic regulator (LQR) [16]	It does not require collocated actuator–sensor pairs for stability. Multiple modes can be simultaneously controlled	It requires measurement of state variables
Linear quadratic Gaussian (LQG) [17]	Full state measurement is not required. It is effective for multimodal control. Does not require collocated actuator–sensor pair	Estimation of modes is needed

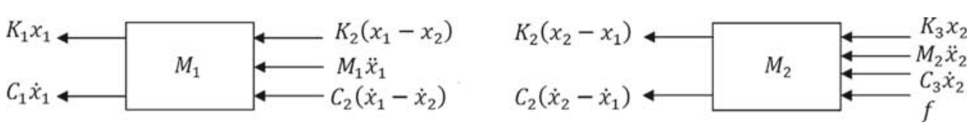

Figure 2. Free body diagrams of the two masses.

an actuator weighing matrix and k_f represents the final location of the vector. Optimal control that optimizes the performance index is given by [23–25]

$$u^*(k) = -L(k)x^*(k) + L_g(k)g(k+1). \quad (8)$$

Quantities with an asterisk represent optimal quantities. $L(k)$ and $L_g(k)$ are control gains and vector $g(k)$ is given as

$$g(k) = F^T\{I - [P^{-1}(k+1) + E]^{-1}E\}g(k+1) + Wz(k) \quad (9)$$

with $g(k_f) = A^T Fz(k_f)$ and $W = C^T Q$. Solution of the following matrix difference Riccati equation gives matrix $P(k)$:

$$P(k) = F^T P(k+1)[I + EP(k+1)]^{-1}F + V \quad (10)$$

with $P(k_f) = C^T SC$, $V = C^T QC$ and $E = GR^{-1} G^T$. Substitution of optimal control law in equation of state gives optimal state x^* as

$$x^*(k+1) = [F - BL(k)]x(k) + BL_g(k)g(k+1). \quad (11)$$

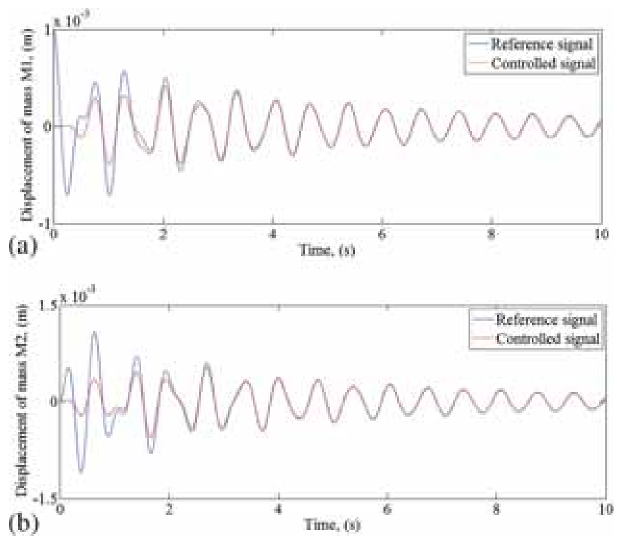

Figure 3. References and controlled time responses of a two-degrees of freedom system.

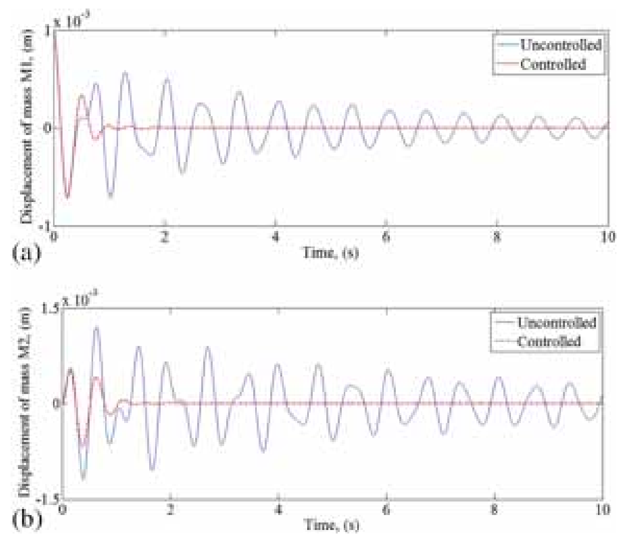

Figure 4. Uncontrolled/controlled time response of a two-degrees of freedom system.

3. Numerical simulations

In this work, a technique based on optimal tracking control is proposed for active vibration control. In this technique, some desired transient signals are simultaneously taken as references which one wishes to achieve. To verify this technique, displacement signals of the test structure have been taken as references and optimal tracking controller is made to track these references. Thereafter in the next step, for active vibration control, the test structure is disturbed with initial displacement and optimal tracking controller is made to track zero references. Two-degrees of freedom system shown in figure 1 with $M_1 = 2M_2 = 10$ kg, $K_1 = 2K_2 = 3K_3 = 1000$ N/m and $C_1 = C_2 = C_3 = 1.1$ Ns/m, is used as a test system. Optimal tracking control is programmed in MATLAB to track the desired references. Corresponding plant matrices for the system shown in figure 1 are computed using sampling time of 0.001 s after as:

$$F = \begin{bmatrix} 0.99 & 2.49\text{e}^{-5} & 9.99\text{e}^{-4} & -8.33\text{e}^{-8} \\ 4.99\text{e}^{-5} & 0.99 & -1.66\text{e}^{-7} & 9.99\text{e}^{-4} \\ -0.15 & 0.05 & 0.99 & -1.74\text{e}^{-4} \\ 0.099 & -0.16 & -3.49\text{e}^{-4} & 0.99 \end{bmatrix}$$

$$G = \begin{bmatrix} -2.70\text{e}^{-11} \\ -4.99\text{e}^{-7} \\ -8.33\text{e}^{-8} \\ -9.99\text{e}^{-4} \end{bmatrix}.$$

Reference signals for the two masses are constructed by obtaining transient responses of the two masses by taking state vector as $x^T = [0.001\,0\,0\,0]$ at time = 0 s. Results shown in figure 3 illustrate that the controller is able to track displacement signals of the test structure.

Optimal tracking problem discussed in this paper is a casual system. The output of this system at any time depends only on the present and the past inputs. The output of this system does not depend on future inputs. Therefore, optimal tracking control discussed in this paper can be practically implemented in AVC applications. To do active vibration suppression based on the proposed method, one can simply use optimal tracking control with reference signal as zero. In figure 4 active vibration control for a two-degrees of freedom system is done using optimal tracking control law. Excellent vibration suppression of both the masses is simultaneously achieved using optimal tracking control.

4. Conclusions

In this paper, a novel technique has been used to control vibrations of a two-degrees of freedom system. First, a mathematical model of the system is derived and then it is converted into a state-space model. Optimal tracking control is then employed to track zero references. Using this technique, one can successfully suppress vibrations at multiple points simultaneously in a structure using a simple procedure.

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